

Block A Unit 2 outline

- It is important to apply the 3 basic laws in circuit theory to analyze and design circuits
- This unit is organized into the following 3 sections
 - › Nodal voltage analysis (application of KCL and Ohm's law)
 - › Mesh current analysis (application of KVL and Ohm's law)
 - › Superposition

Alexander & Sadiku, "Fundamentals of Electric Circuits" 5th Edition Ch 3, 4.3

I. Nodal Voltage Analysis (NVA)

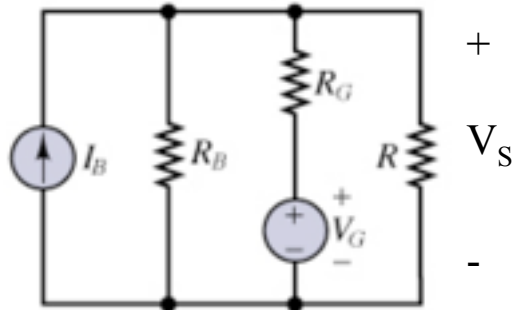
KCL & Ohm's law in action

- The method of nodal voltage analysis is an application of KCL and Ohm's law together
- The unknown variables that you will solve for are **node voltages**
- We will apply KCL at a node and express the unknown currents as unknown node voltages

Example on how to do NVA

Find the voltage across the current source, V_S :

Given: $I_B = 12\text{A}$, $V_G = 12\text{V}$, $R_G = 0.3\Omega$, $R_B = 1\Omega$, $R = 0.23\Omega$



Apply KCL at V_S :

$$I_B = \frac{V_S}{R_B} + \frac{V_S - V_G}{R_G} + \frac{V_S}{R}$$

$$12 = \frac{V_S}{1} + \frac{V_S - 12}{0.3} + \frac{V_S}{0.23}$$

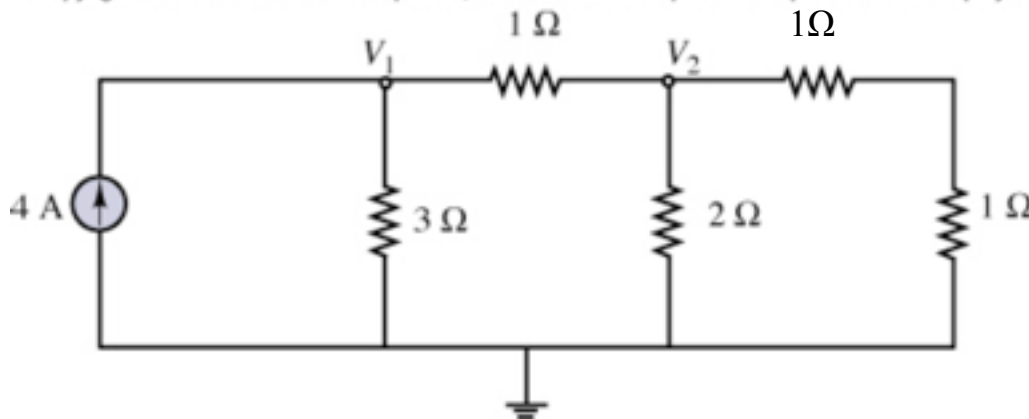
$$V_S = 5.99\text{V}$$

- Note that the negative terminal of V_G is used as a reference (0V) in the example
- The value of V_S is also referenced to this node
- The value of the reference node is not important since we are only interested in the voltage difference

Worked Example on NVA 1

Use nodal voltage analysis to find V_1 and V_2 . (Answer: $V_1 = 4.8\text{V}$, $V_2 = 2.4\text{V}$)

Copyright © The McGraw-Hill Companies, Inc. Permission required for reproduction or display.



First method:

- Apply KCL at V_1 (1)
- Apply KCL at V_2 (2)
- Solve equations (1) and (2)

Alternate method:

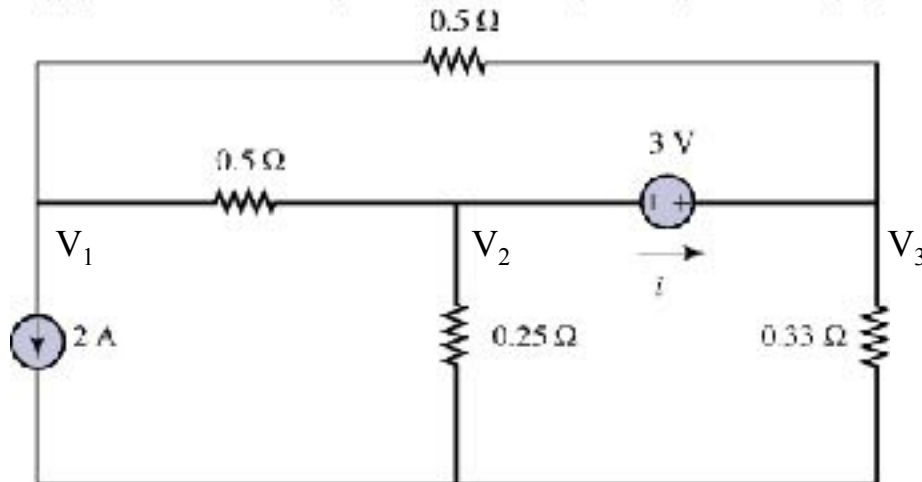
- Combine all the resistors to find V_1 first
- Then combine the 3 resistors on the right followed by voltage divider rule to find V_2

Worked Example on NVA 2

Use nodal voltage analysis to find the current through the 3V source.

(Answer: $i = 8.31 \text{ A}$)

Copyright © The McGraw-Hill Companies, Inc. Permission required for reproduction or display.



Apply KCL at V_1 :

$$2 = \frac{V_2 - V_1}{0.5} + \frac{V_3 - V_1}{0.5}$$

$$V_2 + V_3 - 2V_1 = 1 \quad (1)$$

Apply KCL at V_2 :

$$\frac{V_1 - V_2}{0.5} = \frac{V_2}{0.25} + i$$

$$2V_1 - 6V_2 = i \quad (2)$$

Apply KCL at V_3 :

$$\frac{V_3}{0.33} + \frac{V_3 - V_1}{0.5} = i$$

$$\frac{166}{33} V_3 - 2V_1 = i \quad (3)$$

Since a new variable is introduced in (2), we need one more equation.

$$V_3 - V_2 = 3 \quad (4)$$

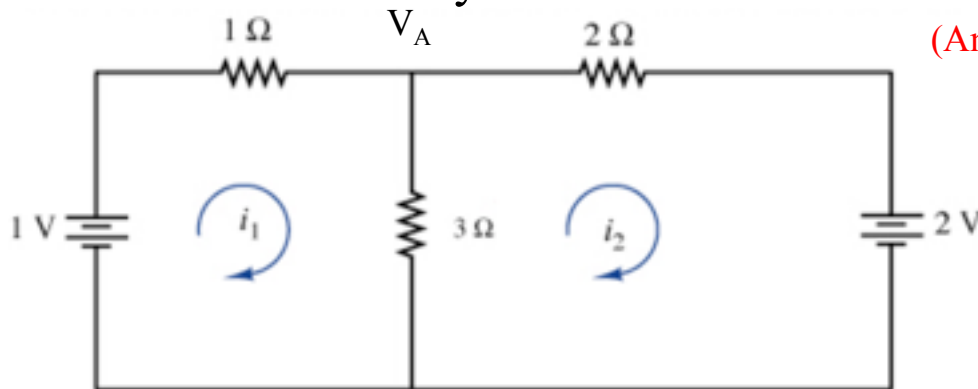
II. Mesh Current Analysis (MCA)

KVL & Ohm's law in action

- The method of mesh current analysis is an application of KVL and Ohm's law together
- The unknown variables that you will solve for are **mesh currents**
- From KVL, the sum of voltage drops and rise must equal zero
- The voltage differences are expressed as the current going through each branch in the mesh

Example on how to do MCA

Use mesh current analysis to find the current through the 3Ω resistor.



(Answer: $i_{3\Omega} = 4/11\text{A}$)

- There are 2 meshes and hence 2 mesh currents
- Apply KVL to each of these 2 meshes

Apply KVL to mesh 1:

$$1 = i_1(1 + 3) - 3i_2$$
$$4i_1 - 3i_2 = 1 \quad (1)$$

Useful tip:

- keep voltages of sources on one side of the equation, and keep voltages of resistors on the other side

Apply KVL to mesh 2:

Follow the defined direction of the mesh current

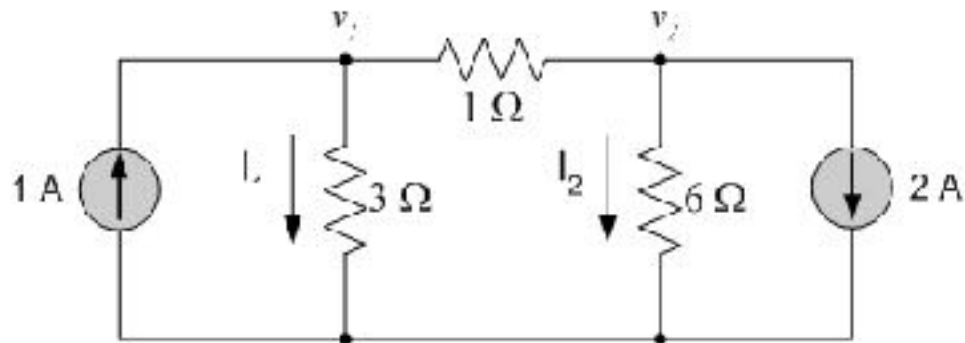
$$-2 = i_2(2 + 3) - 3i_1$$
$$5i_2 - 3i_1 = -2 \quad (2)$$

Now, solve equation (1) and (2):

$$i_{3\Omega} = i_1 - i_2$$

Worked Example on MCA 1

Use mesh current analysis to find currents I_1 and I_2 . (Answer: $I_1 = I_2 = -0.5\text{A}$)



-You see 3 meshes, but only the middle mesh is unknown, the other 2 are defined by the current sources.

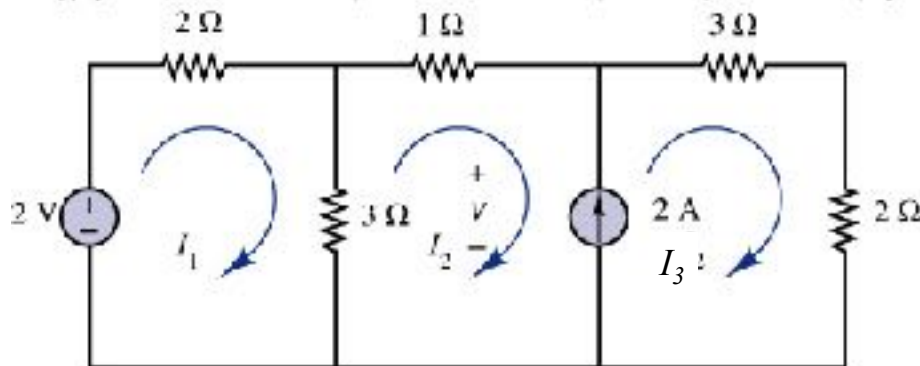
-For simplicity, defined the direction of known mesh currents in the same direction as the sources.

Worked Example on MCA 2

Use mesh current analysis to find the voltage across the current source.

(Answer: $V = 3.89 \text{ V}$)

Copyright © The McGraw-Hill Companies, Inc. Permission required for reproduction or display.



-In the process of deriving equations (2) and (3), a new variable V is introduced.

-Hence, we need one more equation, (4)

Apply KVL at mesh 1:

$$2 = I_1(2+3) - I_2(3)$$

$$\Rightarrow 5I_1 - 3I_2 = 2 \quad (1)$$

Apply KVL at mesh 2:

$$-V = I_2(3+1) - I_1(3)$$

$$\Rightarrow 4I_2 - 3I_1 = -V \quad (2)$$

Apply KVL at mesh 3:

$$V = I_3(3+2)$$

$$\Rightarrow 5I_3 = V \quad (3)$$

One more equation:

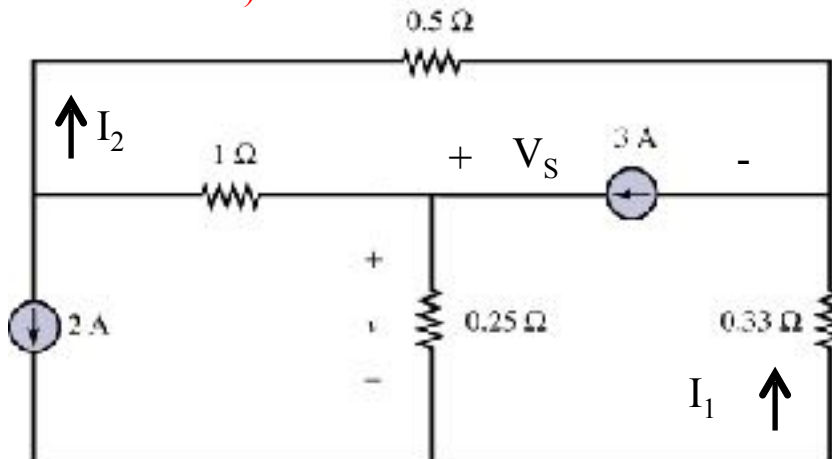
$$I_3 - I_2 = 2A \quad (4)$$

Solve for V , verify by $V = (I_3)(3+2)$

Worked Example on MCA 3

Use mesh current analysis, find the voltage across the $0.25\ \Omega$ resistor.

