

EE3008 Test 1 Answer

Q1.

$$\begin{array}{ll} \text{a). } B = \frac{1}{\tau} = \frac{1}{1ms} = 1000 \text{ Hz} & \text{b). } B = \frac{1}{\tau} = \frac{1}{2ms} = 500 \text{ Hz} \\ \text{c). } B = \frac{1}{\tau} = \frac{1}{1ms} = 1000 \text{ Hz} & \text{d). } B = \frac{1}{\tau} = \frac{1}{1ms} = 1000 \text{ Hz} \end{array}$$

Q2.

(a). Let $s(f)$ denote the spectrum of $S(t)$. According to

$$\text{Time Shift : } S(t - 1) \Leftrightarrow s(f) \exp(-j2\pi f), \quad (1a)$$

$$\text{Time Scaling : } S(at) \Leftrightarrow \frac{1}{|a|} s\left(\frac{f}{a}\right), \quad (1b)$$

we have the spectrum of $x(t) = S(\frac{1}{2}(t - 1))$, i.e., $X(f) = 2s(2f) \exp(-j4\pi f)$. The amplitude spectrum $|X(f)| = 2s(2f)$. Combining $s(t)$ in Fig 1, we can plot the amplitude spectrum $|X(f)| = 2s(2f)$.

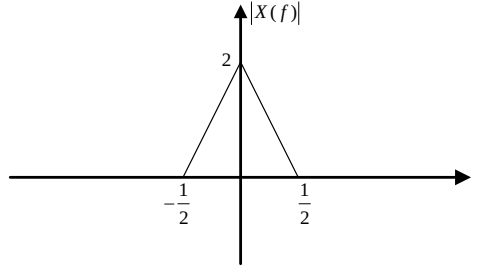


Fig. 1: The amplitude spectrum of $x(t)$ in Q2

(b). Since

$$s(t - 1)\cos(2\pi \times 10^6 \times t) = s(t - 1)\cos(2\pi \times 10^6 \times t - 2\pi \times 10^6) = s(t - 1)\cos(2\pi \times 10^6 \times (t - 1)). \quad (2)$$

According to

$$\text{Time Shift : } s(t - 1) \Leftrightarrow S(f) \exp(-j2\pi f), \quad (3a)$$

$$\text{Modulation 1 : } s(t)\cos(2\pi f_0 t) \Leftrightarrow 0.5[S(f - f_0) + S(f + f_0)], \quad (3b)$$

we have

$$s(t - 1)\cos(2\pi f_0(t - 1)) \Leftrightarrow 0.5[S(f - f_0) \exp(-j2\pi(f - f_0)) + S(f + f_0) \exp(-j2\pi(f + f_0))]. \quad (4)$$

Then the spectrum of $s(t - 1)\cos(2\pi \times 10^6 \times t)$ is

$$0.5[S(f - 10^6) \exp(-j2\pi(f - 10^6)) + S(f + 10^6) \exp(-j2\pi(f + 10^6))]. \quad (5)$$

Q3. (a). The amplitude of the pulse for bipolar is $A = 10^{-6}\text{V}$, thus its average power is $P_1 = \frac{1}{2} \left((10^{-6})^2 + (10^{-6})^2 \right) = 10^{-12}$.

The amplitude of the pulse for 'on' is $A = 2 \times 10^{-6}\text{V}$, thus its average power is $P_2 = \frac{1}{2} \left((2 \times 10^{-6})^2 + 0^2 \right) = 2 \times 10^{-12}$.

(b). $\tau = \frac{1}{R} = 10^{-6}$, and thus the average energies per bit for bipolar is $P_1 \times \tau = 10^{-18}$, and the average energies per bit for 'on-off' is $P_2 \times \tau = 2 \times 10^{-18}$.

Q4. (a). $x(t) = 7\sigma(t)$, (b). infinity

Q5. $R = 2 \times 10^3$ b/s, $\tau = 5 \times 10^{-4}$ s and we have $B = \frac{1}{\tau} = 2000$ Hz. The transmitted spectrum is sketched in Fig. 2

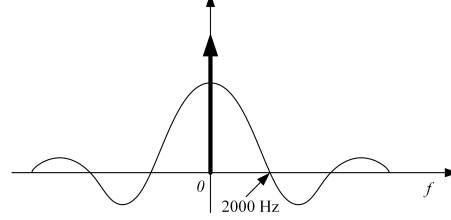


Fig. 2: The transmitted spectrum for Q5

Q6.

(a). The mean is $m = 1$ and the variance is $\sigma^2 = 2$. For a Gaussian distribution, $\Pr(x > x') = Q(\frac{x' - m}{\sigma})$. Let $x' = 0$, we have $\Pr(x > 0) = Q(\frac{0 - 1}{\sqrt{2}}) = Q(\frac{-1}{\sqrt{2}})$.

(b). Since $\sigma^2 = E[(x - E[x])^2] = E[x^2] - (E[x])^2$, the average power of x can be calculated as

$$E[x^2] = (E[x])^2 + \sigma^2 = 1 + 2 = 3. \quad (6)$$

Q7.

(a). Suppose that $x = -1$, if we obtain an incorrect value $x = 1$, we have:

$$\Pr\{-1 \rightarrow 1\} = \Pr(y > 0) = Q\left(\frac{0 - (-1)}{\sigma}\right) = Q\left(\frac{1}{\sigma}\right). \quad (7)$$

Similarly, suppose that $x = 1$, if we obtain an incorrect value $x = -1$, we have

$$\Pr\{1 \rightarrow -1\} = \Pr(y < 0) = 1 - \Pr(y > 0) = 1 - Q\left(\frac{0 - 1}{\sigma}\right) = Q\left(\frac{1}{\sigma}\right). \quad (8)$$

So the error rate is $\frac{1}{2} (\Pr\{-1 \rightarrow 1\} + \Pr\{1 \rightarrow -1\}) = Q\left(\frac{1}{\sigma}\right)$

(b). $\Pr\{y > 0\}$ increases as σ increases.

(c). $\Pr\{y > 0\}$ decreases as σ increases.

Q8.

(a). See Fig. 3.

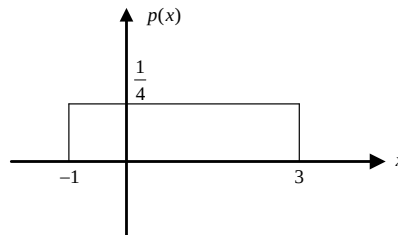


Fig. 3: The PDF of X in Q8

(b). $E[x] = \int_{-\infty}^{\infty} xp(x)dx = \int_{-1}^3 x \cdot \frac{1}{4}dx = \frac{1}{4} \cdot \frac{3^2 - (-1)^2}{2} = 1$ or use $E[x] = \frac{a+b}{2} = 1$, where $a = -1, b = 3$.

(c). $E[x^2] = \int_{-\infty}^{\infty} x^2p(x)dx = \int_{-1}^3 x^2 \cdot \frac{1}{4}dx = \frac{1}{4} \cdot \frac{3^3 - (-1)^3}{3} = \frac{7}{3}$. The variance is $V[x] = E[x^2] - (E[x])^2 = \frac{7}{3} - 1 = \frac{4}{3}$. Or we use $V[x] = \frac{(b-a)^2}{12} = \frac{4}{3}$.