L05-06: Reflection and Transmission from Interfaces

EE3109 Applied Electromagnetics

Lecture Notes

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Jan – Apr 2025

Outline

- Reflection and Transmission
- Normal Incidence
- Oblique Incidence

Review and Preview

- Up till now, we talked about
  - Maxwell's equations, waves equations, potentials
  - Plane waves and its properties (amplitude, phase, directions, polarization, frequency and wavelength)
  - Plane wave propagation in lossless and lossy media

- This class, we will look at
  - How the plane wave transmits and reflects through and interface

Big Picture: Signal Refraction at Boundaries

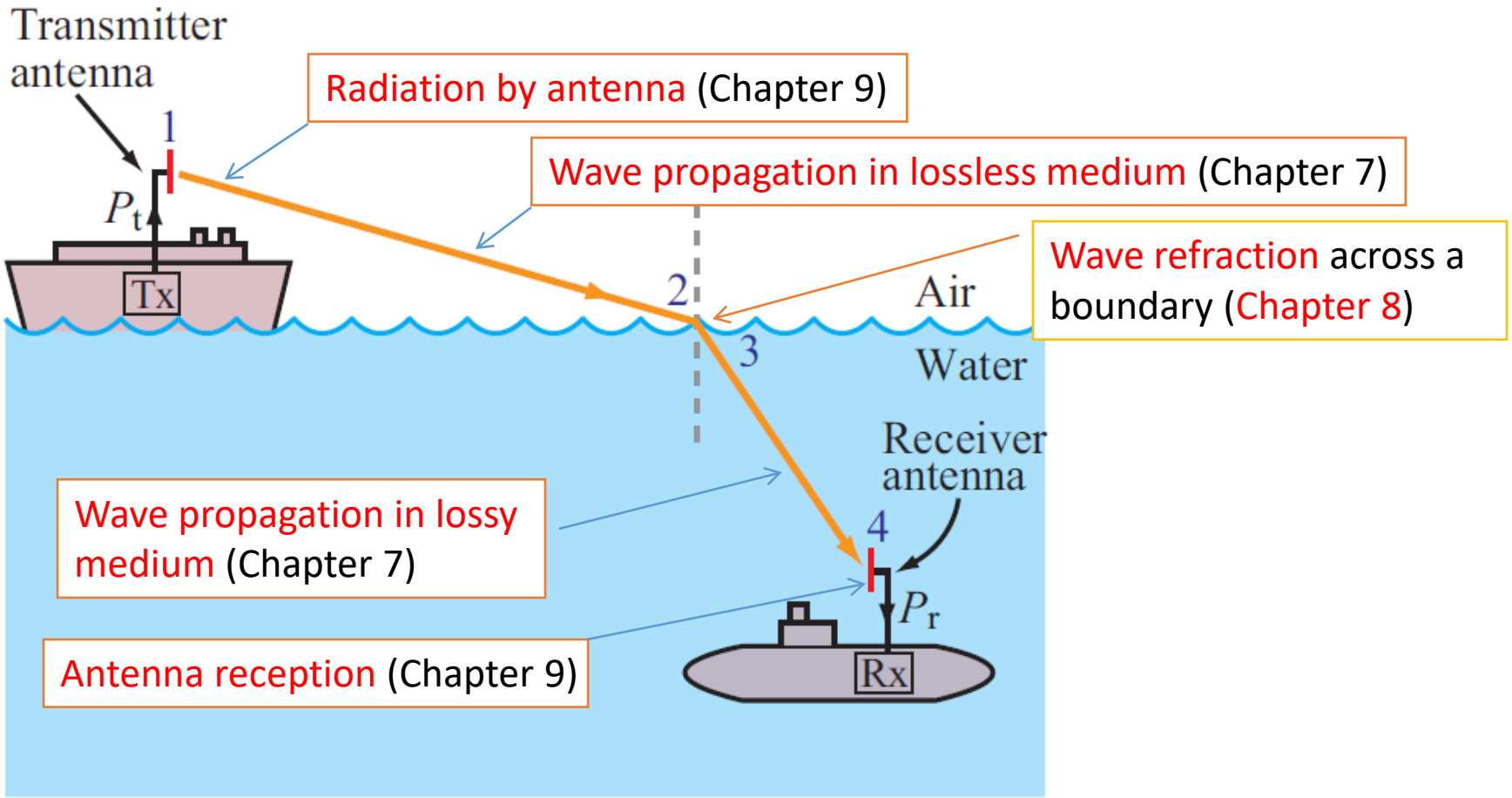

Figure 8-1: Signal path between a shipboard transmitter (Tx) and a submarine receiver (Rx).

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- Reflection and Transmission
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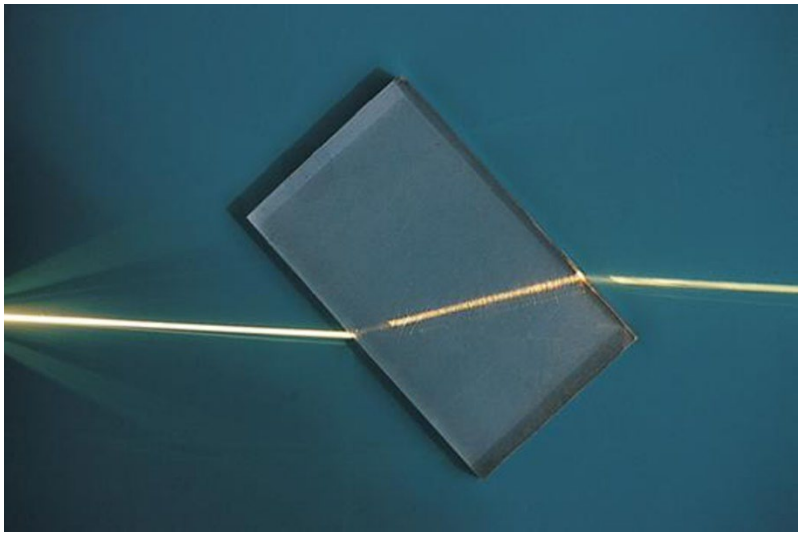
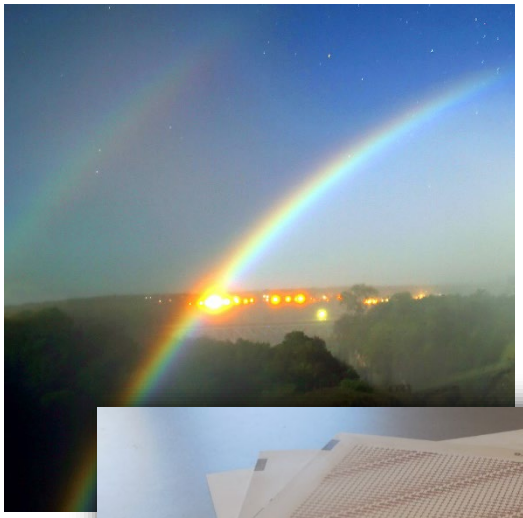
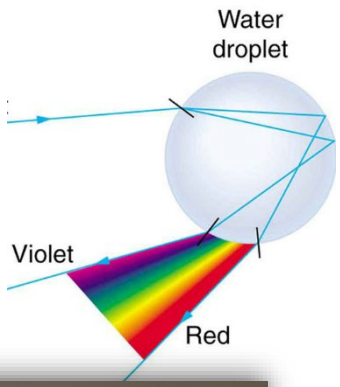
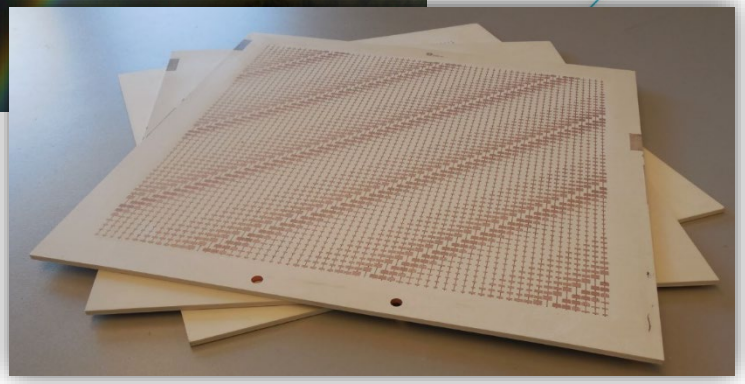
- Oblique Incidence

Examples of EM Wave Reflection and Transmission









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https://upload.wikimedia.org/wikipedia/commons/8/85/Refraction_photo.png
https://upload.wikimedia.org/wikipedia/commons/3/32/Lunar_Rainbow_2_-_Double_Rainbow_Victoria_Falls_-_Calvin_Bradshaw_2.jpg
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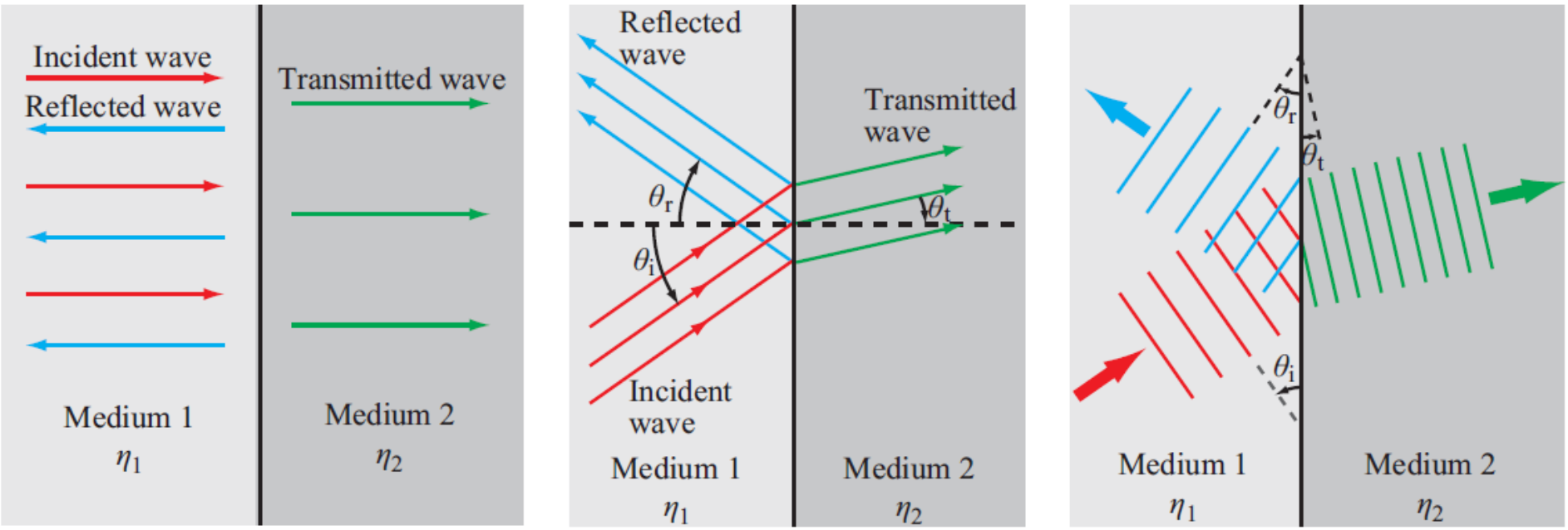
Normal and Oblique Incidence

