

Block A: Unit 3

Q1. [Problem 3.51]

Solution:

- Known quantities:
Circuit shown in Figure P3.51.

- Find:
Thevenin equivalent circuit.

- Analysis:

$$R_{TH} = 1 + \frac{1}{\frac{1}{5} + \frac{1}{4}} = 1 + \frac{20}{9} = \frac{29}{9} \Omega = 3.222 \Omega$$

Voltage divider gives

$$v = \left(\frac{4}{4+5} \right) 36 = 16 \text{ V}$$

KVL:

$$v_{OC} = -0(1) + v = v = 16 \text{ V}$$

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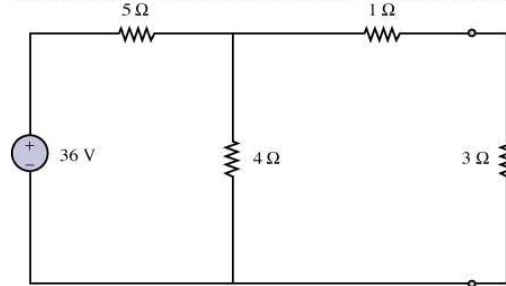


Figure P3.51

Q2. [Problem 3.52]

Solution:

- Known quantities:
Circuit shown in Figure P3.52.

- Find:
Thevenin equivalent circuit and the voltage across resistance 3Ω .

- Analysis:

KVL: $v_3 = 3 + v_2$,

Find v_{oc} (3Ω disconnected)

$$\text{KCL Node 1: } 2 + \frac{1}{2}(v_1 - v_2) + \frac{1}{2}[v_1 - (3 + v_2)] = 0$$

KCL Nodes 2 & 3:

$$\frac{1}{2}(v_2 - v_1) + \frac{1}{4}v_2 + \frac{1}{2}[(3 + v_2) - v_1] + 0 = 0$$

$$\left. \begin{aligned} 1v_1 - 1v_2 &= -\frac{1}{2} \\ -1v_1 + \frac{5}{4}v_2 &= -\frac{3}{2} \end{aligned} \right\} \Rightarrow \begin{aligned} v_1 &= -\frac{17}{2} = -8.5 \text{ V} \\ v_2 &= -8 \text{ V} \end{aligned} \Rightarrow v_{TH} = v_3 = v_2 + 3 = -5 \text{ V}$$

Set all independent sources to zero,

$$R_{TH} = 4\Omega$$

$$v = \frac{3}{4+3}(-5) = -\frac{15}{7} = -2.14 \text{ V}$$

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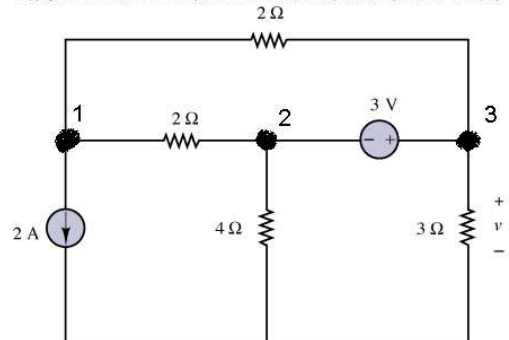


Figure P3.52

Q3. [Problem 3.54]Solution:

• Known quantities:
Circuit shown in Figure P3.54.

• Find:
Norton equivalent circuit.

• Analysis:
 $R_N = 5\Omega \parallel (3\Omega + 2\Omega \parallel 1\Omega) = 2.12\Omega$ (for equivalent circuit, see (a))

Using mesh analysis, see (b):

$$8 - 1(i_1 - i_2) - 2(i_1 - i_{SC}) = 0$$

$$-1(i_2 - i_1) - 5i_2 - 3(i_2 - i_{SC}) = 0$$

$$-2(i_{SC} - i_1) - 3(i_{SC} - i_2) = 0$$

Solving, $i_{SC} = 3.05\text{A} \Rightarrow I_N = 3.05\text{A}$.