(Answer: $v = 0.34\text{V}$)



Method:

-Find the current through 0.25Ω resistor first, then use this to find the voltage.

-Although there are 3 meshes, the bottom left mesh is already known (2A going anti-clockwise).

Apply KVL at mesh 1:

$$V_S = I_2(0.5 + 1) + (2)(1)$$

$$\Rightarrow V_S = 1.5I_2 + 2 \quad (1)$$

Apply KVL at mesh 2:

$$V_S = I_1(0.25 + 0.33) - (2)(0.25)$$

$$\Rightarrow V_S = 0.58I_1 - 0.5 \quad (2)$$

Consider the current source:

$$I_1 + I_2 = 3 \quad (3)$$

Solve for I_1 and I_2

Current through $0.25\ \Omega$ resistor: $I_1 - 2$

$$V = (I_1 - 2)(0.25)$$

Quick note on choosing NVA and MCA

How do you choose between NVA and MCA?

- Your choice should not be due to level of familiarity between the methods.
- One consideration is whichever is simpler to use for a given circuit.
- How then do you decide what is simpler?
- For a start, having fewer equations certainly makes solving easier.

III. Superposition

The method

- This method applies only to circuits that have **multiple sources**
- In such case, it can come in handy (or not)
- In a circuit with multiple sources, superposition considers the current or voltage associated with a given branch for one of the sources, while turning the rest off

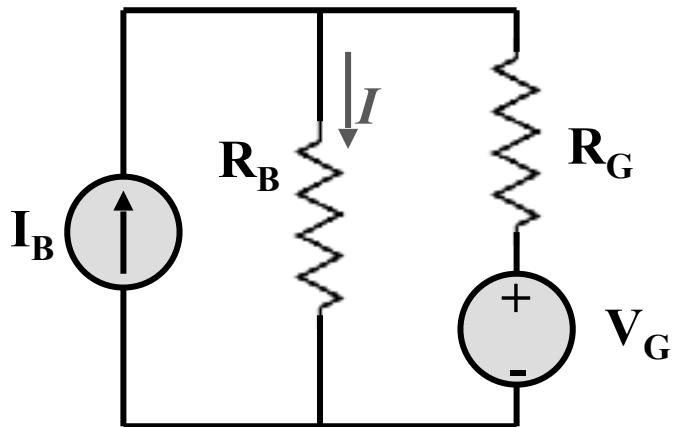


Fig 1: Basic circuit with 2 sources

Aim: Find the current through R_B .

1. Find the current (I_1) through R_B when only I_B is present (V_G is removed)
2. Find the current (I_2) through R_B when only V_G is present (I_B is removed)
3. The net current $I = I_1 + I_2$

How do we “remove” a current source or voltage source?

Disabling sources

Voltage source: If the voltage source does not exist, the voltage across the terminals would be zero. It looks like a short circuit. (see Fig 2a)

Current source: If the current source does not exist, the current through it would be zero. It looks like an open circuit. (see Fig 2b)

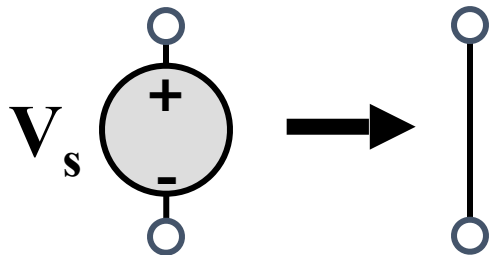


Fig 2a: Disable a voltage source by Replacing with a short circuit.

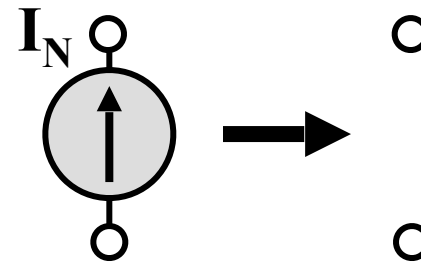


Fig 2b: Disable a current source by Replacing with an open circuit.

Apply the tips to Fig 1

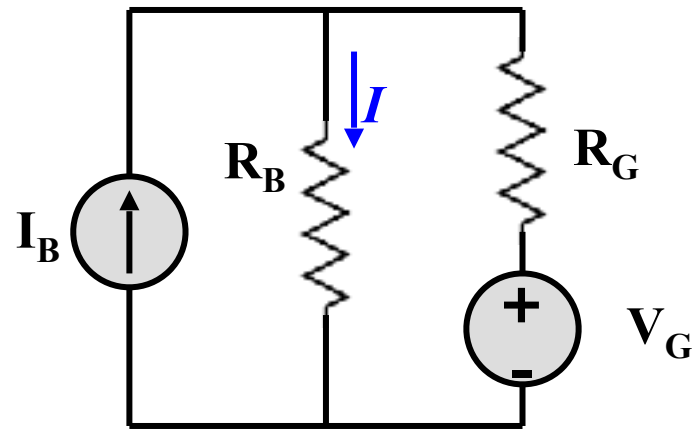
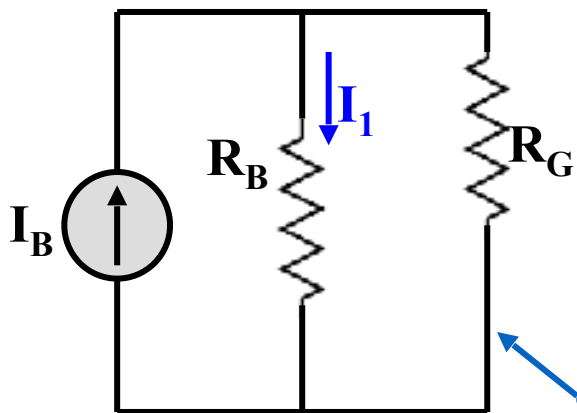
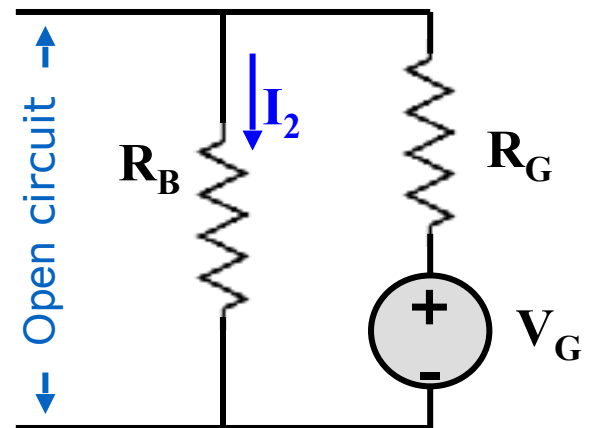


Fig 1: Basic circuit with 2 sources

Disabling V_G , $V_G = 0$



Disabling I_B , $I_B = 0$



$$I = I_1 + I_2$$

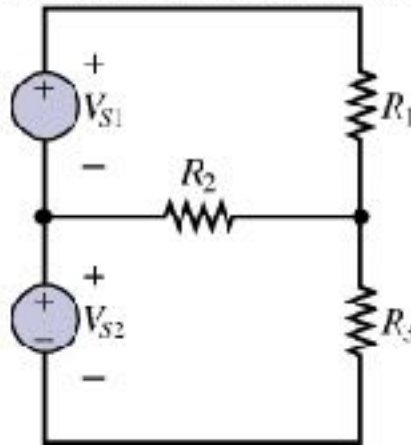
Short circuit

Worked Example on Superposition

Determine the current through R_1 using superposition.

Given $R_1 = 560\Omega$, $R_2 = 3.5k\Omega$, $R_3 = 810\Omega$, $V_{S2} = V_{S1} = 90V$

Copyright © The McGraw-Hill Companies, Inc. Permission required for reproduction or display.



a. First, short V_{S1}
Voltage across R_1 :

$$\begin{aligned} V_{R1a} &= \frac{R_1 \parallel R_2}{(R_1 \parallel R_2) + R_3} V_{S2} \\ &= \left(\frac{560 \parallel 3500}{(560 \parallel 3500) + 810} \right) 90 \\ &= 33.61V \end{aligned}$$

Current through R_1 :

$$\begin{aligned} I_{R1a} &= V_{R1a} / R_{1a} \\ &= 33.61 / 560 \\ &= 0.06A \end{aligned}$$

Worked Example on Superposition

b. Next, short V_{S2}

Current through R_1 :

$$\begin{aligned} I_{R1b} &= \frac{V_{S1}}{R_1 + (R_2 \parallel R_3)} = \frac{90}{560 + (3500 \parallel 810)} \\ &= 0.074A \end{aligned}$$

Take careful note of the directions defined for each of these currents.

Finally,

$$I_{R1} = I_{R1a} + I_{R1b} = 0.134A$$