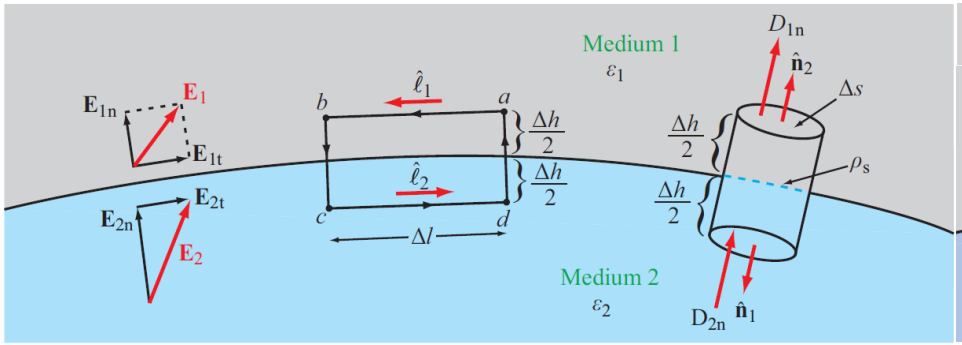


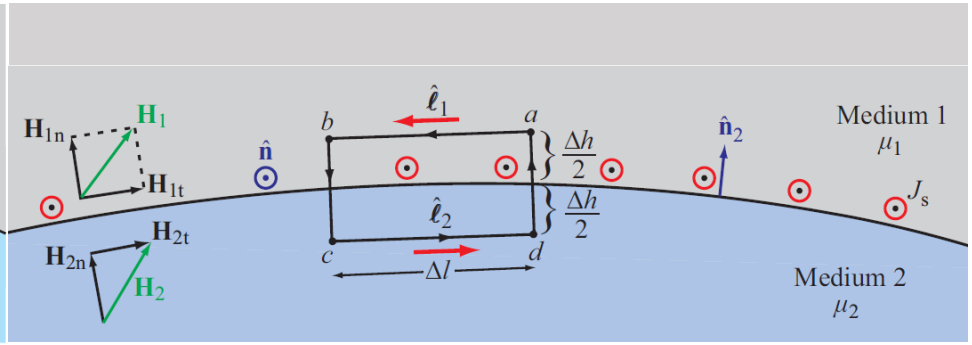
(a) Normal incidence

(b) Ray representation of oblique incidence

(c) Wavefront representation of oblique incidence

Review: Electromagnetic BCs





Tangential Components:

$$\mathbf{E}_{1t} = \mathbf{E}_{2t}\ ;\ \mathbf{H}_{1t} - \mathbf{H}_{2t} = \mathbf{J}_s$$

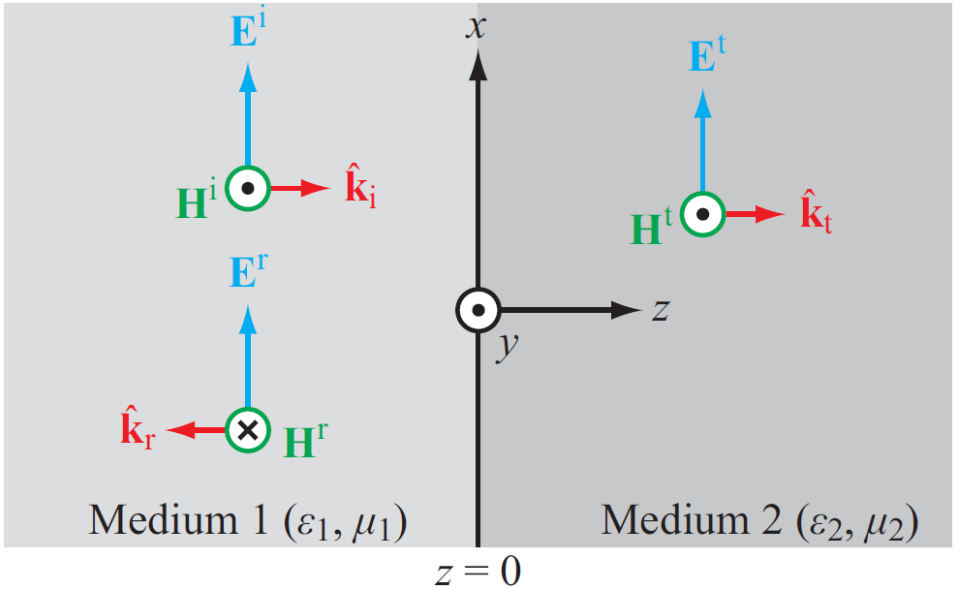
Normal Components:

$$\mathbf{D}_{1n} - D_{2n} = \rho_s\ ;\ \mathbf{B}_{1n} = \mathbf{B}_{2n}$$

Outline

- Reflection and Transmission
- Normal Incidence
- Oblique Incidence

Individual Waves



Note – sign

Incident Wave

$$\mathbf{E}^i(z) = E_0^i\, e^{-jk_1 z}\, \hat{\mathbf{x}}\,,$$

$$\mathbf{H}^i(z) = -\frac{1}{\eta_1}\mathbf{E}^i(z) \times \hat{\mathbf{k}}_i = \frac{E_0^i}{\eta_1}\, e^{-jk_1 z}\, \hat{\mathbf{y}}\,.$$

Note + sign

Note - sign

Reflected Wave

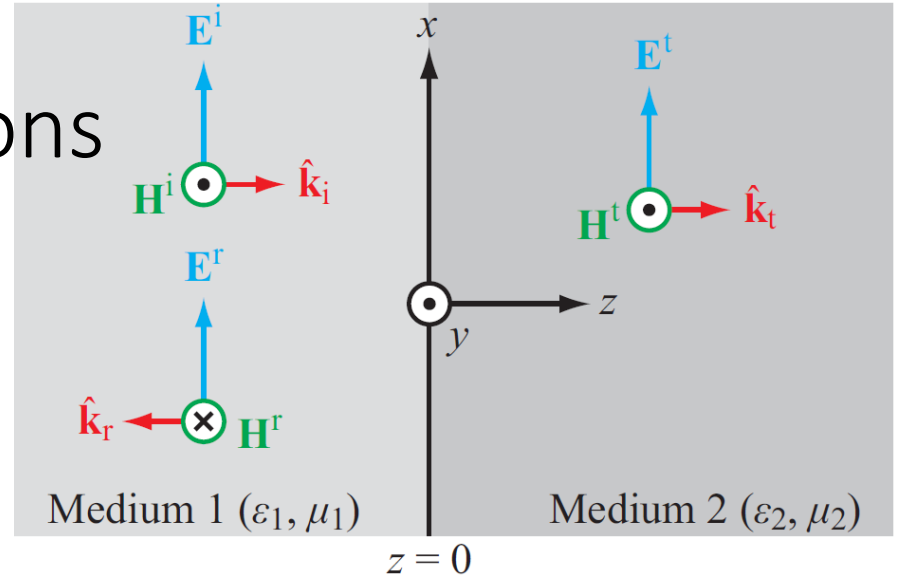
$$\mathbf{E}^r(z) = E_0^r\, e^{+jk_1 z}\, \hat{\mathbf{x}}\,, \mathbf{H}^r(z) = -\frac{1}{\eta_1}\mathbf{E}^r(z) \times \hat{\mathbf{k}}_r = -\frac{E_0^r}{\eta_1}\, e^{+jk_1 z}\, \hat{\mathbf{y}}\,.$$

Note – sign

Transmitted Wave

$$\mathbf{E}^t(z) = E_0^t\, e^{-jk_2 z}\, \hat{\mathbf{x}}\,, \mathbf{H}^t(z) = -\frac{1}{\eta_2}\mathbf{E}^t(z) \times \hat{\mathbf{k}}_t = \frac{E_0^t}{\eta_2}\, e^{-jk_2 z}\, \hat{\mathbf{y}}\,.$$

Add in Boundary Conditions



Total fields

Medium 1

$$\widetilde{\mathbf{E}}_1(z) = \widetilde{\mathbf{E}}^{\mathrm{i}}(z) + \widetilde{\mathbf{E}}^{\mathrm{r}}(z)$$
$$= \hat{\mathbf{x}}(E_0^{\mathrm{i}} e^{-jk_1 z} + E_0^{\mathrm{r}} e^{jk_1 z}), \qquad (8.4a)$$
$$\widetilde{\mathbf{H}}_1(z) = \widetilde{\mathbf{H}}^{\mathrm{i}}(z) + \widetilde{\mathbf{H}}^{\mathrm{r}}(z)$$
$$= \hat{\mathbf{y}} \frac{1}{\eta_1} (E_0^{\mathrm{i}} e^{-jk_1 z} - E_0^{\mathrm{r}} e^{jk_1 z}). \qquad (8.4b)$$

