

Block B: AC circuits**Q1. Problem P4.49**

For the circuit shown in Figure P4.49, the current $i_s(t) = I_0 \cos(\omega t + 30^\circ)$, $I_0 = 13\text{mA}$, $\omega = 1000 \frac{\text{rad}}{\text{s}}$, and the value of the capacitance C is $0.5\mu\text{F}$.

Find:

- The phasor notation for the source current.
- The impedance of the capacitor.
- The voltage across the capacitor, showing all the passages and using phasor notation only.

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Figure P4.49

Solution:

- a) Phasor notation:

$$\mathbf{I}_s = I_0 \angle \phi = 13 \angle 30^\circ \text{ mA}$$

$$\text{b) } Z_C = -jX_C = -j \frac{1}{\omega C} = -j \frac{1}{\left(1000 \frac{\text{rad}}{\text{s}}\right)(0.5 \mu\text{F})} = 0 - j2 \text{ k}\Omega = 2 \angle -90^\circ \text{ k}\Omega$$

- c)

$$\mathbf{V}_C = \mathbf{I}_s \cdot Z_C = (13 \angle 30^\circ \text{ mA})(2 \angle -90^\circ \text{ k}\Omega) = 26 \angle -60^\circ \text{ V}$$

$$v_C(t) = 26 \cos(1000t - 60^\circ) \text{ V}$$

Note that conversion from phasor notation to time notation or vice versa can be done at any time.

Q2. Problem P4.50*(modified)

- a) For the circuit shown in Figure P4.50, find the current $i_3(t)$. Given that

$$i_1(t) = 141.4 \sin(\omega t + 135^\circ) \text{ mA},$$

$$i_2(t) = 50 \cos(\omega t + 53.13^\circ) \text{ mA}, \quad \omega = 377 \frac{\text{rad}}{\text{s}}.$$

- b) If Z_1 is an resistor R in series with an inductor L where $R=50\Omega$ and $L=12.5\text{mH}$, determine the phase voltage V_{Z1} across Z_1 , and its corresponding voltage $v_{Z1}(t)$ in time domain.

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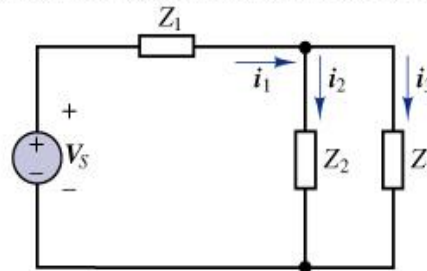


Figure P4.50

Solution:

- a) A solution using trigonometric identities is possible but inefficient, cumbersome, and takes a lot of time. Phasors are better! Note that one current is described with a sine and the other with a cosine function. When using phasors, all currents and voltages must be described with either sine functions or cosine functions. Which does not matter, but it is a good idea to adopt one and use it consistently. Therefore, first converts to cosines.

$$\text{KCL: } -i_1(t) + i_2(t) + i_3(t) = 0 \Rightarrow +i_3(t) = i_1(t) - i_2(t)$$

$$\begin{aligned} i_3(t) &= 141.4 \sin(\omega t + 135^\circ) \text{ mA} - 50 \cos(\omega t + 53.13^\circ) \text{ mA} \\ &= 141.4 \cos(\omega t + 135^\circ - 90^\circ) \text{ mA} - 50 \cos(\omega t + 53.13^\circ) \text{ mA} \end{aligned}$$

$$\begin{aligned} \mathbf{I}_3 &= 141.4 \text{ mA} \angle 45^\circ - 50 \text{ mA} \angle 53.13^\circ \\ &= (99.98 + j \cdot 99.98) \text{ mA} - (30.00 + j \cdot 40.00) \text{ mA} \\ &= (69.98 + j \cdot 59.98) \text{ mA} = 92.17 \angle 40.60^\circ \text{ mA} \end{aligned}$$

$$i_3(t) = 92.17 \cos(\omega t + 40.60^\circ) \text{ mA}$$

If sine functions were used, the result in phasor notation would differ in phase by 90 degrees.

- b) $Z_1 = R + j\omega L = 50 + j(377)(12.5 \times 10^{-3}) = 50 + j 4.71 \Omega$

Therefore

$$\mathbf{V}_{Z1} = \mathbf{I}_3 * \mathbf{Z}_1 = (141.4 \angle 45^\circ) * (50 \angle 5.38^\circ) = 7.1 \text{ V} \angle 50.38^\circ$$

$$\rightarrow v_{Z1}(t) = 7.1 \cos(\omega t + 50.38^\circ) \text{ V}$$

Q3. Problem P4.51* (modified)

For the circuit shown in Figure 5.51, determine the current through Z_3 if

- $V_{S1} = V_{S2} = 170 \cos(377t) \text{ V}$;
- $V_{S1} = 170 \cos(377t) \text{ V}$ and $V_{S2} = 170 \cos(1000t) \text{ V}$;
- Please comment whether a change in the value of Z_1 or Z_2 will affect the answer in part (a) and (b) or not. Justify your answer.

