

In-Class Activity 2 Solutions

Question 1 (Monopoly Profit Maximization): Suppose the only firm manufacturing a certain kind of machine tool can produce up to 100 per week. The demand set for these items is

$$D = \{(q, p) | q + 5p = 850\}$$

The cost of producing q items per week is

$$C(q) = 300 - 10q + q^2$$

How many items should be produced each week in order to maximize profit?

See pages 11-13 of Lecture 4 Part II of II.

Question 2 (Maximize Excise Tax Revenue): The supply and demand sets for a good are

$$S = \{(q, p) | q = bp - a\}, \quad D = \{(q, p) | q = c - dp\},$$

where a, b, c, d are all positive. Suppose the government wishes to raise as much money as possible by imposing an excise tax on the good. What should be the value of the excise tax? What is the resulting government revenue?

When an excise tax is imposed, the selling price at equilibrium, p^T satisfies

$$b(p^T - T) - a = c - dp^T$$

because suppliers are getting a price of $p^T - T$. Solving for p^T , we get

$$p^T = \frac{a + c}{b + d} + \frac{b}{b + d}T$$

Then

$$q^T = c - dp^T = \frac{bc - ad}{b + d} - \frac{bd}{b + d}T$$

Tax revenue is given by

$$R(T) = Tq^T = T \left(\frac{bc - ad}{b + d} - \frac{bd}{b + d}T \right) = \left(\frac{bc - ad}{b + d} \right)T - \left(\frac{bd}{b + d} \right)T^2$$

At the maximum point, T_m , we must have

$$R'(T_m) = 0 \quad \text{and} \quad R''(T_m) < 0$$

$$R'(T_m) = \frac{bc - ad}{b + d} - \frac{2bd}{b + d}T_m = 0 \implies T_m = \frac{bc - ad}{2bd}$$

The second derivative is

$$R''(T_m) = -\frac{2bd}{b + d} < 0,$$

since $b, d > 0$. So, we have a maximum at T_m .

The maximized tax revenue is

$$\begin{aligned} R(T_m) &= \left(\frac{bc - ad}{b + d}\right)T_m - \left(\frac{bd}{b + d}\right)T_m^2 \\ \implies R(T_m) &= \left(\frac{bc - ad}{b + d}\right)\left(\frac{bc - ad}{2bd}\right) - \left(\frac{bd}{b + d}\right)\left(\frac{bc - ad}{2bd}\right)^2 \\ \implies R(T_m) &= \frac{(bc - ad)^2}{2bd(b + d)}\left[1 - \frac{1}{2}\right] = \frac{(bc - ad)^2}{4bd(b + d)} \end{aligned}$$

Question 3 (Perfectly Competitive Firm): Consider a firm in a perfectly competitive market with cost function

$$C(q) = 800 + 70q - 12q^2 + q^3$$

Find the startup and break-even points.

$$C(q) = 800 + 70q - 12q^2 + q^3$$

$$C'(q) = 70 - 24q + 3q^2$$

We know that $p = C'(q)$, so we have

$$\begin{aligned} \Pi(q) &= pq - C(q) = qC'(q) - C(q) = (70 - 24q + 3q^2)q - (800 + 70q - 12q^2 + q^3) \\ \implies \Pi(q) &= -800 - 12q^2 + 2q^3 \end{aligned}$$

At the startup quantity, q_s , we have

$$\Pi(q_s) = \Pi(0) = -800$$

$$\text{or, } -800 - 12q_s^2 + 2q_s^3 = -800$$

$$\implies -12q_s^2 + 2q_s^3 = 0$$

$$\implies 2q_s^2(-6 + q_s) = 0$$

$$\implies q_s = 6 \quad (\text{as } q_s > 0)$$

At the breakeven point, we have

$$\Pi(q_b) = 0$$

$$-800 - 12q^2 + 2q^3 = 0$$

(Note: We don't need to solve a cubic equation for this course. So, we leave this expression as it is at this point.)

Question 4 (Perfectly Competitive Firm): Prove that, at the break-even point for an efficient, perfectly competitive firm, the derivative of the average cost is zero.

If q_b is the breakeven point, then by definition,

$$\Pi(q_b) = 0$$

$$\implies q_b C'(q_b) - C(q_b) = 0$$

Now, average cost is

$$AC(q) = \frac{C(q)}{q}$$

By quotient rule, we get

$$AC'(q) = \frac{qC'(q) - C(q)}{q^2}$$

At the breakeven point,

$$AC'(q_b) = \frac{q_b C'(q_b) - C(q_b)}{q_b^2} = \frac{0}{q_b^2} = 0$$