

Complex Dynamical Networks:

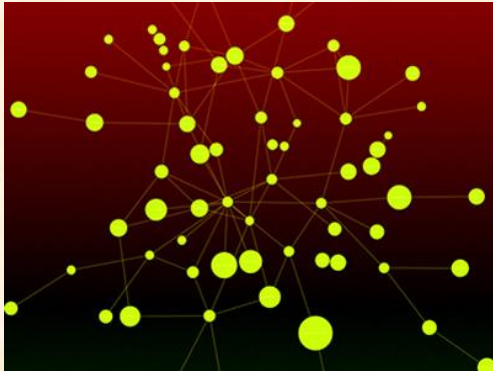
Lecture 9: Network Control

EE 6605

Instructor: G Ron Chen

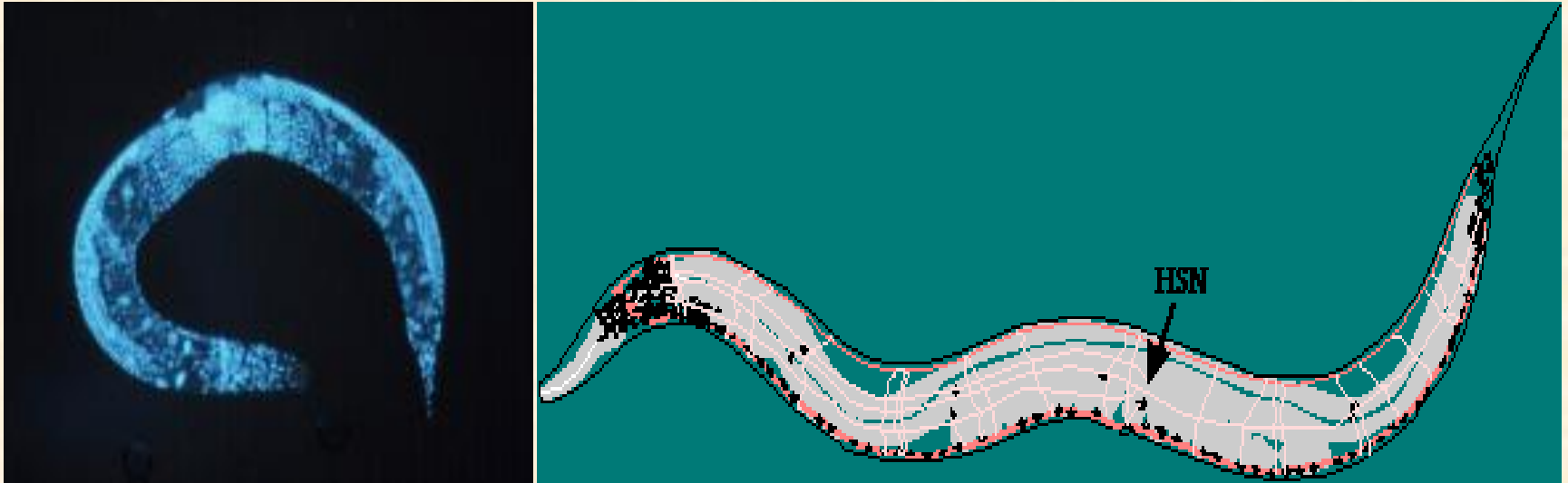


Motivational Examples



Motivation: Example

C. elegans



In its Neural Network:

Neurons: ~ 300 Synapses: ~ 3000

Excerpt

The worm *Caenorhabditis elegans* has 297 nerve cells. The neurons switch one another on or off, and, making 2345 connections among themselves. They form a network that stretches through the nematode's millimeter-long body.

How many neurons would you have to commandeer in order to control the network with complete precision?

The answer is: 49

-- Adrian Cho, *Science*, 13 May 2011, vol. 332, p 777

Here, control = stimuli

Another Example

“ ... very few individuals (approximately 5%) within honeybee swarms can guide the group to a new nest site.”

