

EE3101 Tutorial 4 Solution

(1)

(a) The average signal level for binary “1” at the integrator O/P ($t=T_b$) is:

$$\begin{aligned} \int_0^{T_b} B \frac{At}{T_b} dt &= \frac{BA}{T_b} \frac{t^2}{2} \Big|_0^{T_b} \\ &= \frac{BAT_b}{2} \quad \{\text{Note: the average value of the noise component} = 0\} \end{aligned}$$

Similarly, the average signal level for binary “0” at the integrator O/P is $-\frac{BAT_b}{2}$

The threshold is at 0, the distance between the mean signal level to threshold is $\frac{BAT_b}{2}$

The noise power at the integrator O/P is $\frac{N_0}{2} \int_0^{T_b} B^2 dt = \frac{N_0 B^2 T_b}{2}$

The average prob of bit error is:

$$P_B = Q[\sqrt{(BAT_b/2)^2 / (N_0 B^2 T_b / 2)}] \frac{1}{2} + Q[\sqrt{(BAT_b/2)^2 / (N_0 B^2 T_b / 2)}] \frac{1}{2}$$

$$P_B = Q\left[\sqrt{\frac{A^2 T_b}{2N_0}}\right]$$

In section (8) of the chapter 1 supplement materials, we have $E_b = \frac{A^2 T_b}{3}$

$$\text{i.e. } P_B = Q\left[\sqrt{\frac{3E_b}{2N_0}}\right]$$

(b) If B is a non-zero constant, then it is not an optimum receiver. According to Section 1.6 of the lecture notes, the optimum reference signal should equal to the received signal $s(t)$, or $k s(t)$ where k is a non-zero constant and $s(t)$ is the received signal. The received signals are not the rectangular waveform here, so the reference signal shouldn't be a constant.

$$\begin{aligned} \text{(c) } \psi(t) &= \frac{At}{T_b} \quad \text{for } 0 \leq t \leq T_b \quad \{\text{here we choose } k=1\} \\ \psi(t) &= 0 \quad \text{for other } t \text{ values} \end{aligned}$$

The average signal level for binary “1” at the integrator O/P ($t=T_b$) is:

$$\int_0^{T_b} \left(\frac{At}{T_b}\right)^2 dt = \left(\frac{A}{T_b}\right)^2 \frac{t^3}{3} \Big|_0^{T_b} = \frac{A^2 T_b}{3} \quad (\text{i.e. } E_b)$$

Similarly, the average signal level for binary “0” at the integrator O/P is $-\frac{A^2 T_b}{3}$ (i.e. $-E_b$)

The threshold is at 0, the distance between the mean signal level to threshold is $\frac{A^2 T_b}{3}$

The noise power at the integrator O/P is $\frac{N_0}{2} \int_0^{T_b} \left(\frac{At}{T_b}\right)^2 dt = \frac{N_0 A^2 T_b}{6}$ (i.e. $\frac{N_0}{2} E_b$)

The average prob of bit error is:

$$P_B = Q[\sqrt{(A^2 T_b / 3)^2 / (N_0 A^2 T_b / 6)}] \frac{1}{2} + Q[\sqrt{(A^2 T_b / 3)^2 / (N_0 A^2 T_b / 6)}] \frac{1}{2}$$

$$P_B = Q[\sqrt{\frac{6A^2 T_b}{9N_0}}]$$

$$P_B = Q[\sqrt{\frac{2E_b}{N_0}}]$$

$$\text{For } \frac{E_b}{N_0} = 12dB \rightarrow \frac{E_b}{N_0} = 15.85 \rightarrow \sqrt{\frac{2E_b}{N_0}} = 5.63$$

$$P_B \sim \frac{1}{5.63\sqrt{2\pi}} \exp\left(-\frac{5.63^2}{2}\right) = 9.3 \times 10^{-9}$$

$$(2) Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

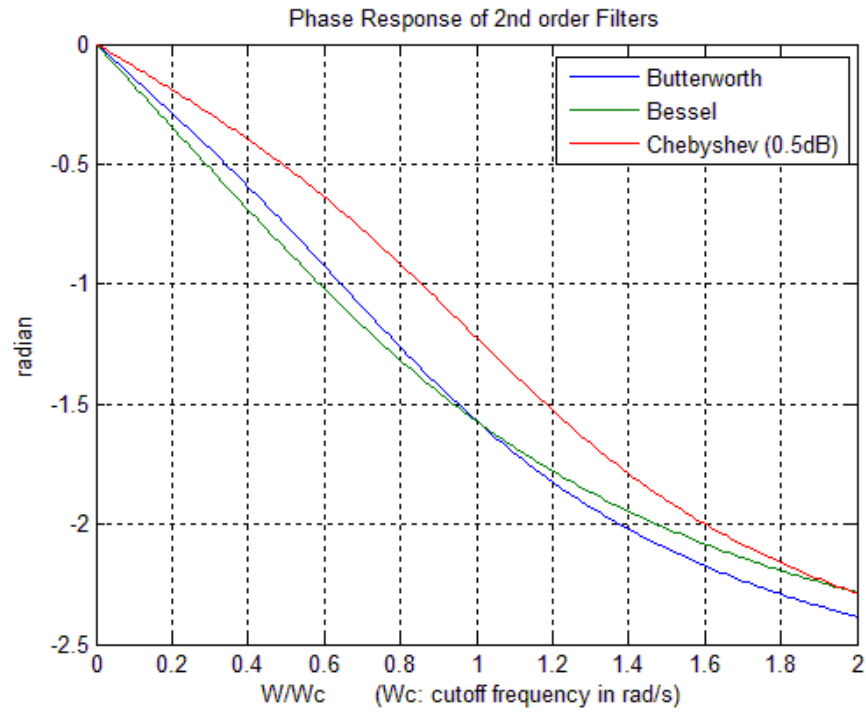
$$(3) Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$$(4) Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

