

GE1359 Algebra
Summer 2022
Mid-term Test

Date: Thursday, 7 July 2022
Time allowed: 90 minutes (14:15 – 15:45)
Submission time: 15 minutes (15:45 – 16:00)

Instructions to candidates:

This paper has FOUR pages (including this cover page).

The formula sheet of trigonometric identities is provided on page 4.

This paper has SEVEN questions.

The questions will begin on page 2.

Answer ALL questions. Show ALL steps to get full mark.

Academic Honesty Pledge

“I pledge that the answers in this examination are my own and that I will not seek or obtain an unfair advantage in producing these answers. Specifically,

- ❖ I will not plagiarize (copy without citation) from any source;
- ❖ I will not communicate or attempt to communicate with any other person during the examination; neither will I give or attempt to give assistance to another student taking the examination; and
- ❖ I will use only approved devices (e.g., calculators) and/or approved device models.
- ❖ I understand that any act of academic dishonesty can lead to disciplinary action.”

Please re-affirm the honesty pledge by writing

**“I pledge to follow the Rules on Academic Honesty and understand that violations may lead to
severe penalties”**

onto the first examination answer sheet.

Please submit your work as a single pdf file to Canvas before 16:00 HK time on 7 July 2022.

Question 1

Find the largest possible domain and largest possible range of the following function. Give your answers in set/interval notations.

$$F(x) = \frac{2}{\sqrt{-x^2 + 4x + 5}}. \quad [16 \text{ marks}]$$

Question 2

Let f , g and h be three real-valued functions defined by

$$f(x) = \frac{x-1}{3-x}, \quad g(x) = \sqrt{x+1} \quad \text{and} \quad h(x) = \frac{1}{x}.$$

For each of the following composite functions, find its expression and largest possible domain. Write the largest possible domains in set/interval notations.

(a) $(h \circ g)(x)$ [5 marks]

(d) $(h \circ f \circ g)(x)$ [10 marks]

Question 3

Consider the real-valued function

$$G(x) = 2x^2 - 12x + 13 \quad \text{for } x \leq 1.$$

(a) Determine whether $G(x)$ is a one-to-one function. Give reasons. [4 marks]

(b) Find $G^{-1}(x)$, the inverse function of $G(x)$, if it exists. [4 marks]

(c) Find the largest possible domain and largest possible range of $G^{-1}(x)$ if it exists. [4 marks]

Question 4

Given that $\sec A = -\frac{\sqrt{10}}{3}$ where $180^\circ < A < 270^\circ$, and $\sin B = -\frac{3}{5}$ where $-90^\circ < B < 0^\circ$.

Without using calculator, find the values of $\sin(2A - B)$ and $\tan\left(\frac{B}{2}\right)$. [18 marks]

Question 5

Find the general solution (in radians) of each of the following equations:

(a) $\csc^2 x = \frac{4}{3}$ [7 marks]

(b) $15 \cos(3x) - 8 \sin(3x) = \frac{17\sqrt{3}}{2}$ [9 marks]

(Hint: In (b), express the left-hand side in the form $R \cos(3x + \theta)$, where $R > 0$ and $0 < \theta < \frac{\pi}{2}$.)

Question 6

Prove that, for any angle α ,

$$\sin \alpha + \sin \left(\alpha + \frac{2\pi}{3} \right) + \sin \left(\alpha + \frac{4\pi}{3} \right) = 0. \quad [6 \text{ marks}]$$

Question 7

Express $\frac{6x^3 - 6x^2 - 3x}{x^4 - x^3 - x + 1}$ into partial fractions. [17 marks]

- End -

Trigonometric identities - Formula sheet

➤ Compound Angle Formulae

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

➤ Double Angle Formulae

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

➤ Half Angle Formulae

$$\cos^2 A = \frac{1}{2}(1 + \cos 2A)$$

$$\sin^2 A = \frac{1}{2}(1 - \cos 2A)$$

➤ Product-to-Sum Formulae

$$\sin A \cos B = \frac{1}{2}[\sin(A + B) + \sin(A - B)]$$

$$\cos A \sin B = \frac{1}{2}[\sin(A + B) - \sin(A - B)]$$

$$\cos A \cos B = \frac{1}{2}[\cos(A + B) + \cos(A - B)]$$

$$\sin A \sin B = -\frac{1}{2}[\cos(A + B) - \cos(A - B)]$$

➤ Sum-to-Product Formulae

$$\sin x + \sin y = 2 \sin\left(\frac{x + y}{2}\right) \cos\left(\frac{x - y}{2}\right)$$

$$\sin x - \sin y = 2 \cos\left(\frac{x + y}{2}\right) \sin\left(\frac{x - y}{2}\right)$$

$$\cos x + \cos y = 2 \cos\left(\frac{x + y}{2}\right) \cos\left(\frac{x - y}{2}\right)$$

$$\cos x - \cos y = -2 \sin\left(\frac{x + y}{2}\right) \sin\left(\frac{x - y}{2}\right)$$