Given that $Z_3 = 17 \angle 11^\circ \Omega$

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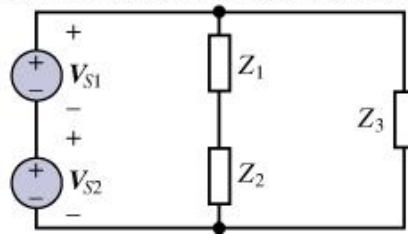


Figure P4.51

Solution:

a)

$$V_{s1} = V_{s2} = 170 \angle 0^\circ \text{ V} = (170 + j0) \text{ V}$$

$$\text{KVL: } -V_{s1} - V_{s2} + I_3 Z_3 = 0$$

$$I_3 = \frac{V_{s1} + V_{s2}}{Z_3} = \frac{170 \angle 0^\circ \text{ V} + 170 \angle 0^\circ \text{ V}}{17 \angle 11^\circ \Omega} = \frac{340 \angle 0^\circ \text{ V}}{17 \angle 11^\circ \Omega} = 20 \angle -11^\circ \text{ A}$$

$$i_3(t) = 20 \cos\left(377 \frac{\text{rad}}{\text{s}} \cdot t - 11^\circ\right) \text{ A}$$

b)

Since the source frequency of V_{S1} and V_{S2} are now different, we cannot just add them together in phasor domain. Instead, we have to apply superposition theorem and determine the effect of each source on Z_3 .

Consider V_{S1} by short circuit V_{S2} ,

$$V_{s1} = 170 \angle 0^\circ \text{ V} = (170 + j0) \text{ V}$$

$$\text{KVL: } -V_{s1} + I_{3a} Z_3 = 0$$

$$I_{3a} = \frac{V_{s1}}{Z_3} = \frac{170 \angle 0^\circ \text{ V}}{17 \angle 11^\circ \Omega} = \frac{170 \angle 0^\circ \text{ V}}{17 \angle 11^\circ \Omega} = 10 \angle -11^\circ \text{ A}$$

$$i_{3a}(t) = 10 \cos\left(377 \frac{\text{rad}}{\text{s}} \cdot t - 11^\circ\right) \text{A}$$

Consider V_{S2} by short circuit V_{S1} ,

$$\mathbf{V}_{s1} = 170 \angle 0^\circ \text{ V} = (170 + j0) \text{ V}$$

$$\text{KVL: } -\mathbf{V}_{s2} + \mathbf{I}_{3b} Z_3 = 0$$

$$\mathbf{I}_{3b} = \frac{\mathbf{V}_{s2}}{Z_3} = \frac{170 \angle 0^\circ \text{ V}}{17 \angle 11^\circ \Omega} = \frac{170 \angle 0^\circ \text{ V}}{17 \angle 11^\circ \Omega} = 10 \angle -11^\circ \text{ A}$$

$$i_{3b}(t) = 10 \cos\left(1000 \frac{\text{rad}}{\text{s}} \cdot t - 11^\circ\right) \text{A}$$

By adding the two current i_{3a} and i_{3b} in time domain, we have

$$i_3(t) = i_{3a}(t) + i_{3b}(t) = 10 \cos\left(377 \frac{\text{rad}}{\text{s}} \cdot t - 11^\circ\right) + 10 \cos\left(1000 \frac{\text{rad}}{\text{s}} \cdot t - 11^\circ\right) \text{A}$$

c) No. It is because Z_3 is connected to voltage sources. As a result, the voltage across Z_3 will be kept constant.

Q4. Problem 4.52

Given that the values of the impedance in the circuit shown in Figure P4.52 are $Z_s = (13000 + j\omega 3)\Omega$, $R = 120\ \Omega$, $L = 19\ \text{mH}$, $C = 220\ \text{pF}$, determine the frequency such that the current I_i and the voltage V_0 are in phase.

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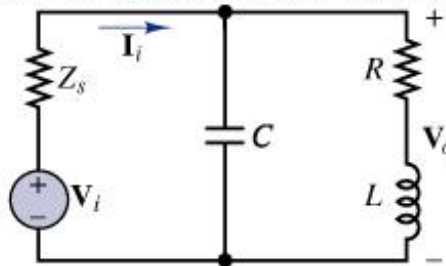


Figure 4.52

Solution:

Z_s is not a factor in this solution. Only R , L , and C will determine if the voltage across this combination is in phase with the current through it. If the voltage and current are in phase, then, the equivalent impedance must have an "imaginary" or reactive part which is zero!

$$Z_{eq} = \frac{V_0}{I_i} = |Z_{eq}| \angle 0^\circ = R_{eq} + jX_{eq}, \quad X_{eq}(\omega) = 0$$

$$\begin{aligned} Z_{eq} &= \frac{(Z_R + Z_L) \cdot Z_C}{Z_R + Z_L + Z_C} = \frac{(R + jX_L) \cdot (-jX_C)}{R + jX_L - jX_C} = \left(\frac{X_L X_C - jRX_C}{R + j(X_L - X_C)} \right) \left(\frac{R - j(X_L - X_C)}{R - j(X_L - X_C)} \right) = \\ &= \frac{[X_L X_C R - jRX_C(X_L - X_C)] - j[R^2 X_C + X_L X_C(X_L - X_C)]}{R^2 + (X_L - X_C)^2} \end{aligned}$$