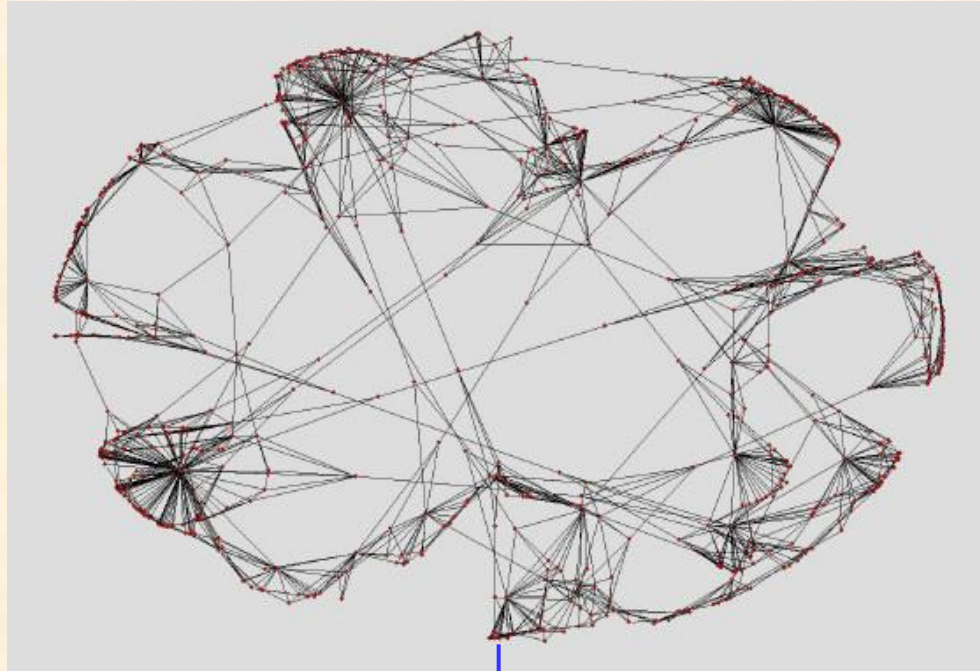
I.D. Couzin et al., *Nature*, 3 Feb 2005, vol. 433, p 513

These 5% of bees can
be considered as
“controlling” or
“controlled” agents

Leader-Follower networks



Now ...

