

Name: _____

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City University of Hong Kong
Department of Electrical Engineering

EE 3121 Differential Equations for Electrical Engineering

Midterm Test 2

NOTE:

1. This is a 1.5-hour, open book, open notes midterm test.
2. There are 4 problems altogether, which amount to 100 points.
3. In addition to final answers, you need to include necessary steps to show your derivations. An answer without any supporting development will unlikely be awarded with a point. You do receive partial credits if you write down the steps despite that you may not complete your solution.
4. Please sign the following statement of pledge of honor. Enjoy the test, and good luck!

I pledge that the answers in this exam/quiz are my own and that I will not seek or obtain an unfair advantage in producing these answers. Specifically,

- *I will not plagiarize (copy without citation) from any source;*
- *I will not communicate or attempt to communicate with any other person during the exam/quiz; neither will I give or attempt to give assistance to another student taking the exam/quiz; and*
- *I will use only approved devices (e.g., calculators) and/or approved device models.*

I understand that any act of academic dishonesty can lead to disciplinary action.

Signature: _____

Question 1 (30 marks) For the following differential equation, assume that the initial conditions are zero:

$$\frac{d^2 y}{dt^2} + 5 \frac{dy}{dt} + 6y = e^{-t}.$$

- (1) Determine the Wronskian $W(\tau)$.
- (2) Find the convolution kernel $K(t - \tau)$.
- (3) Find the solution to the differential equation.

Question 2 (25 marks) Use the Laplace transform to solve the differential equation

$$\frac{d^2 y}{dt^2} + 7 \frac{dy}{dt} + 12y = 1, \quad y(0) = -2, \quad \dot{y}(0) = 1.$$

Question 3 (30 marks) Consider the homogeneous differential equations given in the state space form

$$\dot{x} = \begin{bmatrix} 0 & 0 \\ t & 0 \end{bmatrix} x, \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}.$$

- (1) Find a fundamental matrix for this system.
- (2) Find the state transition matrix $\Phi(t, t_0)$.
- (3) Let $x_1(0) = 0$, $x_2(0) = 1$. Determine the solution to the state space equation.

Question 4 (15 marks) For the system shown in the following figure, obtain the state space description with the state variables as marked in the circuit (i.e., the inductor current and capacitor voltage).