At the resonant frequency the reactive component of this impedance must equal zero:

$$X_{eq}(\omega) = \frac{R^2 X_C + X_L X_C (X_L - X_C)}{R^2 + (X_L - X_C)^2} = 0 \Rightarrow R^2 + X_L (X_L - X_C) = 0, \text{ if } X_C \neq 0$$

$$R^2 + \omega L \left(\omega L - \frac{1}{\omega C} \right) = 0 \Rightarrow \omega^2 L^2 = \frac{L}{C} - R^2$$

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = \sqrt{\frac{1}{(19\text{mH})(220\text{pF})} - \frac{(120\Omega)^2}{(19\text{mH})^2}} = \sqrt{23924\text{G}\frac{\text{rad}}{\text{s}} - 39.89\text{M}\frac{\text{rad}}{\text{s}}} = 4891\text{k}\frac{\text{rad}}{\text{s}}$$

Notes:

1. To separate the equivalent impedance into real (resistive) and "imaginary" (reactive) components, the denominator had to be "rationalized". This was done by multiplying numerator and denominator by the complex conjugate of the denominator, and multiplying term by term. Remember that $j^2 = -1$, etc.
2. The term with R had a negligible effect on the resonant frequency in this case. If R is sufficiently large, however, it will significantly affect the answer.

Q5. Problem 4.53

For the circuit shown in Figure P4.53, determine the current supplied by the source. Given that the values of the impedance, $R_s = 50\Omega$, $R_c = 40\Omega$, $L = 20\ \mu\text{H}$, $C = 1.25\ \text{nF}$, and the voltage applied to the circuit shown in Figure P4.53, $v_s(t) = V_0 \cos(\omega t + 0^\circ)$ $V_0 = 10\ \text{V}$, $\omega = 6\ \text{M}\frac{\text{rad}}{\text{s}}$.

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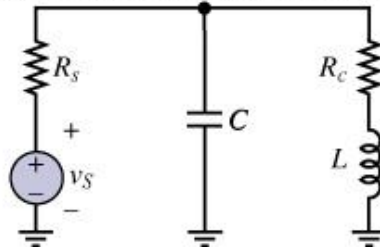


Figure P4.53

Solution:

Assume clockwise currents:

$$X_L = \omega L = \left(6\text{M}\frac{\text{rad}}{\text{s}}\right)(20\ \mu\text{H}) = 120\ \Omega \Rightarrow Z_L = 0 + j120\ \Omega = 120\angle 90^\circ\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{\left(6\text{M}\frac{\text{rad}}{\text{s}}\right)(1.25\ \text{nF})} = 133.3\ \Omega \Rightarrow Z_C = 0 - j133.3\ \Omega = 133.3\angle -90^\circ\ \Omega$$

$$R_C = 40\ \Omega = 40\angle 0^\circ\ \Omega, \quad R_s = 50\ \Omega = 50\angle 0^\circ\ \Omega$$

Equivalent impedances:

$$Z_{eq1} = R_C + Z_L = 40 + j120\ \Omega = 126.5\angle 71.56^\circ\ \Omega$$

$$Z_{eq} = R_s + \frac{R_C \cdot Z_{eq1}}{R_C + Z_{eq1}} = 50 + \frac{(133.3\angle -90^\circ\ \Omega) \cdot (126.5\angle 71.56^\circ\ \Omega)}{133.3\angle -90^\circ\ \Omega + 126.5\angle 71.56^\circ\ \Omega} =$$

$$= 50 + \frac{16.87\angle -18.44^\circ\ \text{k}\Omega^2}{42.161\angle -18.44^\circ\ \Omega} = 50\angle 0^\circ\ \Omega + 400\angle 0^\circ\ \Omega = 450\angle 0^\circ\ \Omega$$

$$\text{OL: } \mathbf{I}_s = \frac{\mathbf{V}_s}{Z_{eq}} = \frac{10\angle 0^\circ\ \text{V}}{450\angle 0^\circ\ \Omega} = 22.22\angle 0^\circ\ \text{mA} \Rightarrow i_s(t) = 22.22\cos(\omega t + 0^\circ)\ \text{mA}$$

Note:

The equivalent impedance of the parallel combination is purely resistive; therefore, the frequency given is the resonant frequency of this network.

Q6. Problem P4.54* (modified)

- a) Determine the current in the circuit shown in Figure P4.54.