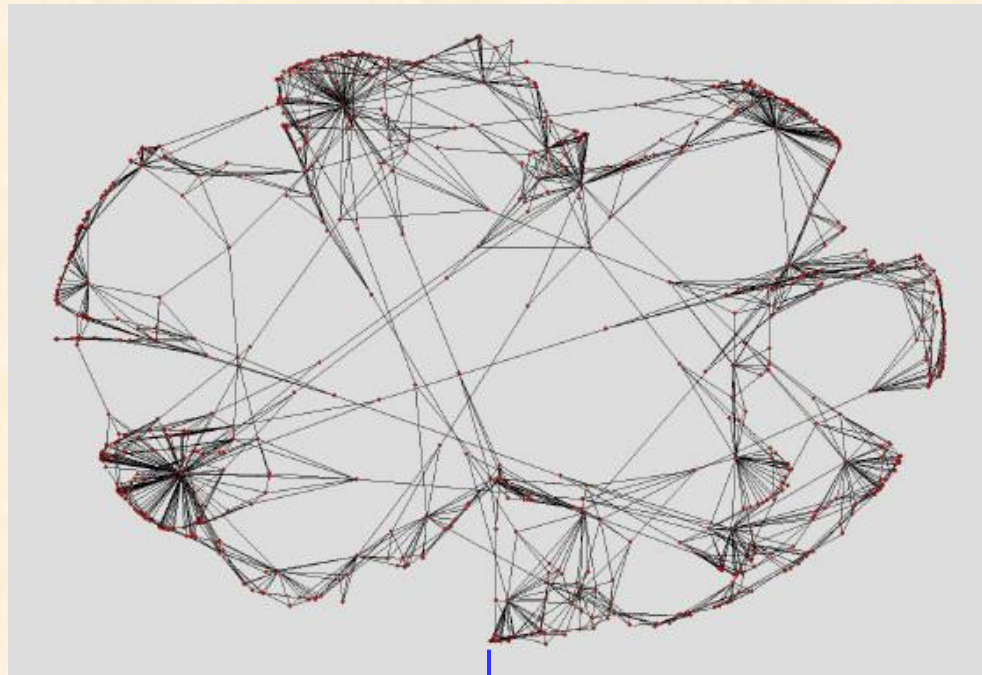


$$\frac{dx_i}{dt} = f(x_i), \quad x_i \in \mathbb{R}^n$$


$$u_i = -H_i x_i$$

- Given a network of dynamical systems (e.g., ODEs)
- Given a specific control objective (e.g., synchronization)
- Assume: a certain class of controllers (e.g., local state-feedback controllers) have been chosen to use

Questions:



$$\frac{dx_i}{dt} = f(x_i), \quad x_i \in R^n$$

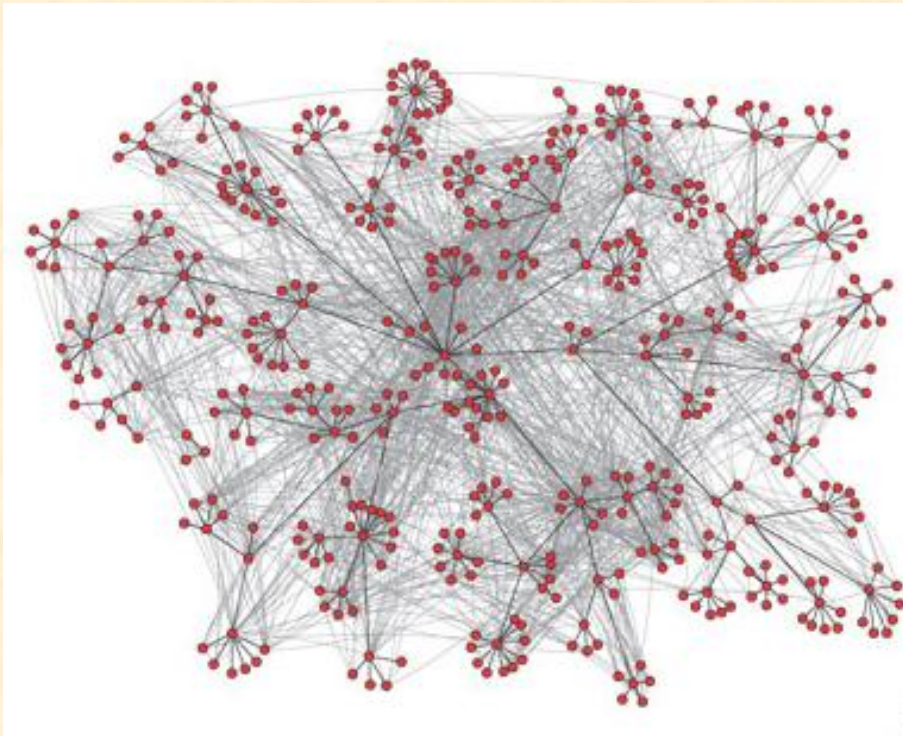

$$u_i = -H_i x_i$$

- How many controllers to use?
- Where to put them (which nodes to “pin”)?