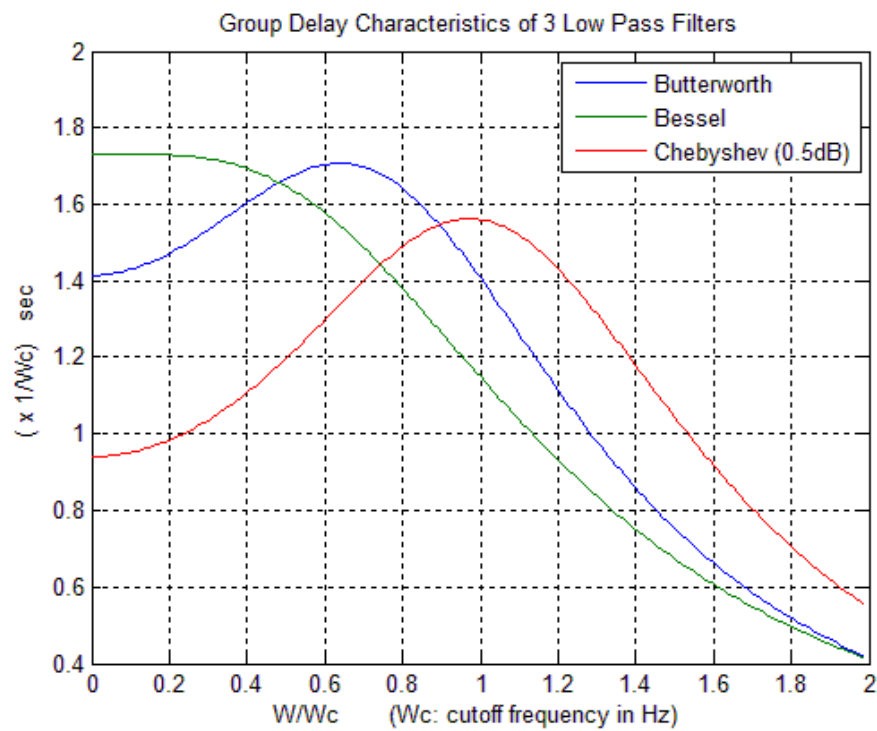
From Section 1.7.1 and 1.7.2 of the lecture notes and tutorial questions (2), (3) and (4), it shows that both receivers in Section 1.7.1 and 1.7.2 are optimum, the probability of error P_B of Unipolar

NRZ and NRZ-L signals are $Q\left(\sqrt{\frac{E_b}{N_0}}\right)$ and $Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$ respectively.

(5) (a)

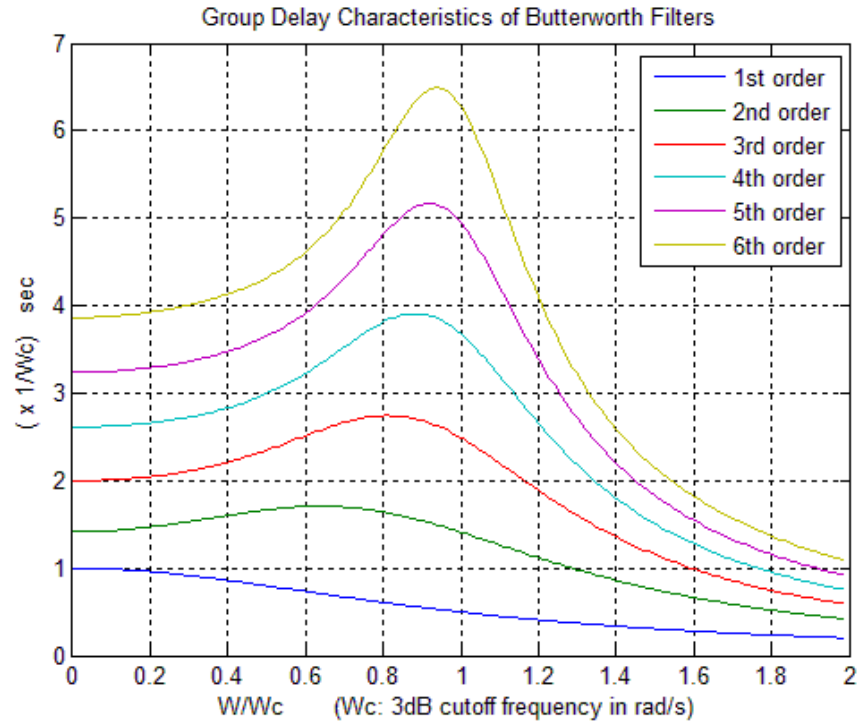


(b)



$GD = \text{constant} \rightarrow \text{linear phase (i.e. no phase distortion)}$

(c)



The change of GD increases as the order increases.