

EE1001 Test (Q&A)
(100 marks, 75 mins in total)

Q1 (10 marks)

Determine the following statements whether they are true or false. (10 marks)

- (i) An argument consisting of all false premises must be an invalid argument.
- (ii) An invalid argument with all true premises can be sound.
- (iii) Given the conclusion of a valid argument is false, the argument must be unsound.
- (iv) “If you are in the campus, can you inform me?” is proposition.
- (v) $\exists x \in \mathbf{R} \exists y \in \mathbf{N}, (H(x,y) \rightarrow K(x,y)) \equiv \exists y \in \mathbf{N} \exists x \in \mathbf{R}, (H(x,y) \rightarrow K(x,y))$

Solution of Q1 (10 marks) – (5 mins)

- (i) False
(A valid argument can go from all false premises to a true conclusion.)
- (ii) False
(An invalid argument can never be sound.)
- (iii) True
(A valid argument with false conclusion means certain premise is false, and thus it must be unsound)
- (iv) False
(A question cannot be an argument)
- (v) True

Q2 (13 marks)

Determine whether the following statement is a tautology, a contradiction, or neither.

$$\sim[\sim u \rightarrow (\sim u \wedge v)] \vee s \rightarrow s$$

(13 marks)

Solution of Q2 (13 marks) – (8 mins)

$$\begin{aligned} & \sim[\sim u \rightarrow (\sim u \wedge v)] \vee s \rightarrow s \\ \equiv & \sim[u \vee (\sim u \wedge v)] \vee s \rightarrow s && \text{(Definition of } \rightarrow \text{)} \\ \equiv & \sim[(u \vee \sim u) \wedge (u \vee v)] \vee s \rightarrow s && \text{(Distributive laws)} \\ \equiv & \sim[\mathbf{t} \wedge (u \vee v)] \vee s \rightarrow s && \text{(Negation laws)} \\ \equiv & \sim[(u \vee v)] \vee s \rightarrow s && \text{(Identity laws)} \\ \equiv & \sim\{\sim[(u \vee v)] \vee s\} \vee s && \text{(Definition of } \rightarrow \text{)} \\ \equiv & [(u \vee v) \wedge \sim s] \vee s && \text{(De Morgan's laws)} \\ \equiv & (u \vee v \vee s) \wedge (\sim s \vee s) && \text{(Distributive laws)} \\ \equiv & (u \vee v \vee s) \wedge \mathbf{t} && \text{(Negation laws)} \\ \equiv & u \vee v \vee s && \text{(Identity laws)} \end{aligned}$$

It's neither a tautology nor contradiction.

Q3 (15 marks)

Citizen A, B, and C are suspected of a crime. Their statements are as follows:

Citizen A: “At least one of the other citizens is innocent.”

Citizen B: “If Citizen C is guilty, then Citizen A is innocent, and if I am innocent, then Citizens A and C are guilty.”

Citizen C: “If Citizens A and B are guilty, then I am innocent”

Let A, B, and C be

A = “Citizen A is innocent”

B = “Citizen B is innocent”

C = “Citizen C is innocent”

- (i) Formulate the statements of Citizens A, B, and C.
- (ii) Given that the innocent told the truth and the guilty lied, who is innocent and who is guilty?

Solution of Q3 (15 marks) – (8 mins)

Q3(i)

The statement of the suspect:

“ $b \vee c$ ” = A’s statement = “At least one of the other citizens is innocent.”

$(\sim c \rightarrow a) \wedge (b \rightarrow \sim a \wedge \sim c)$ = B’s statement = “If Citizen C is guilty, then Citizen A is innocent, and if I am innocent, then Citizens A and C are guilty.”