Objective: To achieve cost-effective control (e.g. synchronization) with good performance

Attempts to Answer

First, consider undirected and unweighted networks



Each node is a higher-dimensional nonlinear dynamical system:

$$\frac{dx_i}{dt} = f(x_i), \quad x \in R^n, \quad i = 1, 2, \dots, N$$

- o **Regular networks**
- o **Random-graph networks**
- o **Small-world networks**
- o **Scale-free networks**
- o

Network Model

Linearly coupled network:

$$\frac{dx_i}{dt} = f(x_i) + c \sum_{j=1}^N a_{ij} H x_j \quad x_i \in R^n \quad i = 1, 2, \dots, N$$

a general assumption is that $f(\cdot)$ is Lipschitz

coupling strength $c > 0$ and

coupling matrices (undirected):

$$A = [a_{ij}]_{N \times N} \quad H = \begin{bmatrix} r_{11} & & & 0 \\ & r_{22} & & \\ & & \ddots & \\ 0 & & & r_{nn} \end{bmatrix}$$

A: If node i connects to node j ($j \neq i$), then $a_{ij} = a_{ji} = 1$; else, $a_{ij} = a_{ji} = 0$; $a_{ii} = 0$

Laplacian matrix: $L = D - A$ $D = \text{diag}\{d_1, \dots, d_N\}$ d_i - degree of node i

Network Model

Linearly coupled network:

$$\frac{dx_i}{dt} = f(x_i) + c \sum_{j=1}^N a_{ij} H x_j \quad x_i \in R^n \quad i = 1, 2, \dots, N$$

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Laplacian matrix: $L = D - A$ $D = \text{diag}\{d_1, \dots, d_N\}$ d_i - degree of node i

For connected networks: $0 = \lambda_1 < \lambda_2 \leq \dots \leq \lambda_N$ $\text{Rank}(L) = N - 1$



Network Synchronization

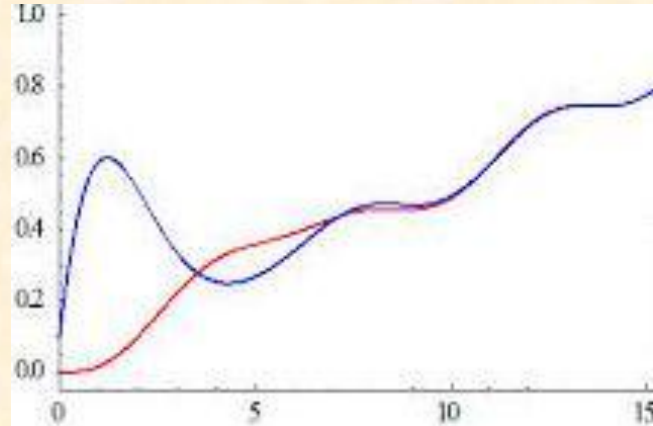
$$\frac{dx_i}{dt} = f(x_i) + c \sum_{j=1}^N a_{ij} H x_j \quad x_i \in R^n \quad i = 1, 2, \dots, N$$

complete state

synchronization:

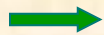
$$\lim_{t \rightarrow \infty} \|x_i(t) - x_j(t)\|_2 = 0, \quad i, j = 1, 2, \dots, N$$

Example:



What kind of controllers? How many? Where?

$$\frac{dx_i}{dt} = f(x_i) + c \sum_{j=1}^N a_{ij} H x_j \quad \leftarrow + u_i \quad i = 1, 2, \dots, N$$



$$(u_i = -H x_i)$$

$$\frac{dx_i}{dt} = f(x_i) + c \sum_{j=1}^N a_{ij} \delta_{ij} H (x_j - x_i) \quad i = 1, 2, \dots, N$$

$$\delta_{ij} = \begin{cases} 1 & \text{if } \text{to-control} \\ 0 & \text{if } \text{not-control} \end{cases}$$

Q: How many $\delta_{ij} = 1$? Which i and j ?

