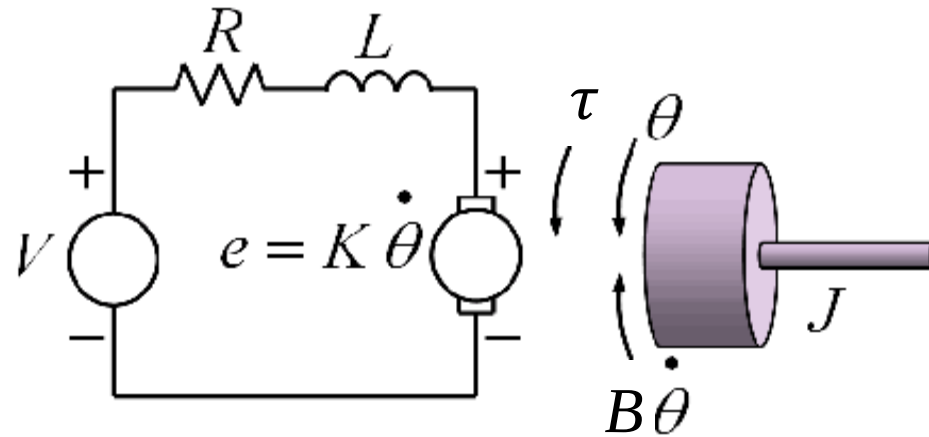


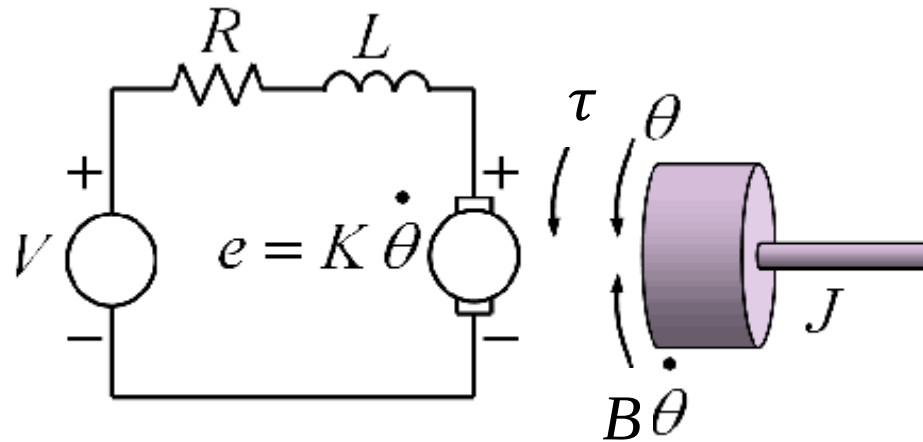
EE3114 Systems and Control

Tutorial - 3

Semester B, 2017-18

DC Motor Control





The DC motor armature circuit is described based on the Kirchhoff voltage law by

$$V(t) = Ri(t) + L \frac{di(t)}{dt} + K\dot{\theta}(t)$$

The mechanical load is described via the torque balance by

$$J\ddot{\theta}(t) = \tau(t) - B\dot{\theta}(t)$$

where

$V(t)$ = Armature Voltage

$i(t)$ = Armature Current

$e(t)$ = Back EMF

$\theta(t)$ = Angular Position

Physical Parameter	Value
Motor constant K	0.01 Nm/A
Rotor inertia J	0.01 kg·m ²
Motor viscous friction constant B	0.1 Nms
Resistance R	1.0 Ω
Inductance L	0.5 H

Motor torque is proportional to the armature current:

$$\tau(t) = K_{\tau} i(t)$$

Back EMF is proportional to the angular velocity of the shaft:

$$e(t) = K \dot{\theta}(t)$$

Assuming that the motor torque and the back EMF constants are equal: $K_{\tau} = K$, the above governing equations become

$$\tau(t) = K i(t)$$

$$e(t) = K \dot{\theta}(t)$$

$$V(t) = R i(t) + L \frac{di(t)}{dt} + K \dot{\theta}(t)$$

$$J \ddot{\theta}(t) = K i(t) - B \dot{\theta}(t)$$

$$\tau(t) = Ki(t)$$

$$e(t) = K\dot{\theta}(t)$$

$$V(t) = Ri(t) + L \frac{di(t)}{dt} + K\dot{\theta}(t)$$

$$J\ddot{\theta}(t) = Ki(t) - B\dot{\theta}(t)$$

(i) Write the Laplace Transforms of the above 4 equations: 5 minutes

$$\tau(t) = Ki(t)$$

$$e(t) = K\dot{\theta}(t)$$

$$V(t) = Ri(t) + L \frac{di(t)}{dt} + K\dot{\theta}(t)$$

$$J\ddot{\theta}(t) = Ki(t) - B\dot{\theta}(t)$$

(i) Write the Laplace Transforms of the above 4 equations:

Solution:

$$T(s) = KI(s)$$

$$E(s) = Ks\Theta(s)$$

$$V(s) = (R + Ls)I(s) + Ks\Theta(s)$$

$$s(Js + B)\Theta(s) = KI(s)$$

(ii) Find the Transfer Function from input $V(s)$ to output $\Theta(s)$: $H(s) = \frac{\Theta(s)}{V(s)} = ?$

10 minutes

(ii) Find the Transfer Function from input $V(s)$ to output $\Theta(s)$:

Solution:

$$I(s) = \frac{V(s) - Ks\Theta(s)}{(R + Ls)}$$

$$s(Js + B)\Theta(s) = K \frac{V(s) - Ks\Theta(s)}{(R + Ls)}$$



$$H(s) = \frac{\Theta(s)}{V(s)} = \frac{K}{s((R + Ls)(Js + B) + K^2)}$$

(iii) Use the following basic building blocks to draw a block diagram for the DC motor from input $V(s)$ to output $\Theta(s)$: **10 minutes**

$$\boxed{\frac{1}{R + Ls}}$$

$$\boxed{\frac{1}{Js + B}}$$

$$\boxed{\frac{1}{s}}$$

$$\boxed{K}$$

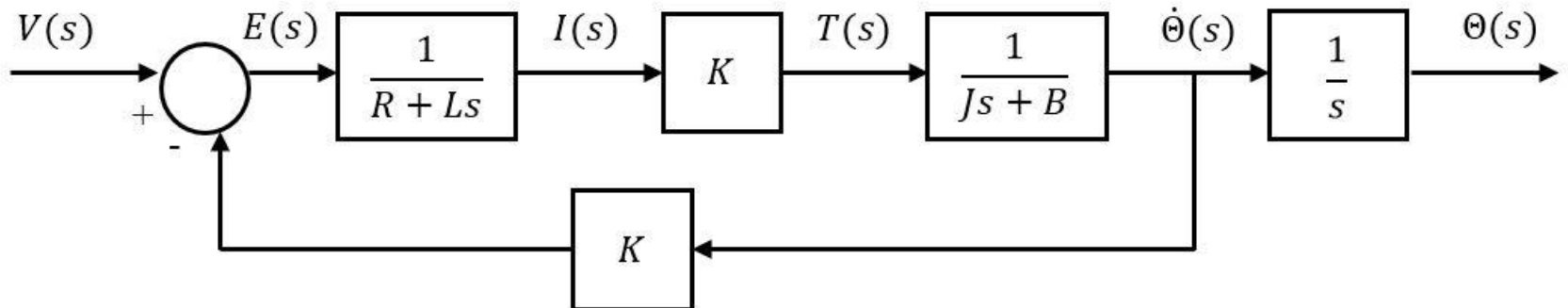
$$I(s) = \frac{V(s) - Ks\Theta(s)}{(R + Ls)}$$

$$s(Js + B)\Theta(s) = K \frac{V(s) - Ks\Theta(s)}{(R + Ls)}$$

$$H(s) = \frac{\Theta(s)}{V(s)} = \frac{K}{s((R + Ls)(Js + B) + K^2)}$$

(iii) Draw a block diagram of the DC motor from input $V(s)$ to output $\Theta(s)$:

Solution: (not unique)



Recall:

$$H(s) = \frac{\Theta(s)}{V(s)} = \frac{K}{s((R + Ls)(Js + B) + K^2)}$$

Physical Parameter	Value
Motor constant K	0.01 Nm/A
Rotor inertia J	0.01 kg·m ²
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Inductance L	0.5 H

(iv) Now, use data →

$$\frac{\Theta(s)}{V(s)} = \frac{0.01}{s((1 + 0.5s)(0.01s + 0.1) + 0.0001)} \cong \frac{2}{s(s^2 + 12s + 20)}$$

and the input is a Unit Impulse function $\delta(t)$

What is the output Unit Impulse Response $h(t)$ =? **10 minutes**

(iv) Input is the Unit Impulse function. The output $h(t) = ?$

Solution:

$$\begin{aligned}\frac{\Theta(s)}{U(s)} &= \frac{2}{s(s^2 + 12s + 20)} = \frac{2}{s(s+2)(s+10)} \\ &= \frac{1}{10s} - \frac{1}{8(s+2)} + \frac{1}{40(s+10)}\end{aligned}$$