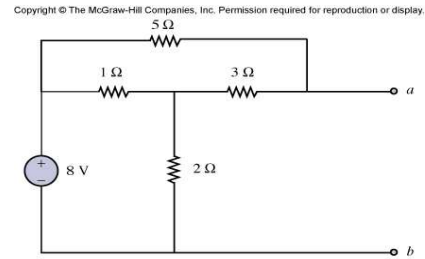
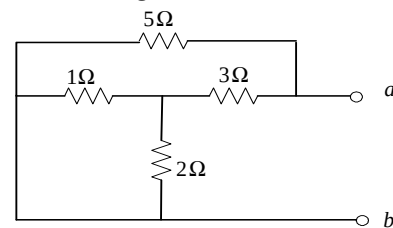
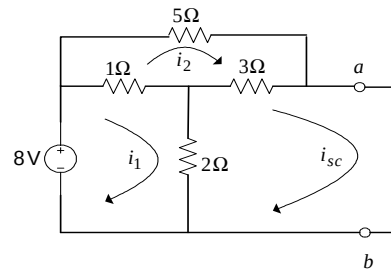


Figure P3.54



(a)



(b)

Q4. [Problem 3.53]Solution:

• Known quantities:
Circuit shown in Figure P3.53.

• Find:
Norton equivalent circuit.

• Analysis:
 $R_N = 3\Omega + 1\Omega + (3\Omega \parallel 1\Omega) = 4.75\Omega$

Using the mesh analysis approach:

$$4i_1 - 3i_2 = 2$$

$$-3i_1 + 4i_2 + 3i_{SC} = 0$$

$$i_2 - i_{SC} = 2$$

Solving, $i_{SC} = -0.42\text{A} \Rightarrow I_N = -0.42\text{A}$

It means the magnitude of i_{SC} is 0.42A and the direction of i_{SC} is count-clockwise.

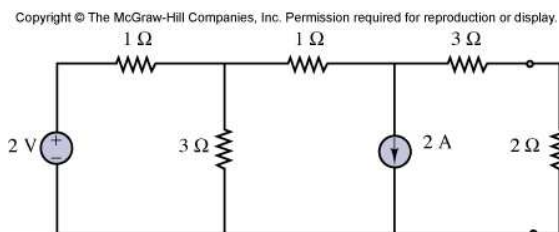
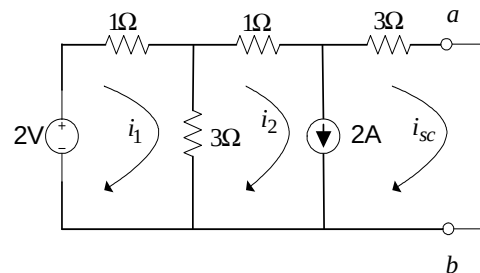


Figure P3.53



Q5. [Problem 3.55]Solution:

- Known quantities:
Circuit shown in Figure P3.55.

- Find:
Thevenin equivalent circuit.

- Analysis:

To find R_{TH} , we zero the two sources by shorting the voltage source and opening the current source. The resulting circuit is shown in the left, see (a).

Therefore, $R_{TH} = 1000 \parallel 1000 + 1 + 3 = 504 \Omega$.

To find V_{oc} , we assume V_b as reference (i.e., zero) and apply nodal analysis, see (b):

$$\frac{V_c - V_a}{1} = 0.01$$

$$\frac{10 - V_c}{1000} = \frac{V_c}{1000} + 0.01$$

Then, $V_a = -0.01V$

Therefore, $V_{OC} = -0.01V$.

It means that a is negative side, b is positive side while the magnitude of V_{oc} is $0.01V$.

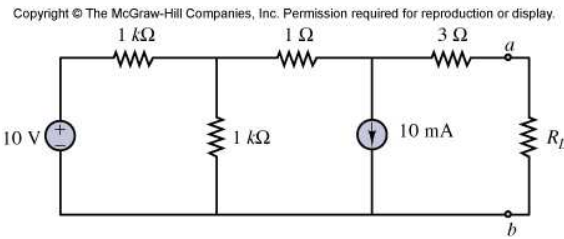
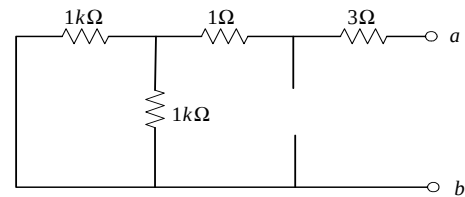
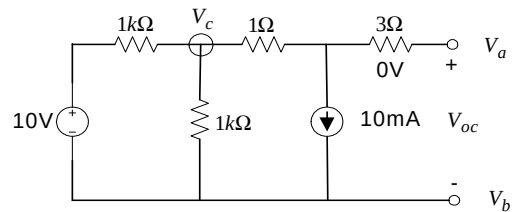


Figure P3.55



(a)



(b)

Q6. [Problem 3.66]Solution:

- Known quantities:
The schematic of the circuit (see Figure P3.66).