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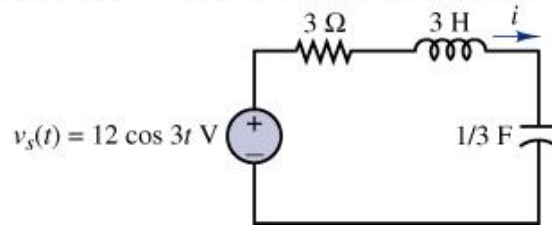


Figure P4.54

- b) If the frequency, ω , of the source is variable, determine at which frequency in rad/s such that the current i is maximum. Under this situation, calculate also the phasor voltage across the 3H inductor and 1/3F capacitor respectively. What can you observe?

Hint: maximum current occurs at which the equivalent impedance seen by the source v_s is resistive. Do you know why? You can use the previous finding to explain it.

Solution:

- a) Assume clockwise currents:

$$\omega = 3 \frac{\text{rad}}{\text{s}}, \quad V_s = 12 \angle 0^\circ \text{ V}$$

$$Z_C = \frac{1}{j\omega C} = -j \Omega, \quad Z_L = j\omega L = j9 \Omega \Rightarrow Z_{total} = 3 + j9 - j = 3 + j8 \Omega$$

$$I = \frac{12}{3 + j8} = 0.4932 - j1.3151 \text{ A} = 1.4045 \angle -69.44^\circ \text{ A}, \quad i(t) = 1.4 \cos(\omega t - 69.4^\circ) \text{ A}$$

- b) Assume the frequency at which the current is maximum is ω_0 .

$$\text{The impedance seen by the source } Z = 3 + j\left(3\omega_0 - \frac{1}{\frac{1}{3}\omega_0}\right) = 3 + j\left(3\omega_0 - \frac{3}{\omega_0}\right)$$

$$\text{For } Z \text{ to be resistive, } \left(3\omega_0 - \frac{3}{\omega_0}\right) = 0 \Rightarrow \omega_0 = 1 \text{ rad/s}$$

$$\text{Under this situation, the current } \mathbf{I} = \frac{12 \angle 0^\circ}{3 \angle 0^\circ} = 4 \angle 0^\circ \text{ A}$$

$$\text{Hence, voltage across the inductor } V_L = 0.5 * I * X_L = j0.5 * 3 * 4 = j6 \text{ V}$$

$$\text{voltage across the capacitor } V_C = 0.5 * I * X_C = -j0.5 * 3 * 4 = -j6 \text{ V}$$

They have the same magnitude but opposite in phase. As a result, all the source voltage will apply only on the resistor, resulting in maximum current flow.

Q7. Problem P4.57* (modified)

Determine the voltage V_2 of the circuit shown in Figure P4.57. It is given that the frequency of the source is 2 rad/s. Calculate also the current through the 6H inductor and its power dissipation. What is the physical meaning if you find the answer (power dissipation) is a purely imaginary value?

Noted: the source voltage is in rms.

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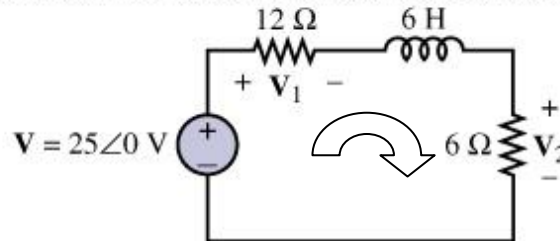


Figure P4.57

Solution:

Specifying the positive directions as in figure P4.57:

$$Z_L = j\omega L = j12\Omega$$

$$V_2 = \frac{R_{6\Omega}}{R_{12\Omega} + Z_L + R_{6\Omega}} V = \frac{6\Omega}{(12 + j12 + 6)\Omega} 25\angle 0^\circ = \frac{150\angle 0^\circ \Omega}{18 + j12\Omega} = 6.93\angle -33.7^\circ \text{ V}$$

The current through the 6H inductor, I_L is given by

$$I_L = \frac{V}{R_{12\Omega} + Z_L + R_{6\Omega}} = \frac{25\angle 0^\circ}{(12 + j12 + 6)} = 1.156\angle -33.7^\circ \text{ V}$$

The corresponding power dissipation by the 6H inductor, Q , is given by

$$\begin{aligned} Q &= I_L * I_L^* * Z_L \quad (\text{because source voltage in rms}) \\ &= (1.156\angle -33.7)(1.156\angle -33.7)^* (j12) \\ &= (1.156\angle -33.7)(1.156\angle +33.7)(j12) \\ &= j16\text{VAr} \end{aligned}$$

The complex j indicates the average energy which is stored by the inductor. It should be noted that a pure inductor does not dissipate energy as heat. The unit VAr is used to differentiate the different in nature to the energy dissipated by resistor. In the later case, we use W to represent it. Please refer to chapter 7 for more concept about complex power

Q8. Problem P4.66

- a) Find the equivalent impedance Z_L shown in Figure P4.66(a), as seen by the source, if the frequency is 377 rad/s
- b) If we want the source to see the load as completely resistive, what value of capacitance should we place between the terminal a and b as shown in Figure P4.66(b)?
- Hint: Find an expression for the equivalent impedance Z , and then find C so that the phase angle of the impedance is zero.
- c) What is the actual impedance that the source sees with the capacitor included in the circuit?

