

EE3114 Systems and Control

Tutorial - 6

Semester B, 2017-18

(i)

$$\mathbf{A} = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 5 & 6 \\ 9 & 8 & 7 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 3 & 0 & 9 \\ 2 & 5 & 8 \\ 1 & 6 & 0 \end{bmatrix}$$

%MatLab code

```
A = [3 2 1; 4 5 6; 9 8 7];
```

```
B = [3 0 9; 2 5 8; 1 6 0];
```

```
C = A*B
```

Find: $\mathbf{C} = \mathbf{AB}$ (3 minutes)

$$\mathbf{A} = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 5 & 6 \\ 9 & 8 & 7 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 3 & 0 & 9 \\ 2 & 5 & 8 \\ 1 & 6 & 0 \end{bmatrix}$$

Solution:

$$\mathbf{C} = \begin{bmatrix} 14 & 16 & 43 \\ 28 & 61 & 76 \\ 50 & 82 & 145 \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -5 & 1 \\ 1 & 5 & 0 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Find: $\mathbf{C} = \mathbf{AB}$ (2 minutes)

$$\mathbf{A} = \begin{bmatrix} 3 & 2 & 1 \\ 4 & 5 & 6 \\ 9 & 8 & 7 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 3 & 0 & 9 \\ 2 & 5 & 8 \\ 1 & 6 & 0 \end{bmatrix}$$

Solution:

$$\mathbf{C} = \begin{bmatrix} 14 & 16 & 43 \\ 28 & 61 & 76 \\ 50 & 82 & 145 \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -5 & 1 \\ 1 & 5 & 0 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Solution:

$$\mathbf{C} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

(ii)

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \text{ Inverse Matrix: } \mathbf{A}^{-1} = \frac{1}{\det(\mathbf{A})} \text{adj}(\mathbf{A})$$

Determinant of $\mathbf{A}$:

$$\det(\mathbf{A}) = |\mathbf{A}| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = +a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

Adjugate of $\mathbf{A}$:

$$\text{adj}(\mathbf{A}) = \begin{pmatrix} + \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & - \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} & + \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \\ - \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} & + \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & - \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} \\ + \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} & - \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} & + \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \end{pmatrix}$$

$$\mathbf{A} = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 9 & -2 \\ 4 & -8 & 11 \end{bmatrix}$$

Find: $\mathbf{A}^{-1} = \frac{1}{\det(\mathbf{A})} \text{adj}(\mathbf{A})$ (10 minutes)

$$\det(\mathbf{A}) = |\mathbf{A}| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = +a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$\text{adj}(\mathbf{A}) = \begin{pmatrix} + \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & - \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} & + \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \\ - \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} & + \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & - \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} \\ + \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} & - \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} & + \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \end{pmatrix}$$

$$\mathbf{A}^{-1} = \frac{1}{53} \begin{bmatrix} 83 & -13 & -25 \\ -19 & 10 & 7 \\ -44 & 12 & 19 \end{bmatrix}$$

%MatLab code

A = [2 -1 3; 1 9 -2; 4 -8 11];

detA = det(A)

invA = inv(A)

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -5 & 1 \\ 1 & 5 & 0 \end{bmatrix}$$

Find: $\mathbf{A}^{-1} = \frac{1}{\det(\mathbf{A})} \text{adj}(\mathbf{A})$ (5 minutes)

Hint: Calculate $\det(\mathbf{A})$ first

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -5 & 1 \\ 1 & 5 & 0 \end{bmatrix} \quad \text{Find: } \mathbf{A}^{-1} = \frac{1}{\det(\mathbf{A})} \text{adj}(\mathbf{A}) \quad (\text{5 minutes})$$

Hint: Calculate $\det(\mathbf{A})$ first

$$\det(\mathbf{A}) = |\mathbf{A}| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = +a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$\text{adj}(\mathbf{A}) = \begin{pmatrix} + \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & - \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} & + \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \\ - \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} & + \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & - \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} \\ + \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} & - \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} & + \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \end{pmatrix}$$

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -5 & 1 \\ 1 & 5 & 0 \end{bmatrix} \quad \text{Find: } \mathbf{A}^{-1} = \frac{1}{\det(\mathbf{A})} \text{adj}(\mathbf{A})$$

Hint: Calculate $\det(\mathbf{A})$ first

$$\det(\mathbf{A}) = |\mathbf{A}| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = +a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$\text{adj}(\mathbf{A}) = \begin{pmatrix} + \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & - \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} & + \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} \\ - \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} & + \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & - \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} \\ + \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} & - \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} & + \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \end{pmatrix}$$

Solution: $\det(\mathbf{A}) = 0 \rightarrow \mathbf{A}^{-1}$ does not exist

(iii)

For an $n \times n$ matrix $\mathbf{A}$, $\det(\lambda \mathbf{I} - \mathbf{A}) = 0$ is called the *characteristic equation* of matrix $\mathbf{A}$, and every root λ of the characteristic equation is called an *eigenvalue* of matrix $\mathbf{A}$.