- Find:
The Thevenin equivalent resistance seen by the 10Ω resistor, the Thevenin (open-circuit) voltage and the Norton (short-circuit) current when the 10Ω resistor is the load. Figure P3.66

- Analysis:

(1) Remove the load, leaving the load terminals open circuited, and the voltage sources. Redraw the circuit, see (a).

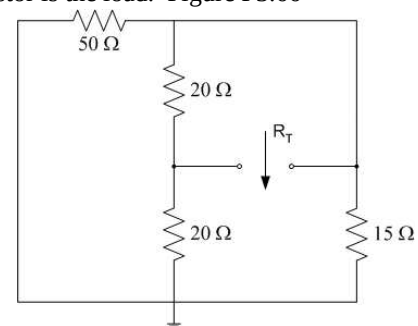
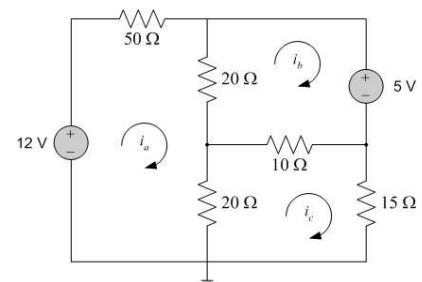
$$R_T = R_2 \parallel (R_3 + (R_1 \parallel R_5)) = 20 \Omega \parallel (20 \Omega + (50 \Omega \parallel 15 \Omega)) = 12.24 \Omega$$

(2) Remove the load, leaving the load terminals open circuited. Redraw the circuit, see (b).

For node #1:

$$\frac{v_1 - 12}{50} + \frac{v_1 - v_2}{20} + i_{5V} = 0$$

For node #2:



(a)

$$\frac{v_2 - v_1}{20} + \frac{v_2}{20} = 0$$

For node #3:

$$\frac{v_3}{15} - i_{5V} = 0$$

For the 5-V voltage source:

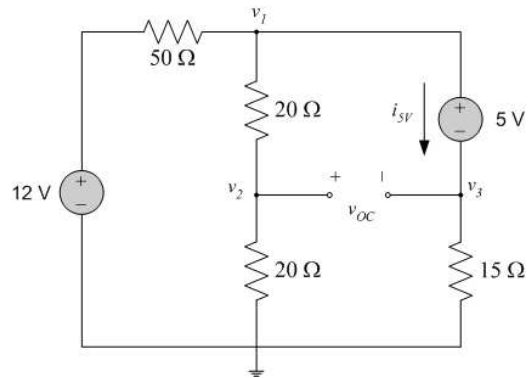
$$v_1 - v_3 = 5$$

Solving the system,

$v_1 = 5.14 \text{ V}$, $v_2 = 2.57 \text{ V}$, $v_3 = 0.13 \text{ V}$ and $i_{5V} = 8.95 \text{ mA}$.

Therefore,

$$v_{OC} = v_2 - v_3 = 2.44 \text{ V}.$$



(b)

(3) Replace the load with a short circuit. Redraw the circuit, see (c).

For mesh (a):

$$i_a(90) - i_b(20) - i_c(20) = 12$$

For mesh (b):

$$-i_a(20) + i_b(20) + 5 = 0$$

For mesh (c):

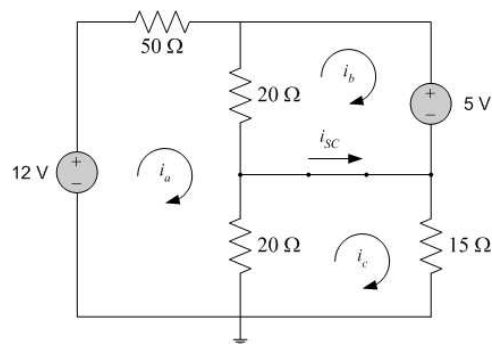
$$-i_a(20) + i_c(35) = 0$$

Solving the system,

$i_a = 119.5 \text{ mA}$, $i_b = -130.5 \text{ mA}$ and $i_c = 68.3 \text{ mA}$.