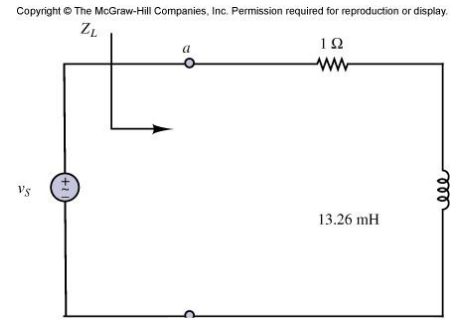


Figure 4.66 (a)

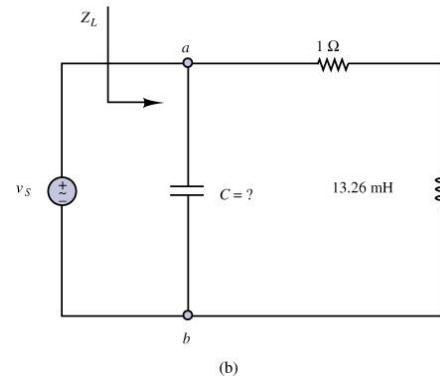


Figure 4.66 (b)

Solution:

a) $Z_L = R + jX_L = 1 + j(377 \times 1326 \times 10^{-3}) = 1 + j5$

b) $Z_L = Z_L || Z_C$, $Z_C = \frac{1}{j\omega C} = \frac{1}{j377C}$

such that $Z_L = \frac{\frac{1}{j377C} (1+j5)}{\frac{1}{j377C} + (1+j5)} = \frac{(1+j5)}{1+j377C(1+j5)} = \frac{1+j5}{1-1885C+j377C}$

The impedance angle is: $\angle Z_L' = \arctan\left(\frac{5}{1}\right) - \arctan\left(\frac{377C}{1-1885C}\right)$

In order to have a strictly real impedance we set $\angle Z_L' = 0$, or:

$\left(\frac{5}{1}\right) = \left(\frac{377C}{1-1885C}\right)$, which leads to: $5 = (377 + 9425)C$ or $C = \frac{5}{9802} = 5101 \mu F$

c) $Z_L = \frac{\frac{1}{j377C} (1+j5)}{\frac{1}{j377C} + (1+j5)} = \frac{\frac{1}{j0.1923} (1+j5)}{\frac{1}{j0.1923} + (1+j5)} = 26$

Q9. Problem P6.1**Find:**

- The frequency response for the circuit of Figure P6.1.
- Plot magnitude and phase of the circuit using a linear scale for frequency.
- Repeat part b., using semilog paper.
- Plot the magnitude response using semilog paper with magnitude in dB.

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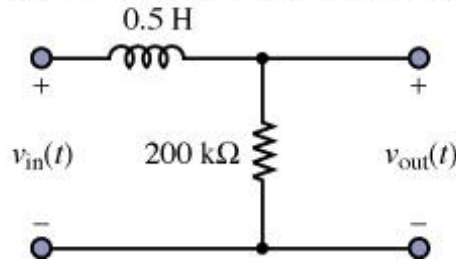


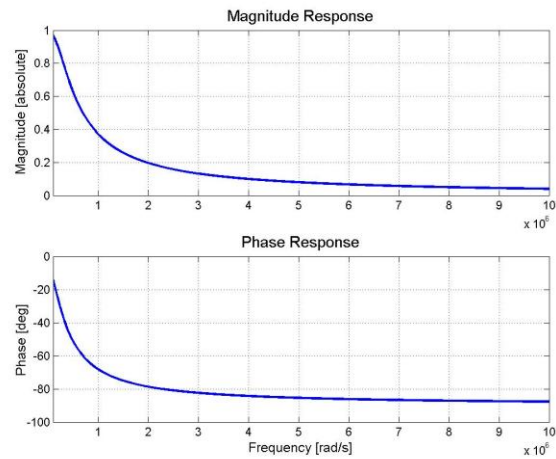
Figure P6.1

Solution:

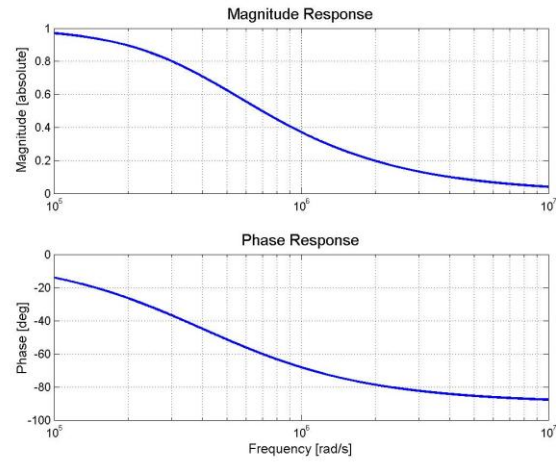
$$\begin{aligned}
 \frac{v_{out}}{v_{in}}(j\omega) &= \frac{R}{R + j\omega L} = \frac{1}{1 + j\omega L / R} = \frac{1}{1 + j(2.5 \times 10^{-6} \omega)} \\
 \text{a) } &= \left(\frac{1}{1 + j(2.5 \times 10^{-6} \omega)} \right) \left(\frac{1 - j(2.5 \times 10^{-6} \omega)}{1 - j(2.5 \times 10^{-6} \omega)} \right) \\
 &= \frac{1 - j(2.5 \times 10^{-6} \omega)}{1 + (2.5 \times 10^{-6} \omega)^2} = \frac{1}{1 + (2.5 \times 10^{-6} \omega)^2} - j \frac{(2.5 \times 10^{-6} \omega)}{1 + (2.5 \times 10^{-6} \omega)^2} \\
 \left| \frac{V_{out}}{V_{in}} \right| &= \sqrt{\left(\frac{1}{1 + (2.5 \times 10^{-6} \omega)^2} \right)^2 + \left(\frac{(2.5 \times 10^{-6} \omega)}{1 + (2.5 \times 10^{-6} \omega)^2} \right)^2} = \frac{1}{\sqrt{1 + 6.25 \times 10^{-12} \omega^2}} \\
 \phi(\omega) &= -\arctan \left(\frac{\frac{(2.5 \times 10^{-6} \omega)}{1 + (2.5 \times 10^{-6} \omega)^2}}{\frac{1}{1 + (2.5 \times 10^{-6} \omega)^2}} \right) = -\arctan(2.5 \times 10^{-6} \omega)
 \end{aligned}$$

The plots obtained using Matlab are shown below:

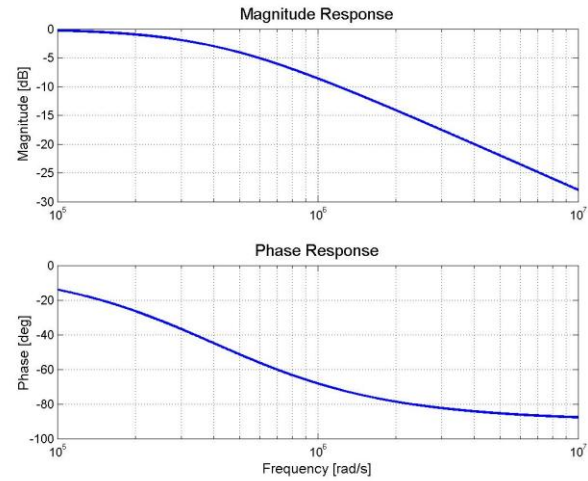
b)



c)



d)



Q10. Problem P6.3**Find:**

- The frequency response for the circuit of Figure P6.3.
- Plot magnitude and phase of the circuit using a linear scale for frequency.
- Repeat part b., using semilog paper.
- Plot the magnitude response using semilog paper with magnitude in dB.

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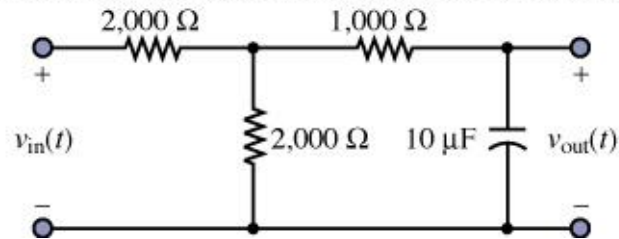


Figure P6.3

Solution:

First, we find the Thévenin equivalent circuit seen by the capacitor:

$$R_T = 2000 \parallel 2000 + 1000 = 2000 \, \Omega$$

and

$$v_{OC} = \frac{2000}{2000 + 2000} v_{in} = \frac{v_{in}}{2}$$

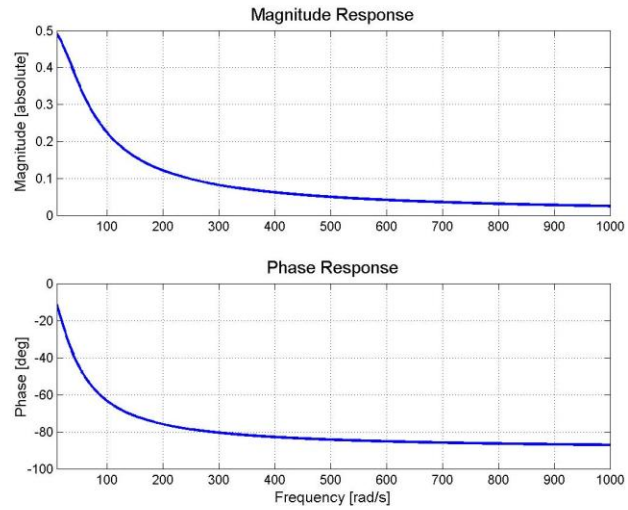
$$a) \quad \frac{v_{out}}{v_{OC}} = \frac{1/j\omega C}{R_T + 1/j\omega C} = \frac{1}{1 + j\omega R_T C} = \frac{1}{1 + j(0.02\omega)}$$

$$\left| \frac{v_{out}}{v_{OC}} \right| = \frac{1}{\sqrt{1 + 4 \times 10^{-4} \omega^2}}$$