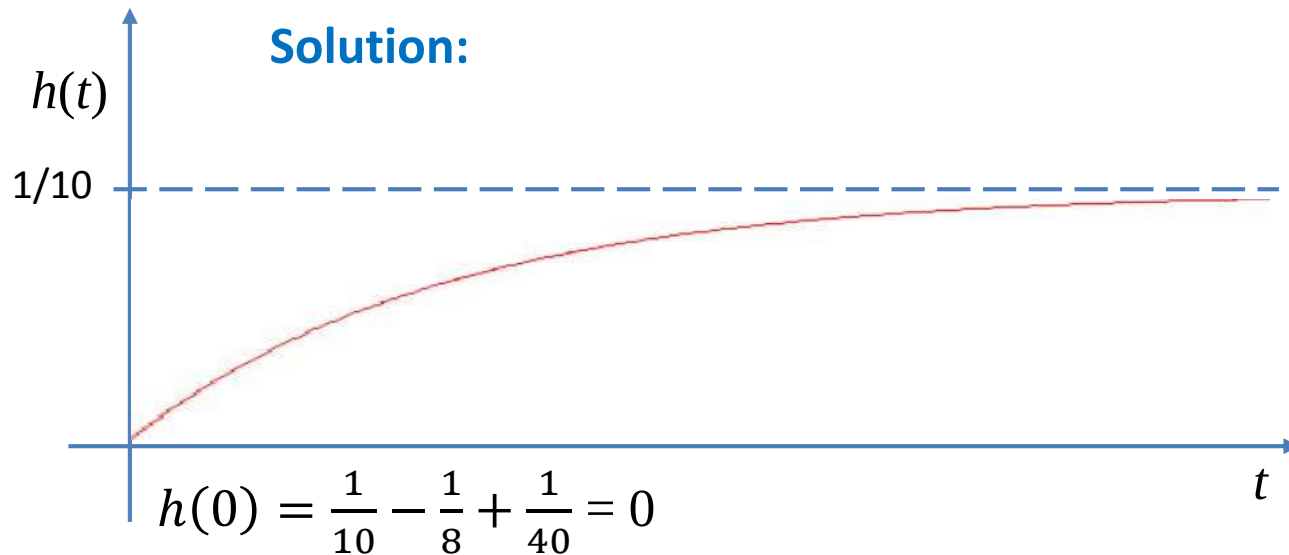
→ $h(t) = \frac{1}{10} - \frac{1}{8}e^{-2t} + \frac{1}{40}e^{-10t}$

(v) Sketch the Unit Impulse Response of the system: **5 minutes**



$$\Rightarrow h(t) = \frac{1}{10} - \frac{1}{8}e^{-2t} + \frac{1}{40}e^{-10t}$$

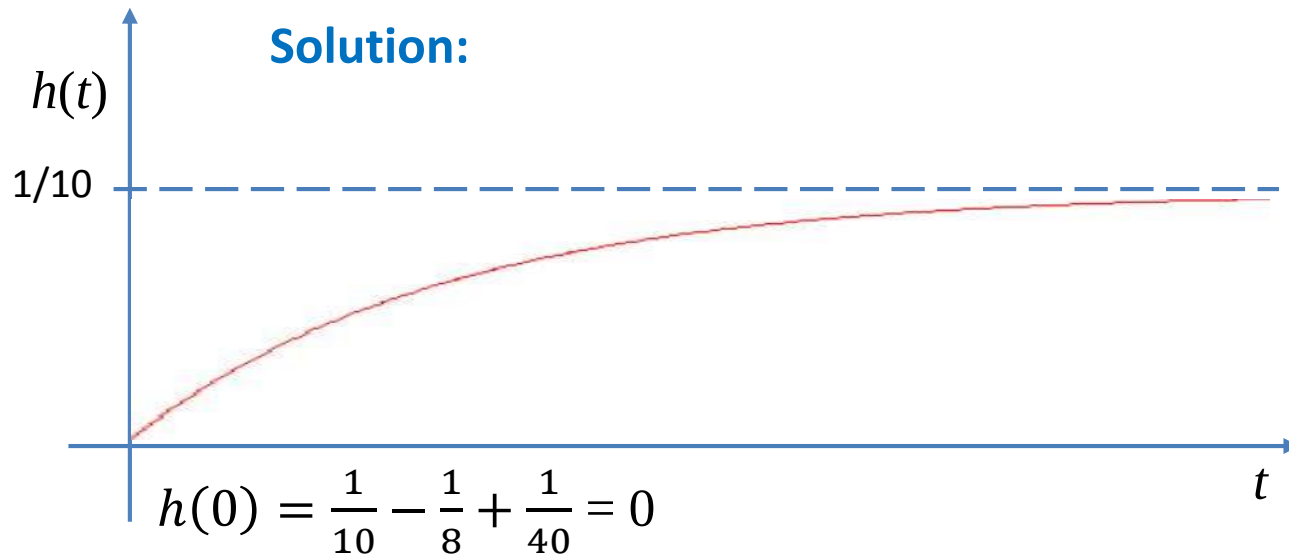
(v) Sketch the Unit Impulse Response of the system:



(vi) Is the system BIBO stable? Why? **5 minutes**

$$\Rightarrow h(t) = \frac{1}{10} - \frac{1}{8}e^{-2t} + \frac{1}{40}e^{-10t}$$

(v) Sketch the Unit Impulse Response of the system:



(vi) Is the system BIBO stable? Why?

Solution:

No, it is only *marginally stable*: its transfer function has a pole on the imaginary axis

$$\frac{2}{s(s+2)(s+10)}$$

END