Therefore,

$$i_{SC} = i_c - i_b = 198.8 \text{ mA}.$$



(c)

Q7. [Problem 3.62]

Solution:

- Known quantities:

The schematic of the circuit (see Figure P3.62). All resistors in the circuit are 100Ω and voltage sources are 5V .

- Find:

The Thevenin equivalent resistance seen by resistor R_3 , the Thevenin (open-circuit) voltage and the Norton (short-circuit) current when R_3 is the load.

- Analysis:

(1) Remove the load, leaving the load terminals open circuited, and the voltage sources. Redraw the circuit, see (a).

$$R_T = R_1 \parallel R_4 + R_2 \parallel R_5 = \frac{R_1 R_4}{R_1 + R_4} + \frac{R_2 R_5}{R_2 + R_5} = 100\Omega$$

(2) Remove the load, leaving the load terminals open circuited. Redraw the circuit, see (b).

$$\text{For node \#1: } \frac{v_1}{R_1} + \frac{v_1 - V_{S1}}{R_4} = 0$$

$$\text{For node \#2: } \frac{v_2}{R_2} + \frac{v_2 + V_{S2}}{R_5} = 0$$

Solving the system,

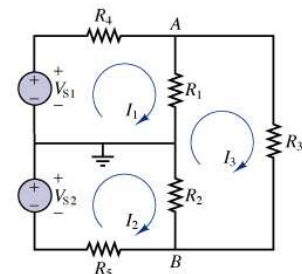
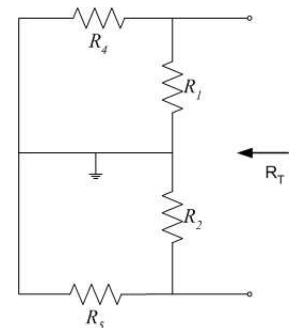


Figure P3.62



(a)

$$v_1 = \frac{R_1}{R_1 + R_4} V_{S1}$$

$$v_2 = -\frac{R_2}{R_2 + R_5} V_{S2}$$

Therefore,

$$v_{OC} = v_1 - v_2 = \frac{R_1}{R_1 + R_4} V_{S1} + \frac{R_2}{R_2 + R_5} V_{S2} = 5V$$

(3) Replace the load with a short circuit. Redraw the circuit, see (c).

For mesh (a): $i_a(R_1 + R_4) - R_1 i_c = V_{S1}$

For mesh (b): $i_b(R_2 + R_5) - R_2 i_c = V_{S2}$

For mesh (c): $-R_1 i_a - R_2 i_b + i_c(R_1 + R_2) = 0$

Solving the system,

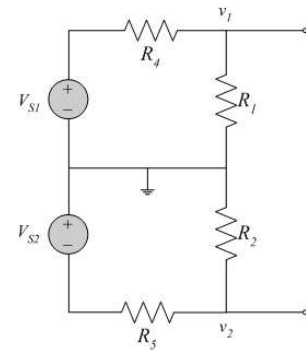
$$i_a = \frac{(R_1 R_2 + R_1 R_5 + R_2 R_5) V_{S1} + R_1 R_2 V_{S2}}{R_1 R_4 (R_2 + R_5) + R_2 R_5 (R_1 + R_4)}$$

$$i_b = \frac{R_1 R_2 V_{S1} + (R_1 R_2 + R_1 R_4 + R_2 R_4) V_{S2}}{R_1 R_4 (R_2 + R_5) + R_2 R_5 (R_1 + R_4)}$$