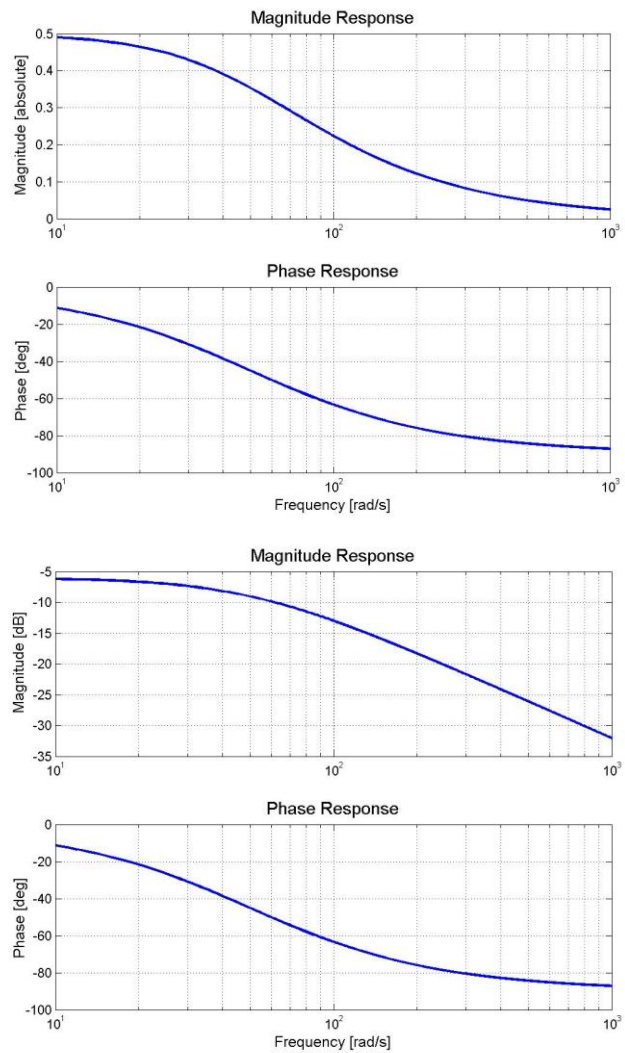
$$\left| \frac{v_{out}}{v_{in}} \right| = \frac{1}{2} \left| \frac{v_{out}}{v_{OC}} \right| = \frac{0.5}{\sqrt{1 + 4 \times 10^{-4} \omega^2}}$$

$$\phi(\omega) = -\arctan(0.02\omega)$$

b) The plots obtained using Matlab are shown below:



c)



Q11. Problem P6.4**Given:** $R_1=500\Omega$, $R_2=1000\Omega$, $L=2H$, $C=100\mu F$ **Find:**

- The frequency response for the circuit of Figure P6.4.
- Plot magnitude and phase of the circuit using a linear scale for frequency.
- Repeat part b., using semilog paper.
- Plot the magnitude response using semilog paper with magnitude in dB.

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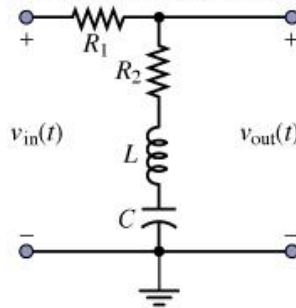
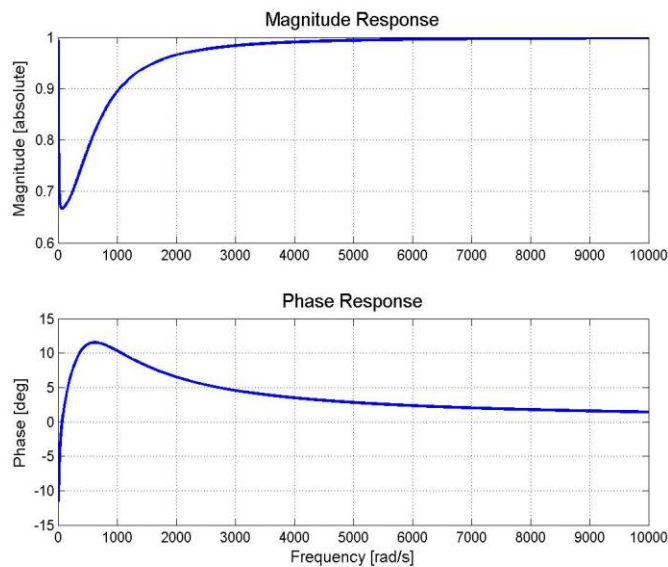