How Many? – One is enough, if ... e.g., c large enough

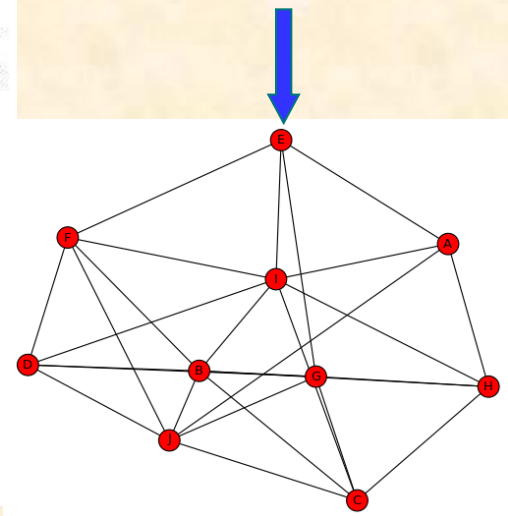
$$\begin{cases} \frac{dx_1(t)}{dt} = f(x_1(t), t) + c \sum_{j=1}^{m_1} a_{1j} x_j(t) - c\varepsilon(x_1(t) - s(t)) \\ \frac{dx_i(t)}{dt} = f(x_i(t), t) + c \sum_{j=1}^{m_1} a_{ij} x_j(t), \quad i = 2, \dots, m_1 \end{cases}$$

Key
Idea

Proposition 1: If $A = (a_{ij})_{i,j=1}^m$ is an irreducible matrix with $\text{Rank}(A) = m - 1$ and satisfying $a_{ij} = a_{ji} \geq 0$, if $i \neq j$, and $\sum_{j=1}^m a_{ij} = 0$, for $i = 1, 2, \dots, m$. Then, all eigenvalues of the matrix

$$\tilde{A} = \begin{pmatrix} a_{11} - \varepsilon & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mm} \end{pmatrix}$$

are negative.



Still yet,

where to apply controllers makes a difference

With control, one can do more, e.g., less energy, faster control

Control Objective: To force the network to synchronize onto the network equilibrium:

$$x_1(t) \rightarrow x_2(t) \rightarrow \dots \rightarrow x_N(t) \rightarrow s, \text{ as } t \rightarrow \infty$$

Here, s is an equilibrium of the network

Pinning Control: (which fraction of nodes to pin?)

Only a small fraction of nodes are selected for control:

1. Selective pinning scheme
2. Random pinning scheme

Example

Consider a scale-free CNN, which has a zero equilibrium ($s = 0$):

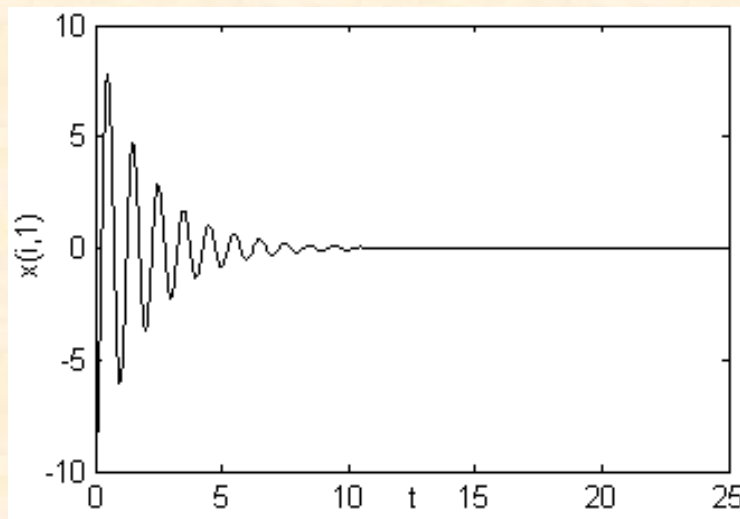
$$\frac{dx_i}{dt} = \begin{pmatrix} \frac{dx_{i1}}{dt} \\ \frac{dx_{i2}}{dt} \\ \frac{dx_{i3}}{dt} \\ \frac{dx_{i4}}{dt} \end{pmatrix} = \begin{pmatrix} -x_{i3} - x_{i4} + c \sum_{j=1}^N a_{ij} x_{j1} \\ 2x_{i2} + x_{i3} + c \sum_{j=1}^N a_{ij} x_{j2} \\ 14x_{i1} - 14x_{i2} + c \sum_{j=1}^N a_{ij} x_{j3} \\ 100x_{i1} - 100x_{i4} \\ \quad + 100(|x_{i4} + 1| - |x_{i4} - 1|) \\ \quad + c \sum_{j=1}^N a_{ij} x_{j4} \end{pmatrix} \quad i = 1, 2, \dots, N = 60$$

