

EE3109 Applied Electromagnetics
2024/25 Semester B
Assignment 2

1. Given the time-domain definition of the Poynting vector is $\mathbf{S}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}, t) \times \mathbf{H}(\mathbf{r}, t)$, show that
- (a) The time-averaged power density of a general EM wave is given by $S_{av}(\mathbf{r}) = \frac{1}{2} \text{Re}\{\mathbf{E}(\mathbf{r}) \times \mathbf{H}(\mathbf{r})^*\}$.
- (b) The time-averaged power density of a plane wave travelling in x-direction in a lossless medium is

$$S_{av}(\mathbf{r}) = \frac{|\mathbf{E}(\mathbf{r})|^2}{2\eta} \hat{\mathbf{x}}.$$

- (c) The time-averaged power density of a plane wave travelling in x-direction in a lossy medium is

$$S_{av}(\mathbf{r}) = \frac{|\mathbf{E}(x=0)|^2}{2|\eta_c|} e^{-2\alpha x} \cos \theta_\eta \hat{\mathbf{x}}.$$

2. The magnetic field of a linearly-polarized incident plane wave is

$$\mathbf{E}^i = \exp(jk_1(mx + pz)) \hat{\mathbf{y}} \text{ [A/m]}.$$

The wave travels in the $\hat{\mathbf{k}} = \frac{1}{2}(\sqrt{3}\hat{\mathbf{x}} - \hat{\mathbf{z}})$ direction. k_1 is the wave number for Medium 1, which is the medium in which the incident wave resides. The plane wave hits a material interface at $z = 0$, where the region $z < 0$ is filled with air, and the region $z > 0$ is filled with a material with $\mu_r = 1.5$, $\epsilon_r = 1$ and $\sigma = 0$.

- (a) What are the values of m and p ?
- (b) Write the wavevectors of the incident wave ($\mathbf{k}^i$) and reflected wave ($\mathbf{k}^r$).
- (c) What is the angle of the transmitted wave in this case?
- (d) What is the transmission coefficient in this case?
- (e) Write the transmitted electric field in phasor form.
- (f) If we change the incidence angle, can we get to a critical angle in this case? If so, what is it?
- (g) Will there be a Brewster angle in this case? If so, what is the Brewster angle?
3. The dispersion relation for a wave in a specific medium is given by
- $$\omega^2 = \omega_0^2 + c^2 k^2,$$
- where ω_0 is the characteristic frequency of the medium, c is the speed of light in vacuum and k is the wavenumber.
- (a) Calculate both the phase velocity and group velocity as functions of k .
- (b) Analyze the behavior of phase and group velocities as the wavenumber approaches zero and infinity. Discuss the physical significance of these limits.

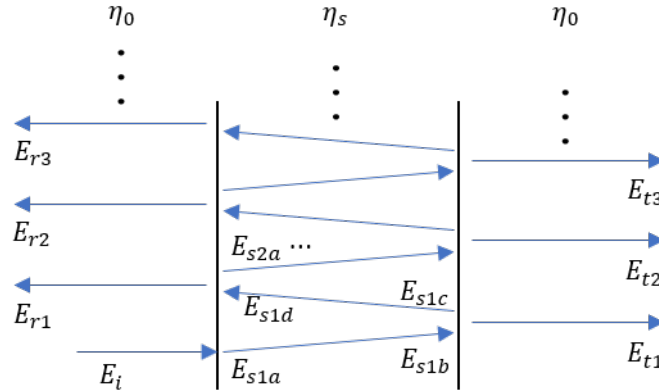
(Choose to answer one of either Q4 or Q5. State clearly which one you choose to answer. If you answer both correctly you may receive up to 2 Bonus Points).

4. Consider a plane wave travelling in the metamaterial with $\mu_r = \epsilon_r = -1$. For simplicity, we define the propagation direction to be along the z-axis, and the electric field to be along the x-axis. The electric field is written below.

$$\mathbf{E}(z, t) = E_{x0} \cos(\omega t - kz + \phi) \hat{\mathbf{x}}$$

- (a) Find the corresponding magnetic field from Maxwell's equations.
- (b) Using Maxwell's equations, show that in this material $\hat{\mathbf{E}} \times \hat{\mathbf{H}} = -\hat{\mathbf{k}}$. (It is due to this relationship that this material is called a "left-handed" metamaterial.)
- (c) Find the refractive index of the plane wave – that is, the phase velocity of an EM wave in free space divided by the phase velocity of the EM wave in the material.
- (d) Find the wavevector of the plane wave.
- (e) Find the Poynting vector (phasor form) of the plane wave.
- (f) If a plane wave in air is incident upon a slab of the material, at 30° from normal, how much of the wave is transmitted into the material, and what will the transmitted angle be? Draw a diagram showing the plane of incidence, the incident wave, the transmitted wave and the angles the two waves make with the surface normal.

5. Consider the reflection and transmission coefficient of a slab of width d , of a material with intrinsic impedance η_s and wave number k_s . At steady state, the reflected and transmitted wave can be viewed as a summation of reflection and transmission components after multiple reflections on the internal surfaces of the slab.



In the diagram above, the electric field at different points of its first round trip is denoted as $E_{s1a}, E_{s1b}, E_{s1c}$ and E_{s1d} . The start of the second round trip is denoted as E_{s2a} , from which point onwards the process repeats, and finding using elementary relationships one can relate one electric field value to the next.

- We will define r and t as the reflection and transmission coefficients of a wave travelling from air to the slab, and r' and t' as the reflection and transmission coefficients of a wave travelling from the slab material into air. Write an expression for all four coefficients as a function of η_s and η_0 .
- We will define “propagation” coefficients p and p' . p corresponds to the multiplicative change in E-field when a positively traveling plane wave propagates for a distance of d within the slab. p' corresponds to the multiplicative change in E-field when a negatively traveling plane wave propagates for a distance of d (ie. displacement of $-d$) within the slab. Write expressions for p and p' as functions of k_s and d .
- Using the above coefficients $\{r, r', t, t', p, p'\}$, write $\{E_{s1a}, E_{s1b}, E_{s1c}, E_{s1d}, E_{s2a}\}$ in terms of E_i . (For example, $E_{s1b} = pE_{s1a} = ptE_i$)
- Write $\{E_{r1}, E_{r2}, E_{r3}\}$ and $\{E_{t1}, E_{t2}, E_{t3}\}$ in terms of E_i .
- As stated in the question, the total reflected electric field is the sum of all reflected electric fields (ie. $E_r = \sum_{n=1}^{\infty} E_{rn}$), and the total transmitted electric field is the sum of all transmitted electric fields. Based on this, find the reflection coefficient $R = E_r/E_i$ and the transmission coefficient $T = E_t/E_i$ for EM wave reflection from and transmission through the slab. Leave your answer in terms of r and t : no need to substitute in the intrinsic impedances.