Medium 2

$$\widetilde{\mathbf{E}}_2(z) = \widetilde{\mathbf{E}}^{\mathrm{t}}(z) = \hat{\mathbf{x}} E_0^{\mathrm{t}} e^{-jk_2 z}, \qquad (8.9a)$$
$$\widetilde{\mathbf{H}}_2(z) = \widetilde{\mathbf{H}}^{\mathrm{t}}(z) = \hat{\mathbf{y}} \frac{E_0^{\mathrm{t}}}{\eta_2} e^{-jk_2 z}. \qquad (8.9b)$$

At the boundary $z = 0$:

$$\widetilde{\mathbf{E}}_1(0) = \widetilde{\mathbf{E}}_2(0) \quad \text{or} \quad E_0^{\mathrm{i}} + E_0^{\mathrm{r}} = E_0^{\mathrm{t}},$$
$$\widetilde{\mathbf{H}}_1(0) = \widetilde{\mathbf{H}}_2(0) \quad \text{or} \quad \frac{E_0^{\mathrm{i}}}{\eta_1} - \frac{E_0^{\mathrm{r}}}{\eta_1} = \frac{E_0^{\mathrm{t}}}{\eta_2}.$$

Solution gives:

$$E_0^{\mathrm{r}} = \left(\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\right) E_0^{\mathrm{i}} = \Gamma E_0^{\mathrm{i}},$$
$$E_0^{\mathrm{t}} = \left(\frac{2\eta_2}{\eta_2 + \eta_1}\right) E_0^{\mathrm{i}} = \tau E_0^{\mathrm{i}},$$

$$\Gamma = \frac{E_0^{\mathrm{r}}}{E_0^{\mathrm{i}}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \quad \text{(normal incidence)},$$
$$\tau = \frac{E_0^{\mathrm{t}}}{E_0^{\mathrm{i}}} = \frac{2\eta_2}{\eta_2 + \eta_1} \quad \text{(normal incidence)}.$$

Reflection and Transmission Coefficients

$$\Gamma = \frac{E_0^{\mathrm{r}}}{E_0^{\mathrm{i}}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} \quad \text{(normal incidence)},$$

$$\tau = \frac{E_0^{\mathrm{t}}}{E_0^{\mathrm{i}}} = \frac{2\eta_2}{\eta_2 + \eta_1} \quad \text{(normal incidence)}.$$

$$\tau = 1 + \Gamma \qquad \text{(normal incidence)}.$$

For non-magnetic, low-loss media:

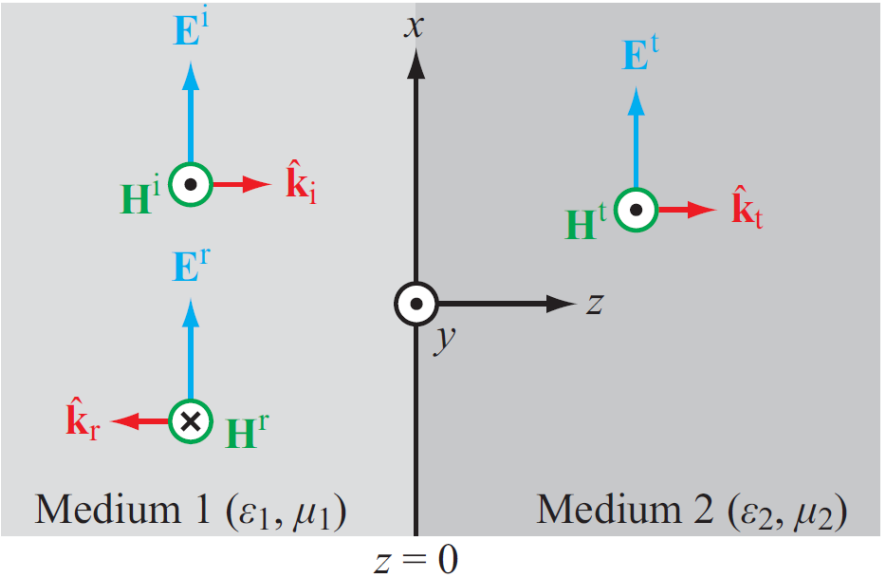
$$\Gamma = \frac{\sqrt{\varepsilon_{r1}} - \sqrt{\varepsilon_{r2}}}{\sqrt{\varepsilon_{r1}} + \sqrt{\varepsilon_{r2}}} = \frac{n_1 - n_2}{n_1 + n_2} \; ; \; \tau = \frac{2n_1}{n_1 + n_2}$$

Where $\boxed{n = \frac{c}{v_p} = \sqrt{\mu_r \varepsilon_r}} \boxed{= \sqrt{\varepsilon_r}}$ is the refractive index.

General When $\mu_r = 1$

Note: $n \neq \eta$!

Summary: EM Fields for Normal Incidence



Incident Wave

$$\mathbf{E}^i = E^i e^{-jk_1 z}\,\hat{\mathbf{x}}$$
$$\mathbf{H}^i = \left(E^i/\eta_1\right) e^{-jk_1 z}\,\hat{\mathbf{y}}$$

Reflected Wave

$$\mathbf{E}^r = \Gamma E^i e^{+jk_1 z}\,\hat{\mathbf{x}}$$
$$\mathbf{H}^r = -\left(\Gamma E^i/\eta_1\right) e^{+jk_1 z}\,\hat{\mathbf{y}}$$

Transmitted Wave

$$\mathbf{E}^t = \tau E^i e^{-jk_2 z}\,\hat{\mathbf{x}}$$
$$\mathbf{H}^t = \left(\tau E^i/\eta_2\right) e^{-jk_2 z}\,\hat{\mathbf{y}}$$

Coefficients

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}\ ; \quad \tau = \frac{2\eta_2}{\eta_2 + \eta_1}\ ; \quad k = \omega\sqrt{\mu\varepsilon}$$

Example 1.

A 10 GHz wave, with amplitude 5 [V/m] travels in air in the +z-direction and forms normal incidence onto a piece of glass ($\mu_r = 1; \varepsilon_r = 2.25; \sigma = 0$). The E-field is linearly polarized in the x-direction. Find the reflection and transmission coefficient; write the electric field and magnetic field in air and in glass.

Example 1 Cont'd

Example 8-3: Normal Incidence on a Metal Surface

A 1-GHz x-polarized plane wave traveling in the $+z$-direction is incident from air upon a copper surface. The air-to-copper interface is at $z = 0$ and copper has $\varepsilon_r = 1$, $\mu_r = 1$, and $\sigma = 5.8 \times 10^7$ S/m. If the amplitude of the electric field of the incident wave is 12 (mV/m), obtain expressions for the instantaneous electric and magnetic fields in the air medium. Assume the metal surface to be several skin depths deep.

Ulaby Example 8-3 (cont.)

Ulaby Example 8-3 (cont.)

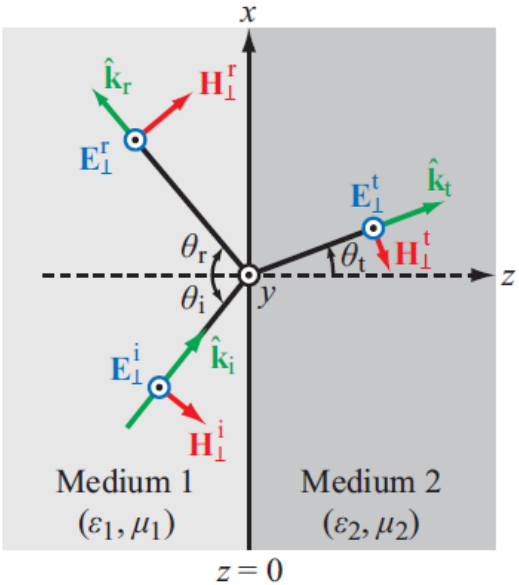
Outline

- Course Matter
- Reflection and Transmission
- Normal Incidence
- Oblique Incidence

Oblique Incidence

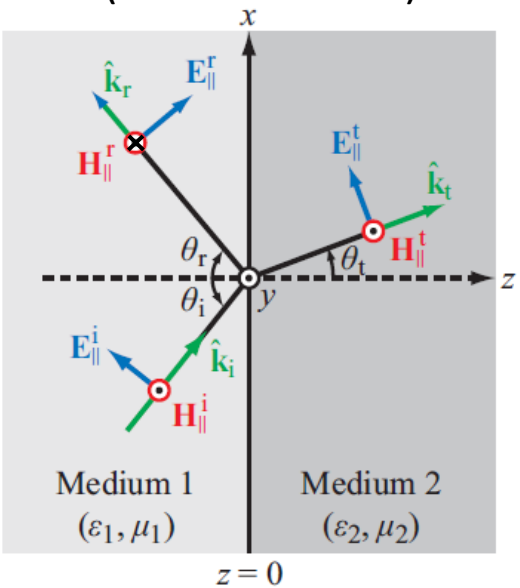
***Plane of incidence** is defined as the plane containing the normal to the boundary and the direction of propagation of the incident wave (x-z plane in the figure).*

A wave of arbitrary polarization may be described as the superposition of two orthogonally polarized waves, one with its electric field parallel to the plane of incidence (***parallel polarization***) and the other with its electric field perpendicular to the plane of incidence (***perpendicular polarization***).