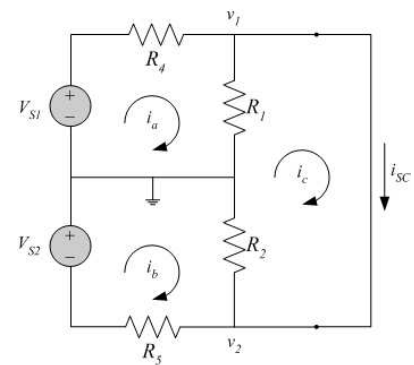
$$i_c = \frac{R_1 (R_2 + R_5) V_{S1} + R_2 (R_1 + R_4) V_{S2}}{R_1 R_4 (R_2 + R_5) + R_2 R_5 (R_1 + R_4)}$$

Therefore,

$$i_{SC} = i_c = \frac{R_1 (R_2 + R_5) V_{S1} + R_2 (R_1 + R_4) V_{S2}}{R_1 R_4 (R_2 + R_5) + R_2 R_5 (R_1 + R_4)} = 50\text{mA}$$



(b)



(c)

Q8. [Problem 19.1 of Alexander & Sadiku]

Solution:

- Known quantities:

The schematic of the circuit (see Figure 19.65).

- Find:

Obtain z parameters for this network.

- Analysis:

To get z_{11} and z_{21} , consider the circuit in (a).

$$z_{11} = \frac{V_1}{I_1} = 1 + 6 \parallel (4 + 2) = 4\Omega$$

$$I_0 = \frac{1}{2} I_1, V_0 = 2I_0 = I_1$$

$$z_{21} = \frac{V_2}{I_1} = 1\Omega$$

To get z_{22} and z_{12} , consider the circuit in (b).

$$z_{22} = \frac{V_2}{I_2} = 2 \parallel (4 + 6) = 1.667\Omega$$

$$I_0' = \frac{2}{10 + 2} I_2 = \frac{1}{6} I_2, V_1 = 6I_0' = I_2$$

$$z_{12} = \frac{V_1}{I_2} = 1\Omega$$

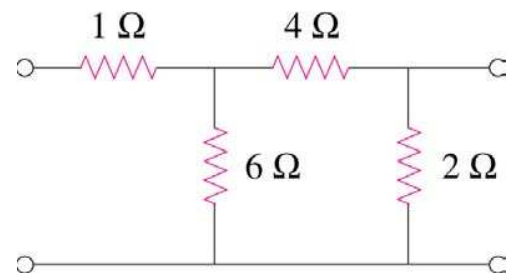
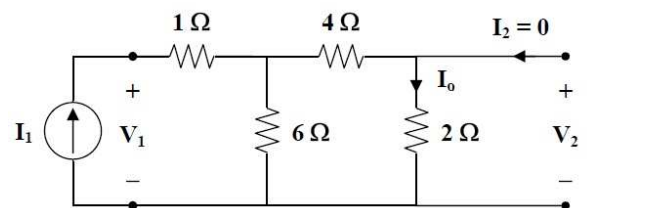
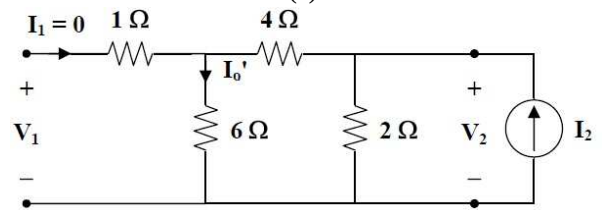


Figure 19.65



(a)



(b)

Hence, $[z] = \begin{bmatrix} 4 & 1 \\ 1 & 1.667 \end{bmatrix} \Omega$.

Q9. [Problem 19.7 of Alexander & Sadiku]

Solution:

- Known quantities:
The schematic of the circuit (see Figure 19.71).

- Find:
Obtain z parameters for this network.