$(\sim a \wedge \sim b) \rightarrow c$ = C’s statement = “If Citizens A and B are guilty, then I am innocent”

Q3(ii)

The situation is satisfied only when the truth values of the suspects’ statuses and the truth values of the suspects’ statements are the same, i.e.,

$[a \leftrightarrow (b \vee c)] \wedge [b \leftrightarrow (\sim c \rightarrow a) \wedge (b \rightarrow \sim a \wedge \sim c)] \wedge [c \leftrightarrow (\sim a \wedge \sim b) \rightarrow c]$ must be true

a	b	c	$b \vee c$	$(\sim c \rightarrow a) \wedge (b \rightarrow \sim a \wedge \sim c)$	$(\sim a \wedge \sim b) \rightarrow c$	$[a \leftrightarrow (b \vee c)] \wedge [b \leftrightarrow (\sim c \rightarrow a) \wedge (b \rightarrow \sim a \wedge \sim c)] \wedge [c \leftrightarrow (\sim a \wedge \sim b) \rightarrow c]$
T	T	T	T	F	T	F
T	T	F	T	F	T	F
T	F	T	T	T	T	F
T	F	F	F	T	T	F
F	T	T	T	F	T	F
F	T	F	T	F	T	F
F	F	T	T	T	T	F
F	F	F	F	F	F	T

Therefore, there is only one possible situation:

1. All Suspects A, B, and C are guilty.

Q4 (12 marks)

- (i) A geometric sequence has the first term $\log 45$ and common ratio $\log h$. (6 marks)
- Find the range of values of h for which the geometric sequence has a sum to infinity.
 - Find the exact value of h for which the sum to infinity of the geometric sequence is $\log 109$.
- (ii) An arithmetic sequence has the first term $\log 45$ and common difference $\log k$. (6 marks)
- Show that the fifth term can be written as $\log(45k^4)$.
 - Given that the fifth term is 8, find the exact value of k .

Solution of Q4 (12 marks) – (8 mins)

Q4(i)(a)

For having a sum of infinity, the common ratio $|r| < 1$ should be smaller than 1.

$$-1 < r < 1$$

$$-1 < \log h < 1$$

$$0.1 < h < 10$$

Q4(i)(b)

$$S = \frac{a}{1-r}$$

$$\frac{\log 45}{1-r} = \log 109$$

$$r = 0.1886$$

$$\log h = 0.1886$$

$$h = 1.544$$

Q4(ii)(a)

$$a_1 = \log 45$$

$$a_n = \log 45 + (n-1)d$$

$$a_5 = \log 45 + (5-1)d = \log 45 + 4d = \log 45 + 4\log k = \log 45 + \log k^4 = \log(45k^4)$$

Q4(ii)(b)

$$a_5 = 8$$

$$\log(45k^4) = 8$$

$$45k^4 = 100,000,000$$

$$k = 38.61$$

Q5 (8 marks)

Let $U = \{x \in \mathbb{N} \mid x^2 - 9x - 10 < 0\}$, $A = \{0, 1, 2, 3, 4, 5\}$, $B = \{2, 4, 6, 8, 10\}$

- What is $A \cap B$? (2 marks)
- What is $U - A$? (2 marks)
- What is $U \cap B$? (2 marks)
- What is $U \cap (A \cup B)$? (2 marks)

Solution of Q5 (8 marks)

$$U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

- (i) $A \cap B = \{2, 4\}$
- (ii) $U - A = \{6, 7, 8, 9\}$
- (iii) $U \cap B = \{2, 4, 6, 8\}$
- (iv) $U \cap (A \cup B) = \{0, 1, 2, 3, 4, 5, 6, 8\}$

Q6 (12 marks)

A company has 10 engineers, 8 programmers, and 5 designers.

- (i) A working group needs to be formed with 2 engineers, 2 programmers, and 1 designer. In how many ways can it be formed? (4 marks)
- (ii) A working group needs to be formed with 3 employees, with no restriction on occupation type. In how many ways can it be formed? (4 marks)
- (iii) A working group needs to be formed with 5 employees. At least 3 engineers are needed. In how many ways can it be formed? (4 marks)

Solution of Q6 (12 marks)

(i) ${}_{10}C_2 * {}_8C_2 * {}_5C_1 = 6300$

(ii) ${}_{23}C_3 = 1771$

or

$${}_{10}C_3 + {}_{10}C_2 * ({}_8C_1 + {}_5C_1) + {}_{10}C_1 * ({}_8C_2 + {}_5C_2 + {}_5C_1 * {}_8C_1) + {}_8C_3 + {}_8C_2 * {}_5C_1 + {}_8C_1 * {}_5C_2 + {}_5C_3 \\ = 120 + 585 + 780 + 56 + 140 + 80 + 10 = 1771$$