(a) Perpendicular polarization

(TE Polarization)

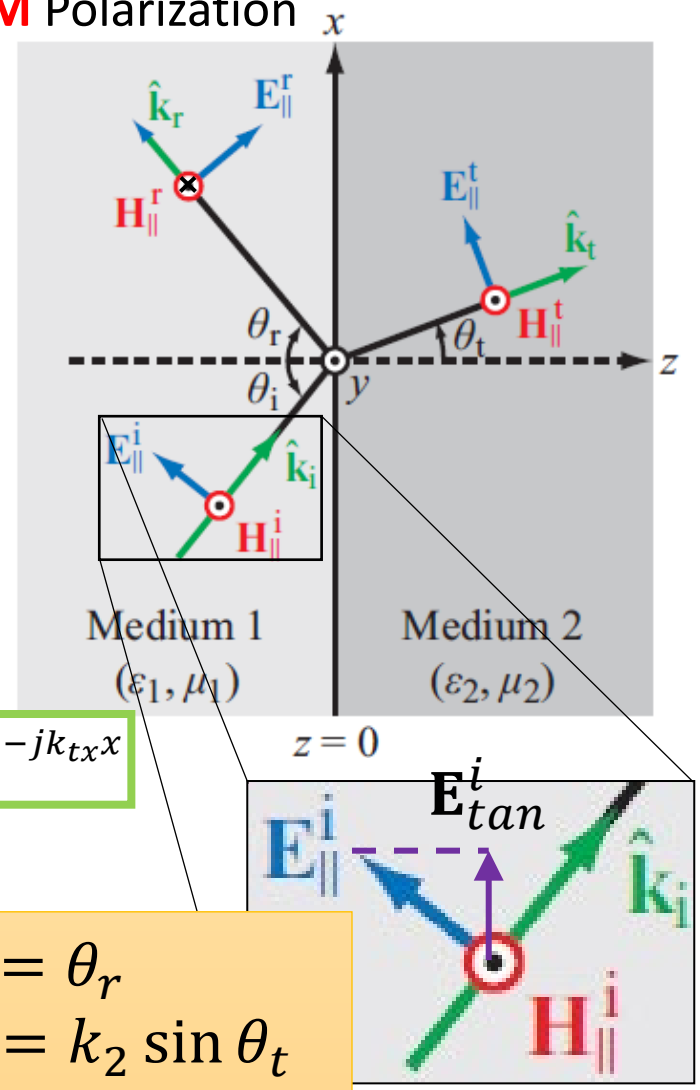


(b) Parallel polarization

(TM Polarization)

Snell's Law

TM Polarization



Consider the tangential E-field TM case:

$$\mathbf{E}_{tan}^{i} = \cos\theta_i \, E_0^{\mathrm{i}} e^{-jk_{ix}x} \, \hat{\mathbf{x}} \qquad k_{ix} = k_1 \sin\theta_i$$

$$\mathbf{E}_{tan}^{r} = \cos\theta_r \, E_0^{\mathrm{r}} e^{-jk_{rx}x} \, \hat{\mathbf{x}} \qquad k_{rx} = k_1 \sin\theta_r$$

$$\mathbf{E}_{tan}^{t} = \cos\theta_t \, E_0^{\mathrm{t}} e^{-jk_{tx}x} \, \hat{\mathbf{x}} \qquad k_{tx} = k_2 \sin\theta_t$$

$$\mathbf{E}_{tan}^{i} + \mathbf{E}_{tan}^{r} = \mathbf{E}_{tan}^{t} \Rightarrow \boxed{C_{1i} e^{-jk_{ix}x} + C_{1r} e^{-jk_{rx}x} = C_2 e^{-jk_{tx}x}}$$

(Must be true for all x)

From $\boxed{}$ and $\boxed{}$:

$$k_{ix} = k_{rx} = k_{tx} \Rightarrow$$

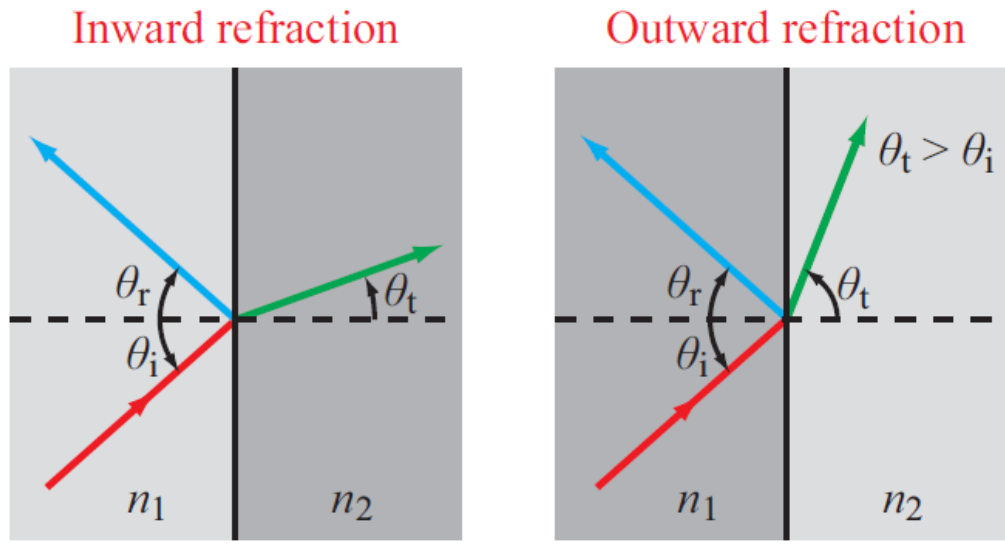
$$\theta_i = \theta_r$$
$$k_1 \sin\theta_i = k_2 \sin\theta_t$$
$$(\text{or } n_1 \sin\theta_i = n_2 \sin\theta_t)$$

(Snell's Law)

Critical Angle

Snell's Law:

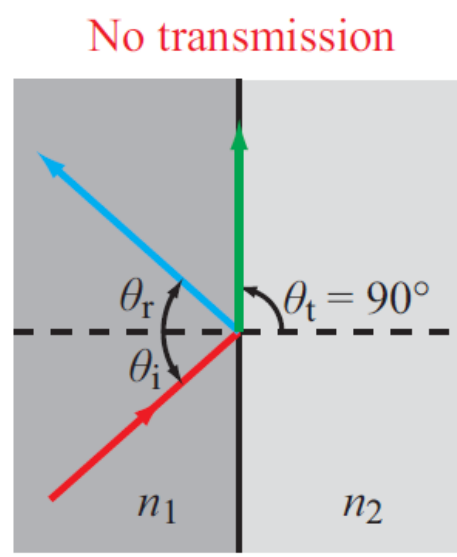
$$\theta_i = \theta_r$$
$$n_1 \sin\theta_i = n_2 \sin\theta_t$$



(a) $n_1 < n_2$ (b) $n_1 > n_2$

When the refraction angle is 90°, the corresponding incidence angle is called the critical angle.

$$\sin\theta_c = \frac{n_2}{n_1}\sin\theta_t\Big|_{\theta_t=\pi/2} = \frac{n_2}{n_1}$$
$$= \sqrt{\frac{\varepsilon_{r_2}}{\varepsilon_{r_1}}} \qquad (\text{for } \mu_1 = \mu_2).$$



(c) $n_1 > n_2$ and $\theta_i = \theta_c$

Example 8-4: Light Beam Passing through a Slab

A dielectric slab with index of refraction n_2 is surrounded by a medium with index of refraction n_1, as shown in Fig. 8-11. If $\theta_i < \theta_c$, show that the emerging beam is parallel to the incident beam.

Oblique Incidence for TE Polarization

Notation:
$k_i = k_r = k_1;\ \ k_t = k_2$
$\theta_i = \theta_r = \theta_1;\ \ \theta_t = \theta_2$

$k_{1x} = k_1 \sin\theta_1$
$k_{2x} = k_2 \sin\theta_2$
$k_{1z} = k_1 \cos\theta_1$
$k_{2z} = k_2 \cos\theta_2$

Plane Waves (Signs read from diagram)

$$\mathbf{E}^i = E_0^i e^{-j(k_{1x}x + k_{1z}z)}\hat{\mathbf{y}} = E_y^i\,\hat{\mathbf{y}} \qquad \mathbf{H}^i = \left(E_y^i/\eta_1\right)\left(-\cos\theta_1\,\hat{\mathbf{x}} + \sin\theta_1\,\hat{\mathbf{z}}\right)$$

$$\mathbf{E}^r = E_0^r e^{-j(k_{1x}x - k_{1z}z)}\,\hat{\mathbf{y}} = E_y^r\,\hat{\mathbf{y}} \qquad \mathbf{H}^r = \left(E_y^r/\eta_1\right)\left(\cos\theta_1\,\hat{\mathbf{x}} + \sin\theta_1\,\hat{\mathbf{z}}\right)$$

$$\mathbf{E}^t = E_0^t e^{-j(k_{2x}x + k_{2z}z)}\,\hat{\mathbf{y}} = E_y^t\,\hat{\mathbf{y}} \qquad \mathbf{H}^t = \left(E_y^t/\eta_2\right)\left(-\cos\theta_2\,\hat{\mathbf{x}} + \sin\theta_2\,\hat{\mathbf{z}}\right)$$

Tangential Components at $z = 0$

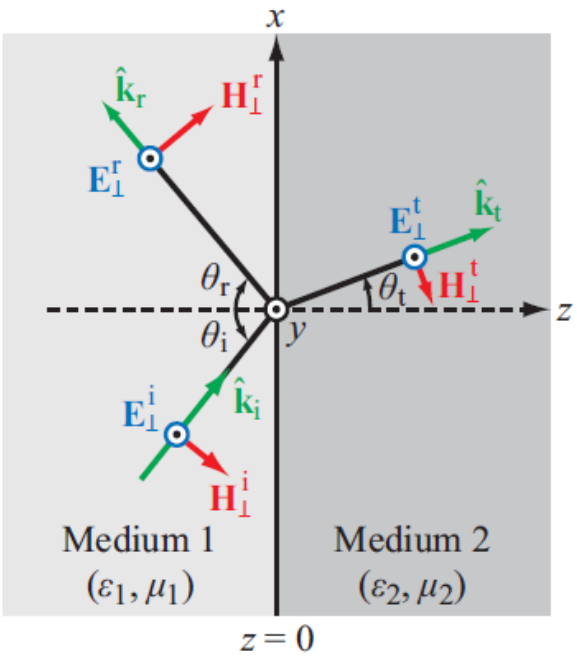
$$\mathbf{E}_{tan}^i = E_0^i e^{-jk_{1x}x}\,\hat{\mathbf{y}} \qquad \mathbf{H}_{tan}^i = -(\cos\theta_1/\eta_1)E_0^i e^{-jk_{1x}x}\,\hat{\mathbf{x}}$$

$$\mathbf{E}_{tan}^r = E_0^r e^{-jk_{1x}x}\,\hat{\mathbf{y}} \qquad \mathbf{H}_{tan}^r = (\cos\theta_1/\eta_1)E_0^r e^{-jk_{1x}x}\,\hat{\mathbf{x}}$$

$$\mathbf{E}_{tan}^t = E_0^t e^{-jk_{2x}x}\,\hat{\mathbf{y}} \qquad \mathbf{H}_{tan}^t = -(\cos\theta_2/\eta_2)E_0^t e^{-jk_{2x}x}\,\hat{\mathbf{x}}$$