- Analysis:

To get z_{11} and z_{21} , consider the circuit in (a).

$$\frac{V_1 - V_x}{20} = \frac{V_x}{50} + \frac{V_x + 12V_x}{160} \rightarrow V_x = \frac{40}{121} V_1$$

$$I_1 = \frac{V_1 - V_x}{20} = \frac{81}{121} \left(\frac{V_1}{20} \right) \rightarrow z_{11} = \frac{V_1}{I_1} = 29.88$$

$$V_2 = 60 \left(\frac{13V_x}{160} \right) - 12V_x = -\frac{57}{8} V_x = -\frac{57}{8} \left(\frac{40}{121} \right) V_1$$

$$= -\frac{57}{8} \left(\frac{40}{121} \right) \frac{20 \times 121}{81} I_1 = -70.37 I_1 \rightarrow z_{21} = \frac{V_2}{I_1} = -70.37$$

To get z_{22} and z_{12} , consider the circuit in (b).

$$V_x = \frac{50}{100 + 50} V_2 = \frac{1}{3} V_2$$

$$I_2 = \frac{V_2}{150} + \frac{V_2 + 12V_x}{60} = 0.09 V_2 \rightarrow z_{22} = \frac{V_2}{I_2} = 1 / 0.09 = 11.11$$

$$V_1 = V_x = \frac{1}{3} V_2 = \frac{11.11}{3} I_2 = 3.704 I_2 \rightarrow z_{12} = \frac{V_1}{I_2} = 3.704$$

Thus,

$$[z] = \begin{bmatrix} 29.88 & 3.704 \\ -70.37 & 11.11 \end{bmatrix} \Omega$$

(b)

Q10. [Problem 19.30 of Alexander & Sadiku]

Solution:

- Known quantities:
The schematics of the circuit (see Figure 19.87).

- Find:
Obtain h parameters for these networks.

- Analysis:

(a) Convert to z parameters; then, convert to h parameters.

$$z_{11} = z_{12} = z_{21} = 60 \Omega, \quad z_{22} = 100 \Omega$$

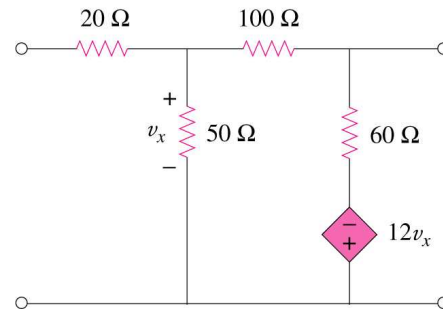
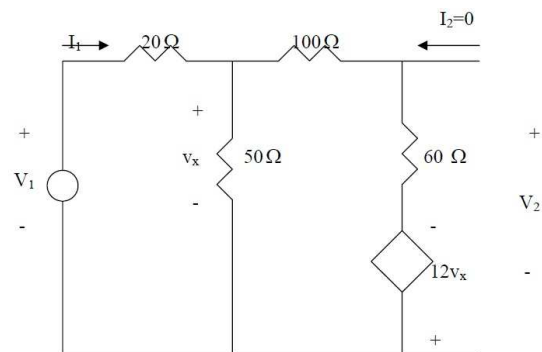
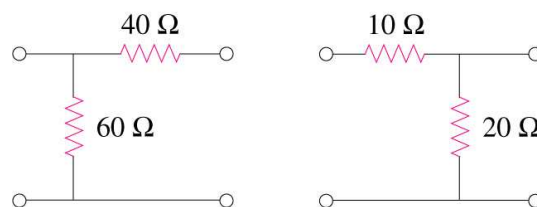
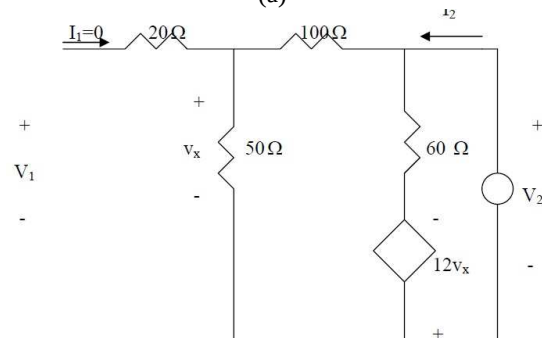


Figure 19.71



(a)



(a)

(b)

Figure 19.87

$$\Delta_z = z_{11}z_{22} - z_{12}z_{21} = 6000 - 3600 = 2400,$$

$$h_{11} = \frac{\Delta_z}{z_{22}} = \frac{2400}{100} = 24, \quad h_{12} = \frac{z_{12}}{z_{22}} = \frac{60}{100} = 0.6$$

$$h_{21} = \frac{-z_{21}}{z_{22}} = -0.6, \quad h_{22} = \frac{1}{z_{22}} = 0.01$$

$$\text{Thus, } [h] = \begin{bmatrix} 24\Omega & 0.6 \\ -0.6 & 0.01S \end{bmatrix}$$