(iii) ${}_{10}C_3 * {}_{13}C_2 + {}_{10}C_4 * {}_{13}C_1 + {}_{10}C_5 = 9360 + 2730 + 252 = 12342$

Q7 (8 marks)

- (i) Find the first 3 terms of $(2 - 5x)^5$, in ascending powers of x , giving each term in its simplest form. (4 marks)
- (ii) The coefficient of x^2 in the expansion of $(1 + 3x)^n$ is 9. Given that n is a positive integer, find the value of n . (4 marks)

Solution of Q7 (8 marks)

(i) $32 - 400x + 2000x^2$

(ii) ${}_nC_2 (3x)^2 = 9x^2 \rightarrow n = 2$ or -1 ; positive solution is required, $n = 2$

Q8 (10 marks)

A class has 58 students. Each of the student masters at least one skill. Among them, 36 have Skill A, 28 have Skill B, 20 have skill C, and 12 have Skill A and B simultaneously. Several Employers are calling for part-time employees from the class. The details are as follows.

Employer	Work time	Requirement	Reward
Museum	April 6	Skill A	\$100
Library	April 7	Skill B	\$100
Television Station	April 8	Skill C	\$200

Assume that the students will participate all the part-time jobs if they satisfy the requirement. Each job can be participated once. Finally, there are 5 students who have a reward of \$400.

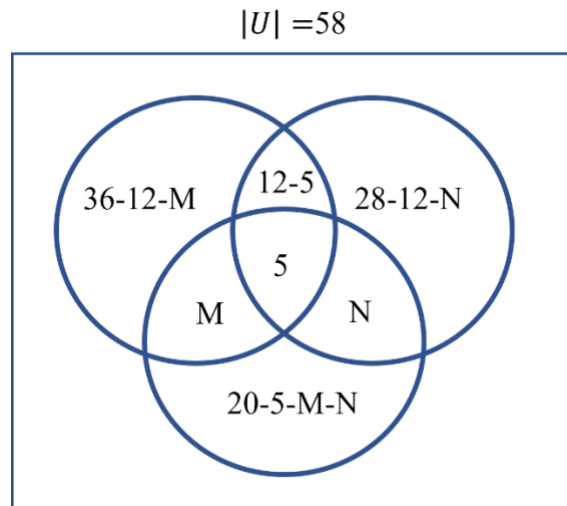
How many students will be rewarded \$300 in total?

Solution of Q8 (10 marks)

A student can only obtain \$300 by working at Museum + Television Station or Library + Television Station. For the former, students have Skill A and C, but lack of Skill B. For the latter, students have Skill B and C, but lack of Skill A.

Let the set of students who master skill A as Set A, the set of students who master skill B as Set B, and the set of students who master skill C as Set C.

The Venn diagram can be obtained as



M represents the number of students who have Skill A and C, but lack of Skill B.

N represents the number of students who have Skill B and C, but lack of Skill A.

Then, by the inclusion-exclusion principle:

$$\begin{aligned}
 |A \cup B \cup C| &= |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C| \\
 &= 36 + 28 + 20 - 12 - (5 + M) - (5 + N) + 5 = 58
 \end{aligned}$$

Then $M+N=9$

So there are 9 students who are rewarded \$300.

Q9 (12 marks)

- (i) Consider that there are 100,000 people gathering at the Time Square on New Year. Then there must be a birthday that at least X people share. What is the value of X?
(4 marks)
- (ii) There are 6 pairs of blue socks, 4 pairs of white socks, and 3 pairs of black socks. How many socks must be taken out to ensure that he has at least two socks in the same color?
(4 marks)
- (iii) Four coins are tossed simultaneously. In how many outcomes at most two coins will turn up as heads?
(4 marks)

Solution of Q9 (12 marks)

- (i) There are 366 possible birthday dates. $\lceil 100000/366 \rceil = 274$. So, X equals to 274.
- (ii) 4
- (iii) If there are no coin heads on: ${}_4C_0 = 1$ outcome
If there are one coin head on: ${}_4C_1 = 4$ outcome
If there are two coin heads on: ${}_4C_2 = 6$ outcome
 $1+4+6=11$.

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