Boundary Conditions

$$k_{1x} = k_{2x} = k_x;$$
$$\mathbf{E}_{tan}^i + \mathbf{E}_{tan}^r = \mathbf{E}_{tan}^t;$$
$$\mathbf{H}_{tan}^i + \mathbf{H}_{tan}^r = \mathbf{H}_{tan}^t.$$



(a) Perpendicular polarization
(TE Polarization)

Oblique Incidence for TE Polarization

Substitute E and H fields into B.C.'s to get:

$$E_0^i + E_0^r = E_0^t$$

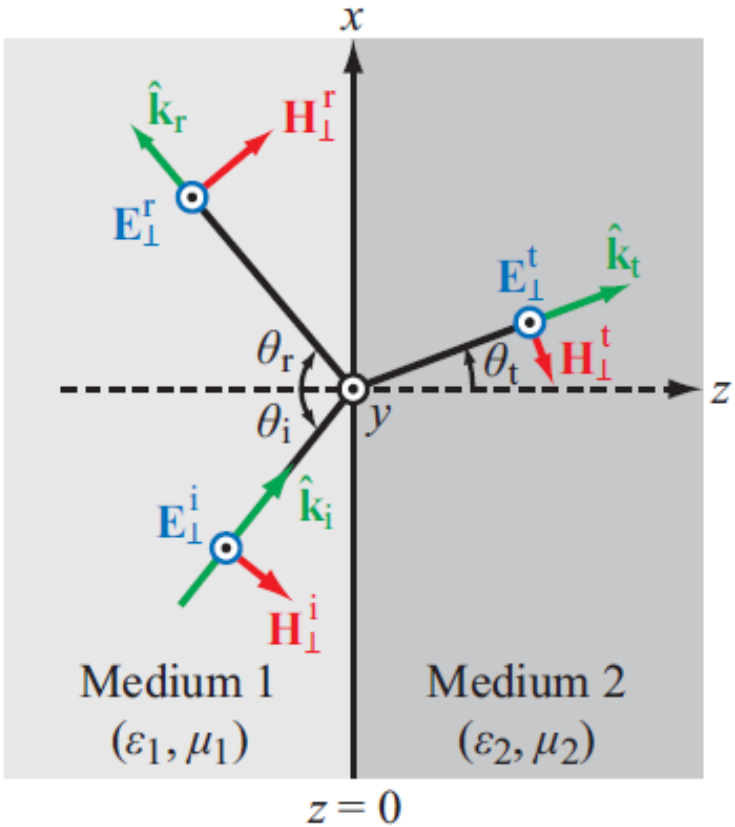
$$\frac{\cos\theta_1}{\eta_1} E_0^i - \frac{\cos\theta_1}{\eta_1} E_0^r = \frac{\cos\theta_2}{\eta_2} E_0^t$$

Rearrange to get:

$$\frac{E_0^r}{E_0^i} \equiv \Gamma_{\mathrm{TE}} = \frac{\eta_2 \cos\theta_1 - \eta_1 \cos\theta_2}{\eta_2 \cos\theta_1 + \eta_1 \cos\theta_2} \; ;$$

$$\frac{E_0^t}{E_0^i} \equiv \tau_{TE} = \frac{2\eta_2 \cos\theta_1}{\eta_2 \cos\theta_1 + \eta_1 \cos\theta_2} \; .$$

(You can do the same for the TM polarization)



(a) Perpendicular polarization

(TE Polarization)

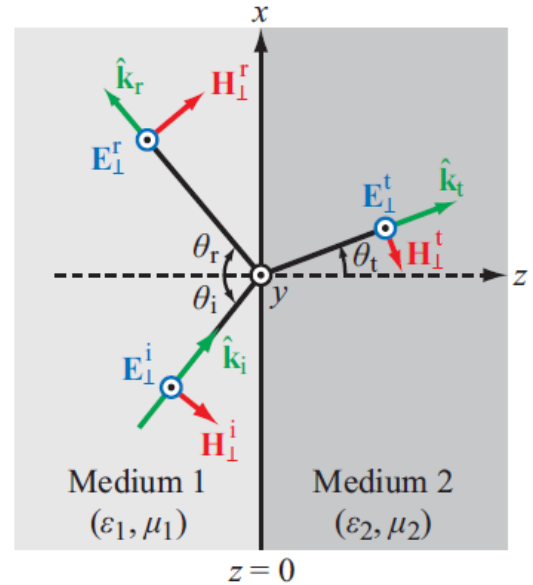
Oblique Incidence

TE Polarization:

$$\Gamma_{\mathrm{TE}} = \frac{\eta_2 \cos\theta_1 - \eta_1 \cos\theta_2}{\eta_2 \cos\theta_1 + \eta_1 \cos\theta_2} \; ;$$

$$\tau_{\mathrm{TE}} = \frac{2\eta_2 \cos\theta_1}{\eta_2 \cos\theta_1 + \eta_1 \cos\theta_2} \; ;$$

$$\tau_{\mathrm{TE}} = 1 + \Gamma_{\mathrm{TE}} \, .$$



(a) Perpendicular polarization

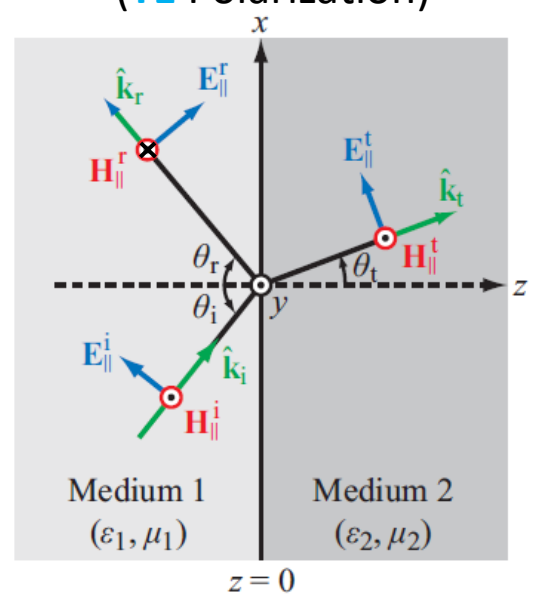
(TE Polarization)

TM Polarization:

$$\Gamma_{\mathrm{TM}} = \frac{\eta_2 \cos\theta_2 - \eta_1 \cos\theta_1}{\eta_2 \cos\theta_2 + \eta_1 \cos\theta_1} \; ;$$

$$\tau_{\mathrm{TM}} = \frac{2\eta_2 \cos\theta_1}{\eta_2 \cos\theta_2 + \eta_1 \cos\theta_1} \; ;$$

$$\tau_{\mathrm{TM}} = (1 + \Gamma_{\mathrm{TM}}) \frac{\cos\theta_1}{\cos\theta_2} \, .$$

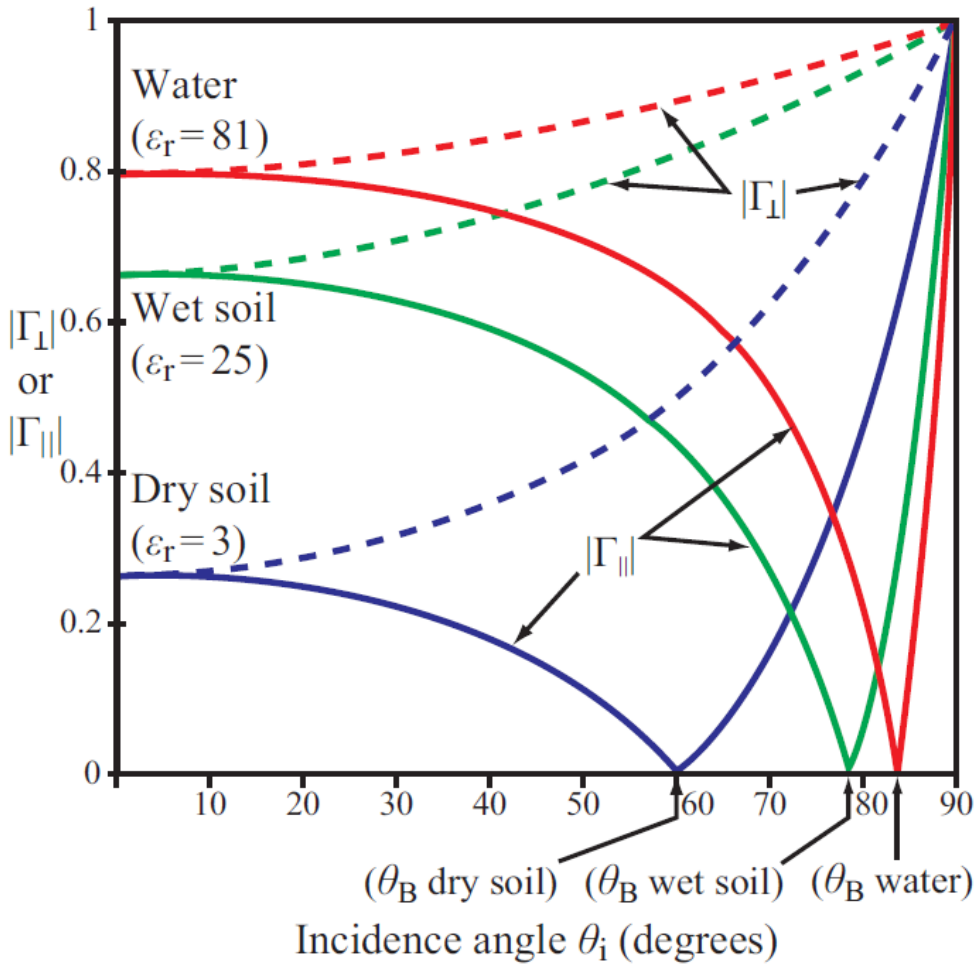


(b) Parallel polarization

(TM Polarization)

Reflection Coefficient vs. Angle

Ulaby Fig. 8-17.



$$\Gamma_{TE} = \frac{\eta_2 \cos\theta_1 - \eta_1 \cos\theta_2}{\eta_2 \cos\theta_1 + \eta_1 \cos\theta_2} \; ;$$

$$\Gamma_{TM} = \frac{\eta_2 \cos\theta_2 - \eta_1 \cos\theta_1}{\eta_2 \cos\theta_2 + \eta_1 \cos\theta_1} \; .$$

Brewster Angle

The Brewster angle θ_B is defined as the incidence angle θ_i at which the Fresnel reflection coefficient $\Gamma = 0$.