Equivalently, a number λ is an *eigenvalue* of $\mathbf{A}$ if and only if there is a nonzero n -vector $\mathbf{X}$ such that $\mathbf{AX} = \lambda \mathbf{X}$, and $\mathbf{A}$ is called the *eigenvector* associated with λ .

Note that λ could be real or complex, depending on the matrix $\mathbf{A}$.

%MatLab code

```
A = [ x x x ; x x x ; x x x ];
```

```
Lambda = eig(A)
```

%MatLab code

```
A = [ x x x ; x x x ; x x x ];
```

```
[V, D] = eig(A)
```

%V has eigenvectors

%D has eigenvalues in diagonal

(iii)

For an $n \times n$ matrix $\mathbf{A}$, $\det(\lambda \mathbf{I} - \mathbf{A}) = 0$ is called the *characteristic equation* of matrix $\mathbf{A}$, and every root λ of the characteristic equation is called an *eigenvalue* of matrix $\mathbf{A}$.

Equivalently, a number λ is an *eigenvalue* of $\mathbf{A}$ if and only if there is a nonzero n -vector $\mathbf{X}$ such that $\mathbf{A}\mathbf{X} = \lambda\mathbf{X}$, and $\mathbf{A}$ is called the *eigenvector* associated with λ .

Note that λ could be real or complex, depending on the matrix $\mathbf{A}$.

Find: The *characteristic equation and eigenvalues* of matrix

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} \quad (10 \text{ minutes})$$

(iii)

For an $n \times n$ matrix $\mathbf{A}$, $\det(\lambda \mathbf{I} - \mathbf{A}) = 0$ is called the *characteristic equation* of matrix $\mathbf{A}$, and every root λ of the characteristic equation is called an *eigenvalue* of matrix $\mathbf{A}$.

Equivalently, a number λ is an *eigenvalue* of $\mathbf{A}$ if and only if there is a nonzero n -vector $\mathbf{X}$ such that $\mathbf{A}\mathbf{X} = \lambda\mathbf{X}$, and $\mathbf{A}$ is called the *eigenvector* associated with λ .

Find: The *characteristic equation and eigenvalues* of matrix

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$$

Solutions:

$$\lambda^2 - 2\lambda + 3 = 0$$

$$\lambda_1 = 1 + j\frac{\sqrt{8}}{2}$$

$$\lambda_2 = 1 - j\frac{\sqrt{8}}{2}$$

(iii)

For an $n \times n$ matrix $\mathbf{A}$, $\det(\lambda \mathbf{I} - \mathbf{A}) = 0$ is called the *characteristic equation* of matrix $\mathbf{A}$, and every root λ of the characteristic equation is called an *eigenvalue* of matrix $\mathbf{A}$.

Equivalently, a number λ is an *eigenvalue* of $\mathbf{A}$ if and only if there is a nonzero n -vector $\mathbf{X}$ such that $\mathbf{AX} = \lambda \mathbf{X}$, and $\mathbf{A}$ is called the *eigenvector* associated with λ .

Find: The *characteristic equation and eigenvalues* of matrix

$$\mathbf{A} = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

(10 minutes)

%MatLab code

A = [4 0 1; -2 1 0; -2 0 1];

p = poly(A)

r = roots(p)

(iii)

For an $n \times n$ matrix $\mathbf{A}$, $\det(\lambda \mathbf{I} - \mathbf{A}) = 0$ is called the *characteristic equation* of matrix $\mathbf{A}$, and every root λ of the characteristic equation is called an *eigenvalue* of matrix $\mathbf{A}$.

Equivalently, a number λ is an *eigenvalue* of $\mathbf{A}$ if and only if there is a nonzero n -vector $\mathbf{X}$ such that $\mathbf{AX} = \lambda \mathbf{X}$, and $\mathbf{A}$ is called the *eigenvector* associated with λ .

Find: The *characteristic equation and eigenvalues* of matrix

$$\mathbf{A} = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

Solutions:

$$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$$

$$\lambda = 1 \quad \lambda = 2 \quad \lambda = 3$$

END