X.F. Wang, G.R. Chen, Physica A (2002)

1 Selective Pinning Control

Here, network size $N = 60$, coupling strength $c = 8$ and number of controlled nodes is $m = 15$, by $u_i = -hx_i$

Pin the first 15 largest nodes:



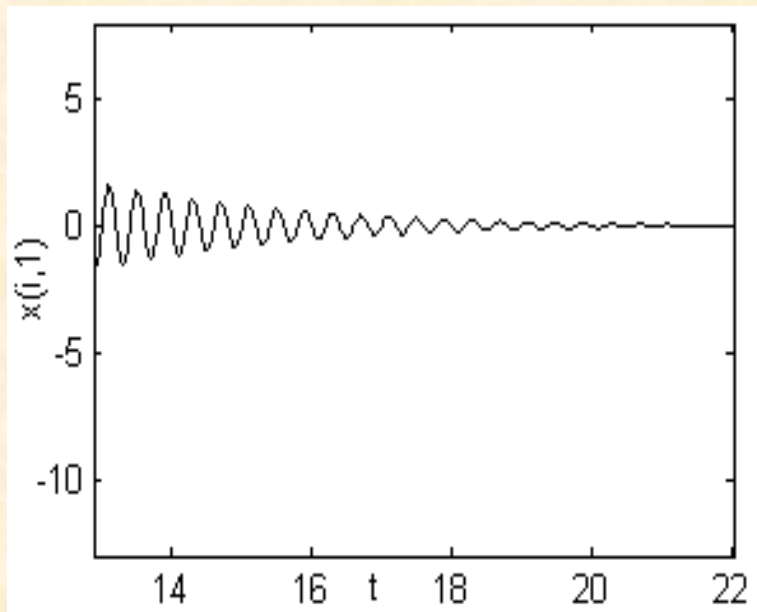
The controlled state x_1

Control gain: $h \approx 30$

Settling time = 10

2 Random Pinning Control

Randomly pin 15 nodes.



The controlled state x_i

Comparison:

1. Control gain is much larger:

$$h \approx 513$$

Recall the last one:

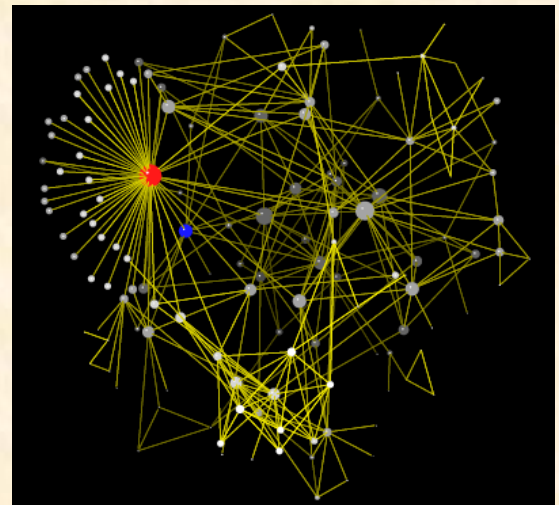
$$h \approx 30$$

2. It takes twice longer time to synchronize the network:

Settling time = 20

Recall the last one: 10

Controllability Theory ?



In retrospect, ...

J.S.I.A.M. CONTROL

Ser. A, Vol. 1, No. 2

Printed in U.S.A., 1963

MATHEMATICAL DESCRIPTION OF LINEAR DYNAMICAL SYSTEMS*

R. E. KALMAN†



Abstract. There are two different ways of describing dynamical systems: (i) by means of state variables and (ii) by input/output relations. The first method may be regarded as an axiomatization of Newton's laws of mechanics and is taken to be the basic definition of a system.