Perpendicular Polarization

$\theta_{B\perp}$ ***does not exist for nonmagnetic materials.*** **(Why?)**

Parallel Polarization

$$\theta_{B\parallel} = \sin^{-1}\sqrt{\frac{1}{1+(\varepsilon_1/\varepsilon_2)}}$$

$$= \tan^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}} \qquad (\text{for } \mu_1 = \mu_2).$$

Summary For Reflection and Transmission

Table 8-2: Expressions for Γ, τ, R, and T for wave incidence from a medium with intrinsic impedance η_1 onto a medium with intrinsic impedance η_2. Angles θ_i and θ_t are the angles of incidence and transmission, respectively.

Property	Normal Incidence $\theta_i = \theta_t = 0$	Perpendicular Polarization	Parallel Polarization
Reflection coefficient	$\Gamma = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$	$\Gamma_\perp = \dfrac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$	$\Gamma_\parallel = \dfrac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$
Transmission coefficient	$\tau = \dfrac{2\eta_2}{\eta_2 + \eta_1}$	$\tau_\perp = \dfrac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$	$\tau_\parallel = \dfrac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$
Relation of Γ to τ	$\tau = 1 + \Gamma$	$\tau_\perp = 1 + \Gamma_\perp$	$\tau_\parallel = (1 + \Gamma_\parallel)\dfrac{\cos\theta_i}{\cos\theta_t}$
Reflectivity	$R = \lvert\Gamma\rvert^2$	$R_\perp = \lvert\Gamma_\perp\rvert^2$	$R_\parallel = \lvert\Gamma_\parallel\rvert^2$
Transmissivity	$T = \lvert\tau\rvert^2 \left(\dfrac{\eta_1}{\eta_2}\right)$	$T_\perp = \lvert\tau_\perp\rvert^2 \dfrac{\eta_1 \cos\theta_t}{\eta_2 \cos\theta_i}$	$T_\parallel = \lvert\tau_\parallel\rvert^2 \dfrac{\eta_1 \cos\theta_t}{\eta_2 \cos\theta_i}$
Relation of R to T	$T = 1 - R$	$T_\perp = 1 - R_\perp$	$T_\parallel = 1 - R_\parallel$

Notes: (1) $\sin\theta_t = \sqrt{\mu_1\varepsilon_1/\mu_2\varepsilon_2}\,\sin\theta_i$; (2) $\eta_1 = \sqrt{\mu_1/\varepsilon_1}$; (3) $\eta_2 = \sqrt{\mu_2/\varepsilon_2}$; (4) for nonmagnetic media, $\eta_2/\eta_1 = n_1/n_2$.

Summary

- In this class, we considered how plane waves behave in the presence of a boundary
  - We showed how plane waves reflect and refract
  - We calculated the electromagnetic field across the reflection boundary
  - We proved Snell's Law, examined the critical angle and the Brewster angle.

Generalized Snell's Law?

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Light Propagation with Phase Discontinuities: Generalized Laws of Reflection and Refraction

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Conventional optical components rely on gradual phase shifts accumulated during light propagation to shape light beams. New degrees of freedom are attained by introducing abrupt phase changes over the scale of the wavelength. A two-dimensional array of optical resonators with spatially varying phase response and subwavelength separation can imprint such phase discontinuities on propagating light as it traverses the interface between two media. Anomalous reflection and refraction phenomena are observed in this regime in optically thin arrays of metallic antennas on silicon with a linear phase variation along the interface, which are in excellent agreement with generalized laws derived from Fermat's principle. Phase discontinuities provide great flexibility in the design of light beams, as illustrated by the generation of optical vortices through use of planar designer metallic interfaces.

The shaping of the wavefront of light with optical components such as lenses and prisms, as well as diffractive elements such as gratings and holograms, relies on gradual phase changes accumulated along the optical path. This approach is generalized in transformation optics (*1*, *2*), which uses metamaterials to bend light in unusual ways, achieving such phenomena as negative refraction, subwavelength-focusing, and cloaking (*3*, *4*) and even to explore unusual geometries of space-time in the early universe (*5*). A new degree of freedom of controlling wavefronts can be attained by introducing abrupt phase shifts over the scale of the wavelength along the optical path, with the propagation of light governed by Fermat's principle. The latter states that between two media; $\Phi(\vec{r}_s)$ depends on the coordinate $\vec{r}_s$ along the interface. Then, the total phase shift $\Phi(\vec{r}_s) + \int_A^B \vec{k} \cdot d\vec{r}$ will be stationary for the actual path that light takes; $\vec{k}$ is the wave vector of the propagating light. This provides a generalization of the laws of reflection and refraction, which is applicable to a wide range of subwavelength structured interfaces between two media throughout the optical spectrum.

Generalized laws of reflection and refraction. The introduction of an abrupt phase shift, denoted as phase discontinuity, at the interface between two media allows us to revisit the laws of reflection and refraction by applying Fermat's principle. Consider an incident plane wave at an angle θ_i. Assuming that the two paths are infi-

gles for total internal reflection, provided that $n_t < n_i$:

$$\theta_c = \arcsin\left(\pm\frac{n_t}{n_i} - \frac{\lambda_o}{2\pi n_i}\frac{d\Phi}{dx}\right) \qquad (3)$$

Similarly, for reflection we have

$$\sin(\theta_r) - \sin(\theta_i) = \frac{\lambda_o}{2\pi n_i}\frac{d\Phi}{dx} \qquad (4)$$

where θ_r is the angle of reflection. There is a nonlinear relation between θ_r and θ_i, which is markedly different from conventional specular reflection. Equation 4 predicts that there is always a critical angle of incidence

$$\theta_c' = \arcsin\left(1 - \frac{\lambda_o}{2\pi n_i}\left|\frac{d\Phi}{dx}\right|\right) \qquad (5)$$

above which the reflected beam becomes evanescent.

In the above derivation, we have assumed that Φ is a continuous function of the position along the interface; thus, all the incident energy is transferred into the anomalous reflection and refraction. However, because experimentally we use an array of optically thin resonators with subwavelength separation to achieve the phase change along the interface, this discreteness implies that there are also regularly reflected and refracted beams, which follow conventional laws of reflection and refraction ($d\Phi/dx = 0$ in Eqs. 2 and 4). The separation between the resonators controls the amount of energy in the anomalously reflected and refracted beams. We have also assumed that the amplitudes of the scattered radiation by each resonator are identical, so that the reflected and refracted beams are plane waves. In the next section, we will show with simulations—which represent numerical solutions of Maxwell's

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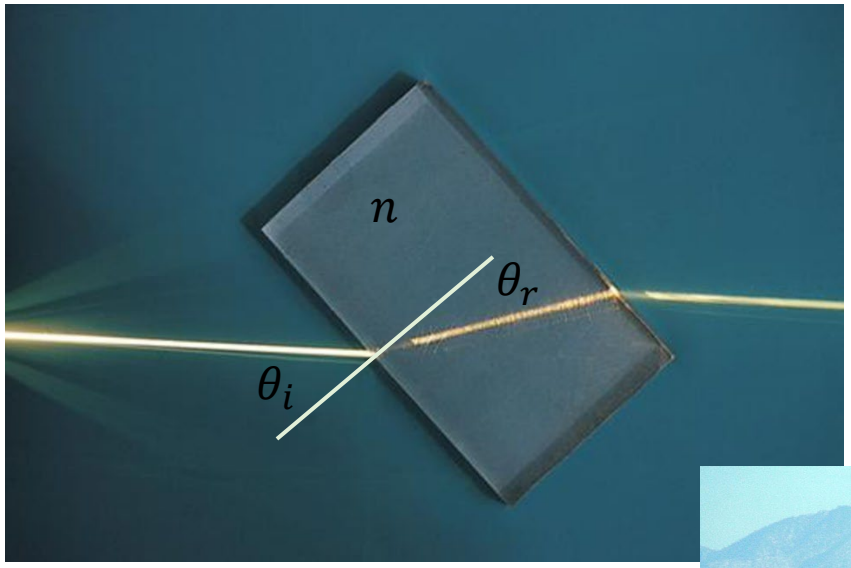
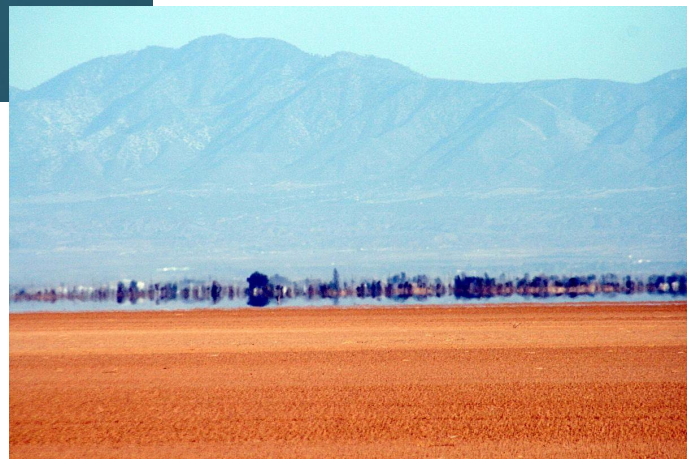
Snell's Law: Bending Light with Material