Figure P6.4

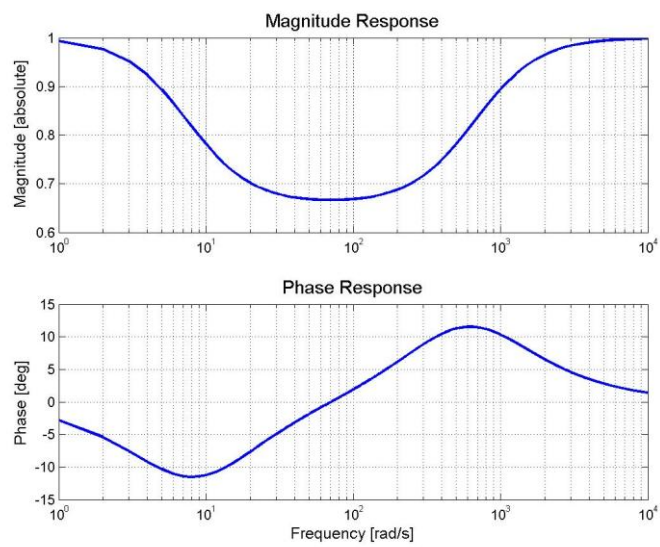
Solution:

$$a) \frac{V_{out}(j\omega)}{V_{in}} = \frac{R_2 + j\omega L + \frac{1}{j\omega C}}{R_1 + R_2 + j\omega L + \frac{1}{j\omega C}} = \frac{1 + j\omega CR_2 + (j\omega)^2 LC}{1 + j\omega C(R_1 + R_2) + (j\omega)^2 LC} = \frac{1 - 0.0002\omega^2 + j(0.1)\omega}{1 - 0.0002\omega^2 + j(0.15)\omega}$$

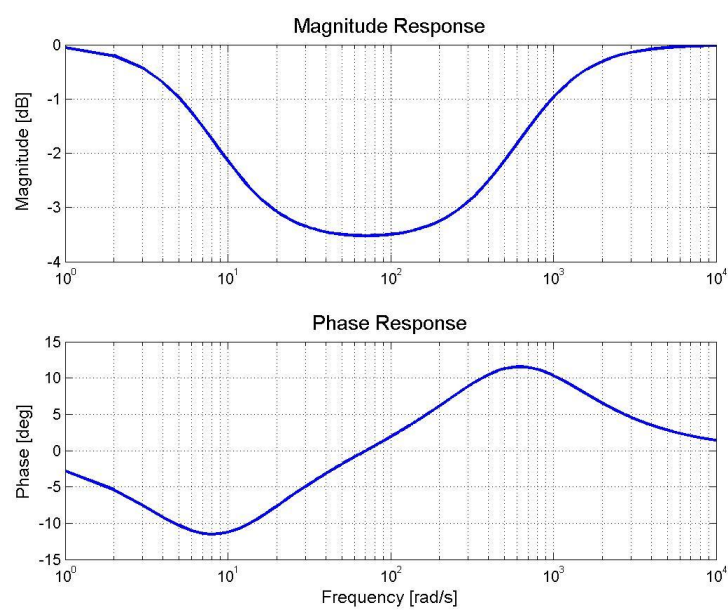
- The plots obtained using Matlab are shown below:



c)



d)



Q12. Problem 6.12

Compute and plot the frequency response function for the circuit shown in Figure 6.12 using semilog in frequency with magnitude in dB (Bode Plot).

Given that $R_1 = 100 \, \Omega$; $R_L = 100 \, \Omega$; $R_2 = 50 \, \Omega$; $C = 80 \, \text{nF}$

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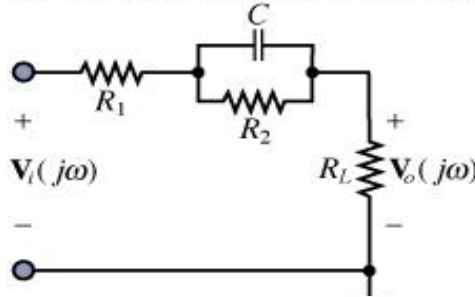


Figure P6.12

Solution:

Using voltage division:

$$Z_{eq} = \frac{Z_{R2} Z_C}{Z_{R2} + Z_C} = \frac{R_2 \frac{1}{j\omega C}}{R_2 + \frac{1}{j\omega C}} \cdot \frac{j\omega C}{j\omega C} = \frac{R_2}{1 + j\omega R_2 C}$$

$$\begin{aligned} VD: H_v[j\omega] &= \frac{V_o[j\omega]}{V_i[j\omega]} = \frac{Z_{RL}}{Z_{R1} + Z_{eq} + Z_{RL}} = \frac{R_L}{R_1 + \frac{R_2}{1 + j\omega R_2 C} + R_L} \cdot \frac{1 + j\omega R_2 C}{1 + j\omega R_2 C} = \\ &= \frac{R_L [1 + j\omega R_2 C]}{R_1 + R_2 + R_L + j[R_1 + R_L]\omega R_2 C} = \frac{R_L}{R_1 + R_2 + R_L} \cdot \frac{1 + j\omega R_2 C}{1 + j \frac{[R_1 + R_L]\omega R_2 C}{R_1 + R_2 + R_L}} \end{aligned}$$

Plotting the response in a Bode Plot:

