

HOMEWORK PROBLEMS #5

5-1 Answer the following True or False questions, with a few words or counterexamples to support your answers (to avoid trivial cases, assume that all graph sizes $N \geq 3$):

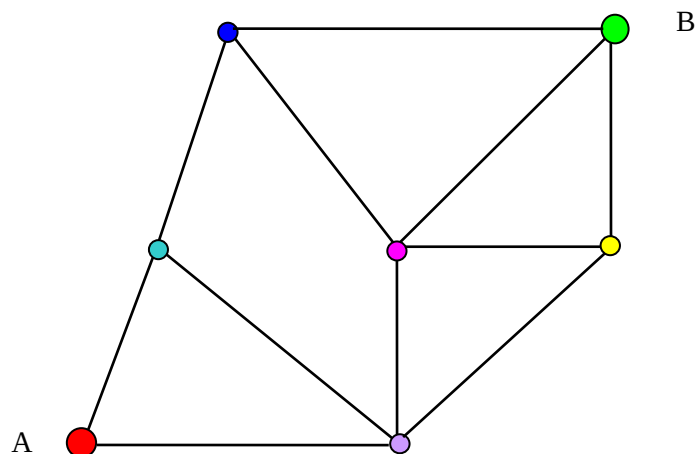
- (i) A directed Eulerian graph may have a Hamiltonian underlying graph, and vice versa.
- (ii) A directed Eulerian graph may not be directed Hamiltonian.
- (iii) A directed Hamiltonian graph may not be directed Eulerian.

5-2 For a given connected graph of size $N \geq 3$, its *connectivity* d_c is defined to be the minimum number of nodes whose removal will disconnect the graph. On the other hand, let d_m be the smallest node degree of the given graph.

- (i) Show a simple example for which $d_c \leq d_m$ is true.
- (ii) Is $d_c \leq d_m$ always true? If so, explain why; if not, show a counterexample.

5-3 For a given fully-connected network with M edges, find a formula for computing the average degree $\langle k \rangle$ of the network. Verify your formula for two special cases: $M = 1$ and $M = 10$.

5-4 Consider the following undirected and unweighted network:



An undirected and unweighted network

- (i) Convert the network to be a weighted network, in such a way that the weight of the edge connecting node i and node j is defined as $w_{ij} = k_i k_j$, where k_i is the degree of node i and k_j is the degree of node j . For every edge in the network, mark its weight nearby.
- (ii) For the weighted network you obtained in part (i), a “distance” between two adjacent nodes may be defined as the reciprocal of the edge weight between them, namely, $d_{ij} = 1/w_{ij}$. Convert all the weights to distances on the above-obtained weighted network.
- (iii) Describe one real-life example which shows that the above-defined “distance” is indeed meaningful.