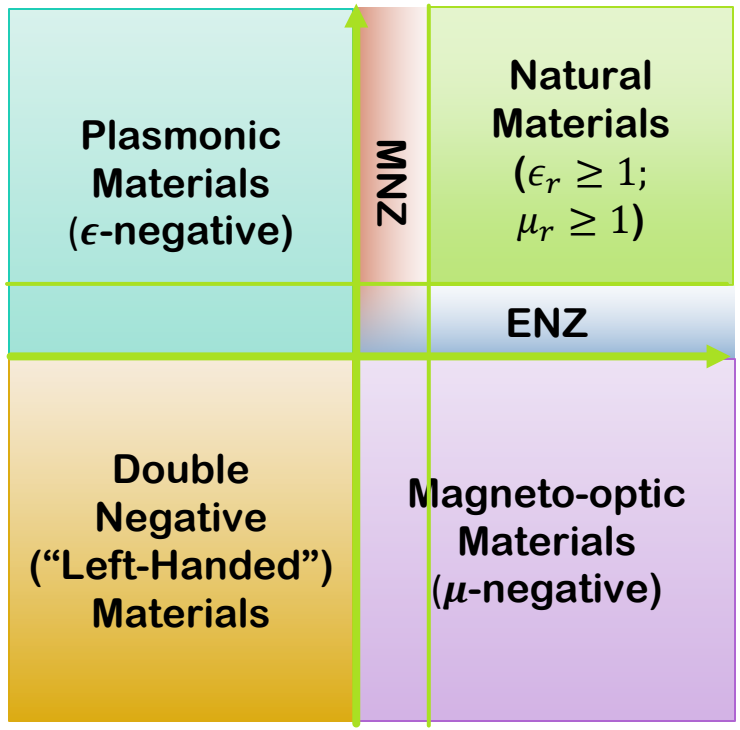


$$\sin\theta_i = n\sin\theta_r$$

($n \cong 1.5$ for glass)



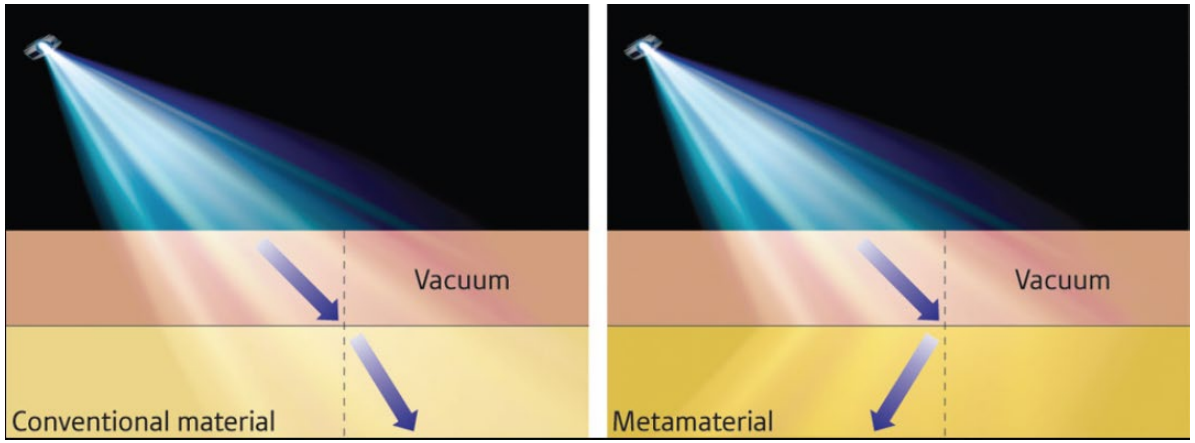
Metamaterials



- **Permittivity (ϵ, "epsilon")** and **permeability (μ, "mu")** denote a material's response to **electric** and **magnetic** fields
- By engineering the material structure, one can achieve material properties unfound in nature

Negative refraction effects

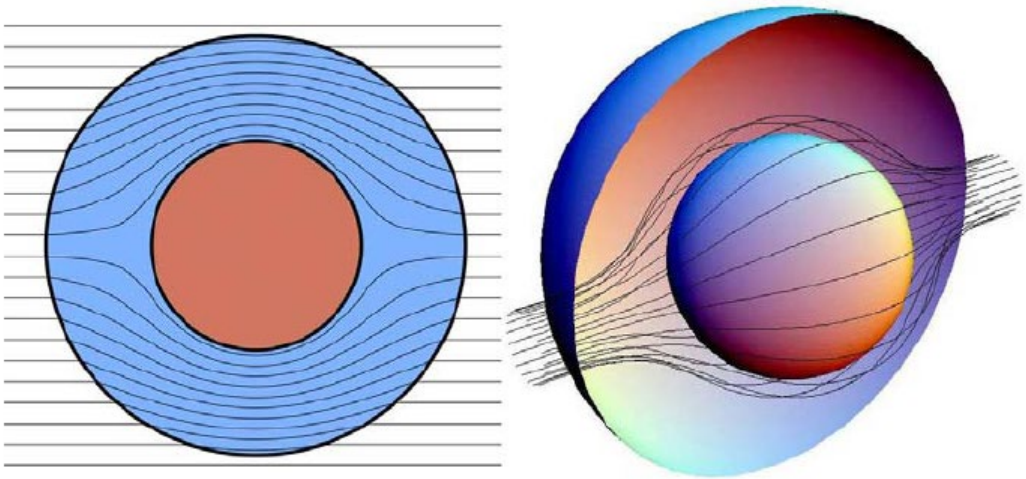
- Weird things happen with light propagation in double negative materials (a.k.a. negative refractive index materials)



V. M. Shalaev, U of T Plasmonic Summer School (2013)

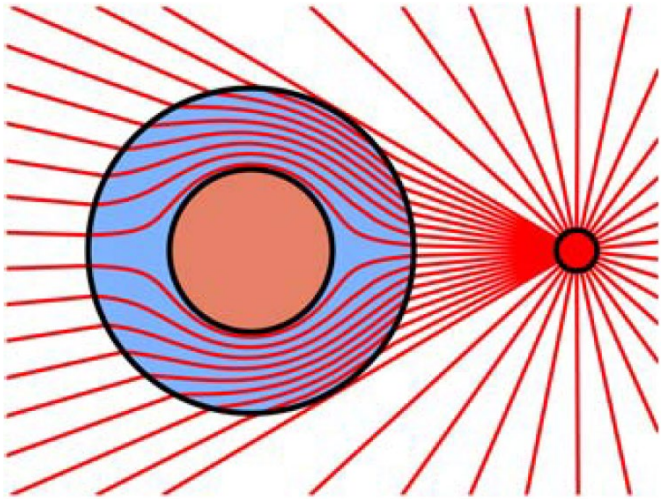
Electromagnetic Cloaking

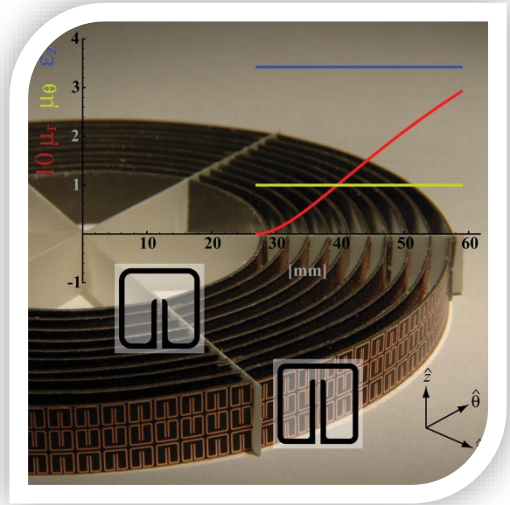
- If we can make materials at will, we can also bend EM waves at will….

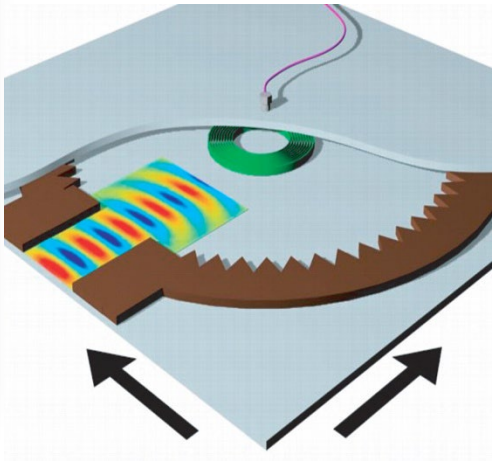


Pendry et. al., Science, vol.312, p.1780, 2006

From idea to reality



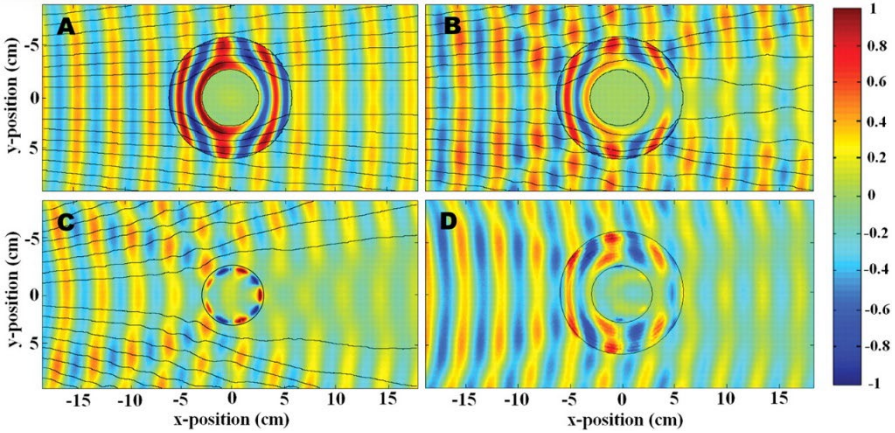




$$\varepsilon'_{r'} = \mu'_{r'} = \frac{R_2}{R_2 - R_1}\frac{(r' - R_1)^2}{r'^2},$$