(b) Similarly,

$$z_{11} = 30\Omega, \quad z_{12} = z_{21} = z_{22} = 20\Omega$$

$$\Delta_z = 600 - 400 = 200,$$

$$h_{11} = \frac{200}{20} = 10, \quad h_{12} = \frac{20}{20} = 1$$

$$h_{21} = -1, \quad h_{22} = \frac{1}{20} = 0.05$$

$$\text{Thus, } [h] = \begin{bmatrix} 10\Omega & 1 \\ -1 & 0.05S \end{bmatrix}$$

Q11. [Problem 19.31 of Alexander & Sadiku]

Solution:

- Known quantities:

The schematic of the circuit (see Figure 19.88).

- Find:

Determine h parameters for this network.

- Analysis:

We get $\mathbf{h}_{11}$ and $\mathbf{h}_{21}$ by considering the circuit in (a).

At node 1,

$$I_1 = \frac{V_3}{2} + \frac{V_3 - V_4}{2} \rightarrow 2I_1 = 2V_3 - V_4$$

At node 2,

$$\frac{V_3 - V_4}{2} + 4I_1 = \frac{V_4}{1}$$

$$8I_1 = -V_3 + 3V_4 \rightarrow 16I_1 = -2V_3 + 6V_4$$

Add (1) and (2),

$$18I_1 = 5V_4 \rightarrow V_4 = 3.6I_1$$

$$V_3 = 3V_4 - 8I_1 = 2.8I_1$$

$$V_1 = V_3 + I_1 = 3.8I_1$$

$$h_{11} = \frac{V_1}{I_1} = 3.8\Omega$$

$$I_2 = \frac{-V_4}{1} = -3.6I_1 \rightarrow h_{21} = \frac{I_2}{I_1} = -3.6$$

To get $\mathbf{h}_{22}$ and $\mathbf{h}_{12}$, refer to the circuit in (b). The dependent current source can be replaced by an open circuit since $4I_1 = 0$.

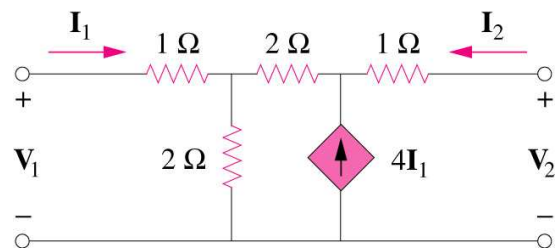
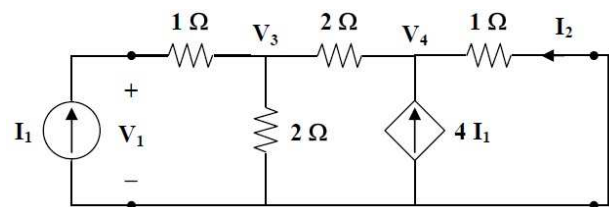
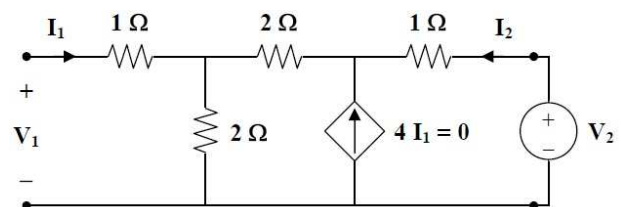


Figure 19.88



(a)



(b)

$$V_1 = \frac{2}{2+2+1}V_2 = \frac{2}{5}V_2 \rightarrow h_{12} = \frac{V_1}{V_2} = 0.4$$

$$I_2 = \frac{V_2}{2+2+1} = \frac{V_2}{5} \rightarrow h_{22} = \frac{I_2}{V_2} = \frac{1}{5} = 0.2\text{S}$$

$$\text{Thus, } [h] = \begin{bmatrix} 3.8\Omega & 0.4 \\ -3.6 & 0.2\text{S} \end{bmatrix}$$