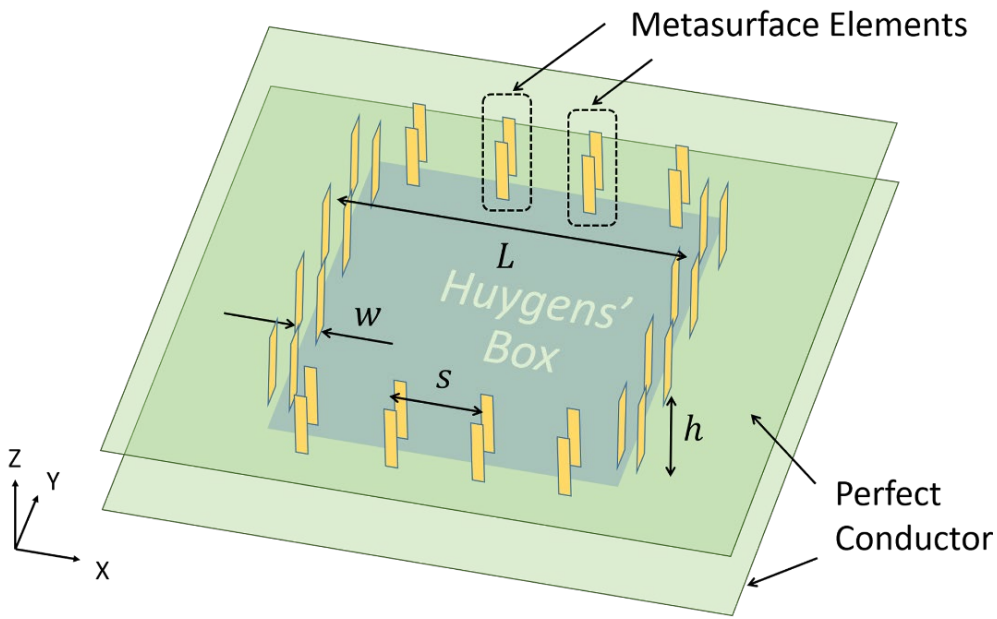
$$\varepsilon'_{\theta'} = \mu'_{\theta'} = \frac{R_2}{R_2 - R_1},$$

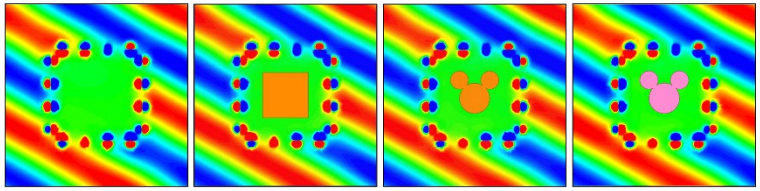
$$\varepsilon'_{\phi'} = \mu'_{\phi'} = \frac{R_2}{R_2 - R_1}$$

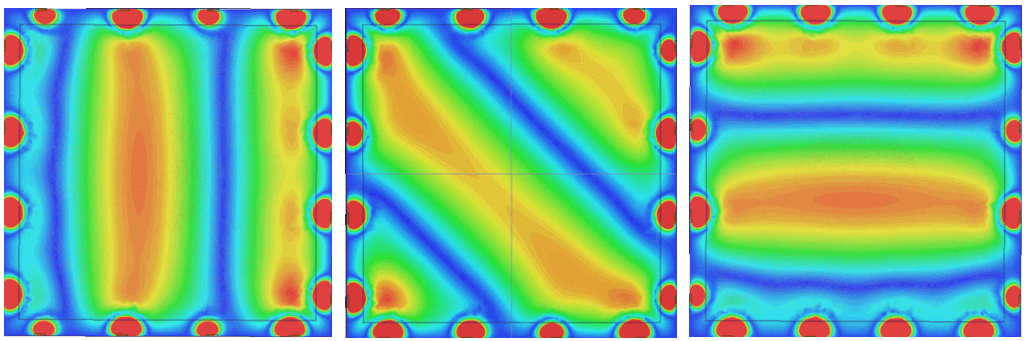


Pendry et al., Science, vol.312, p.1780, 2006; Schurig et al., Science, vol. 314, p. 977, 2006.

Active Cloaking with a *Huygens' Box*

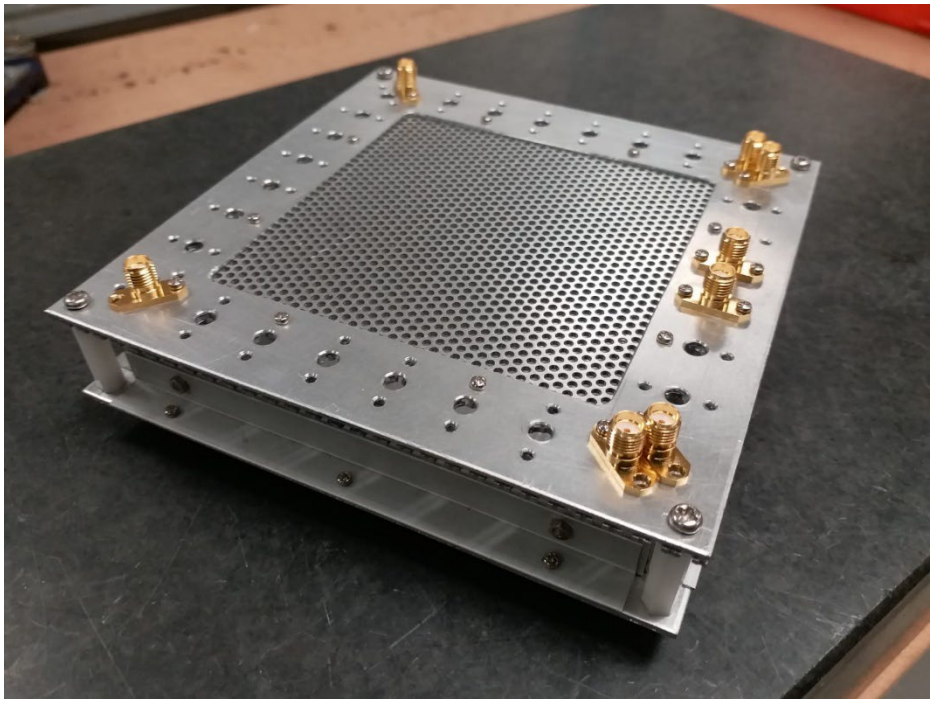


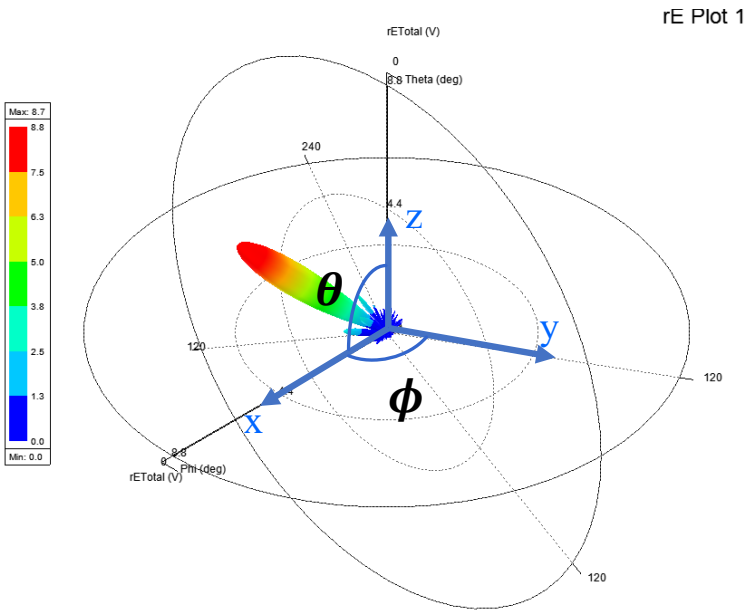


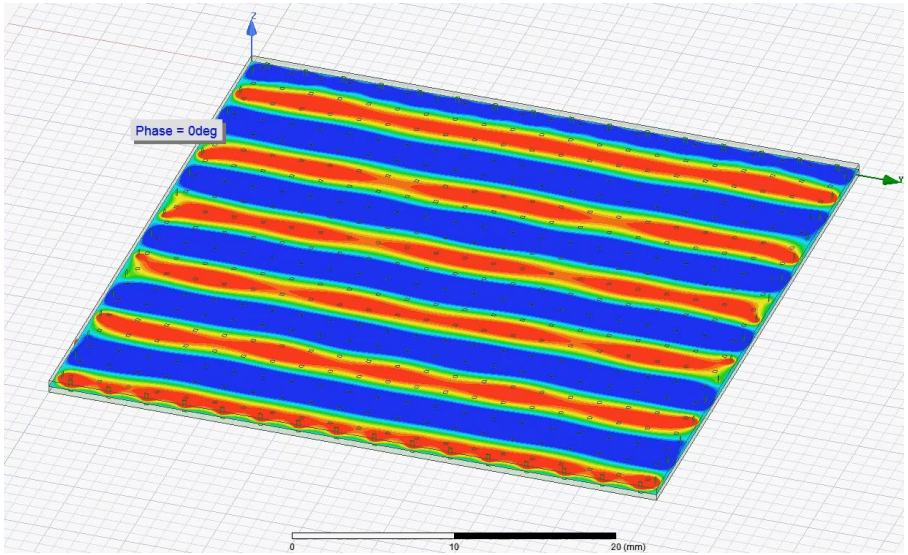


Wong and Eleftheriades, IEEE TAP, Aug. 2020.

The Huygens' Box Antenna







K.A. Oyesina and A.M.H. Wong, IEEE AP-S, July 2019.
K.A. Oyseina and A.M.H. Wong, IEEE *Antennas Wirel. Propag. Lett. 19*, 2020.
K.A. Oyesina and A.M.H. Wong, IEEE AP-S, July 2022.