It is then shown (in the linear case) that the input/output relations determine only one part of a system, that which is completely observable and completely controllable. Using the theory of controllability and observability, methods are given for calculating irreducible realizations of a given impulse-response matrix. In particular, an explicit procedure is given to determine the minimal number of state variables necessary to realize a given transfer-function matrix. Difficulties arising from the use of reducible realizations are discussed briefly.

Controllability

Linear Time-Invariant (LTI) system

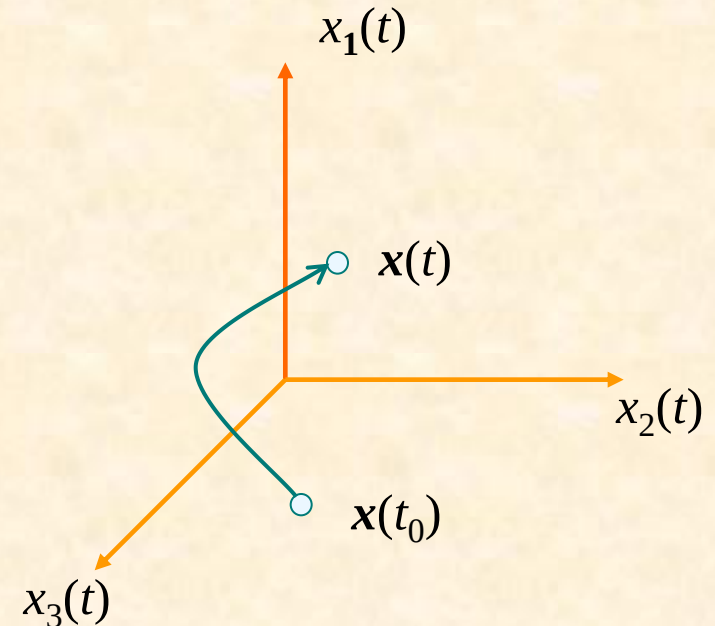
$$\dot{x}(t) = Ax(t) + Bu(t)$$

$x \in R^N$: state vector

$u \in R^p$: control input

$A \in R^{N \times N}$: state matrix

$B \in R^{N \times p}$: control input matrix



Controllable: The system orbit can be driven by an input from any initial state to any target state in finite time.

Classical Controllability Theorems

$$\dot{x}(t) = Ax(t) + Bu(t)$$

(i) Rank Criterion

The controllability matrix Q is of full row rank:

$$Q = [B \ AB \ \cdots \ A^{N-1}B]$$

(ii) Eigenvector Test

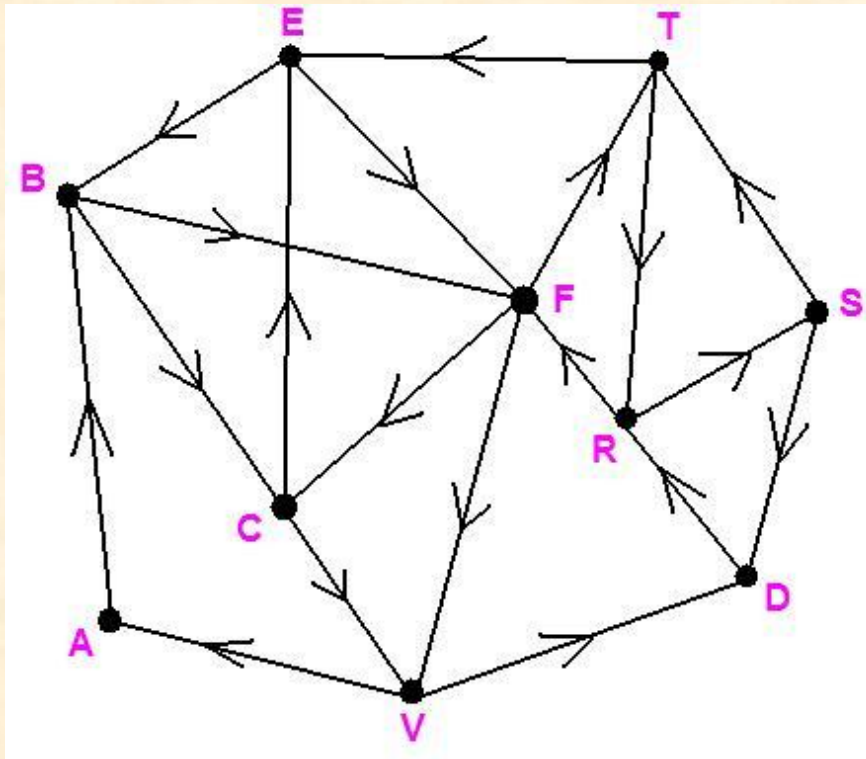
The following relationship holds:

$$v^T A = \lambda v^T, \quad v^T B \neq \mathbf{0}^T$$

λ : eigenvalue of A

v : nonzero left eigenvector of A

What About Directed Networks?



Controllability of Directed Networks

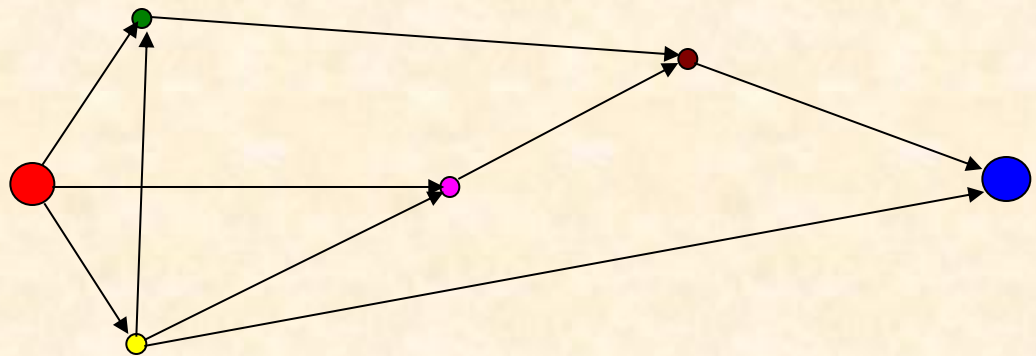
Undirected networks:

Linear systems – Necessary and Sufficient condition:
the controllability matrix has full row rank

Nonlinear systems – By case studies

Directed networks: More challenging

Where to pin is
extremely important



Some earlier attempts

- **Leader-follower multi-agent systems**

H.G. Tanner, *CDC* , 2004

...

- **Pinning controllability of complex networks**

F. Sorrentino, M. di Bernardo, F. Garofalo, G. Chen, *Phys. Rev. E*, 2007

...

- **Structural controllability of complex networks**

Y.Y. Liu, J.J. Slotine, A.L. Barabási, *Nature*, 2011

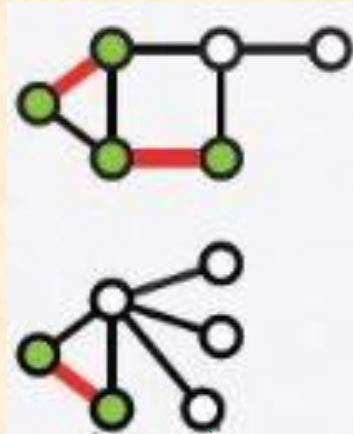
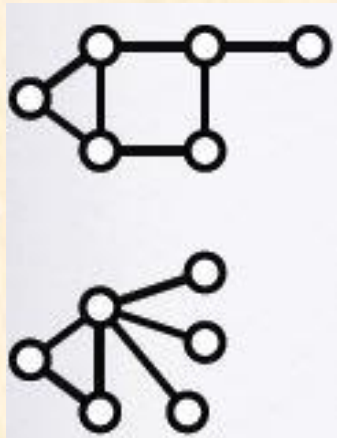
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Matching Theory (review)

Undirected
network

Matching:
a set of edges without
common nodes

Maximum matching
(Perfect matching):
matching of largest size



Matched
node

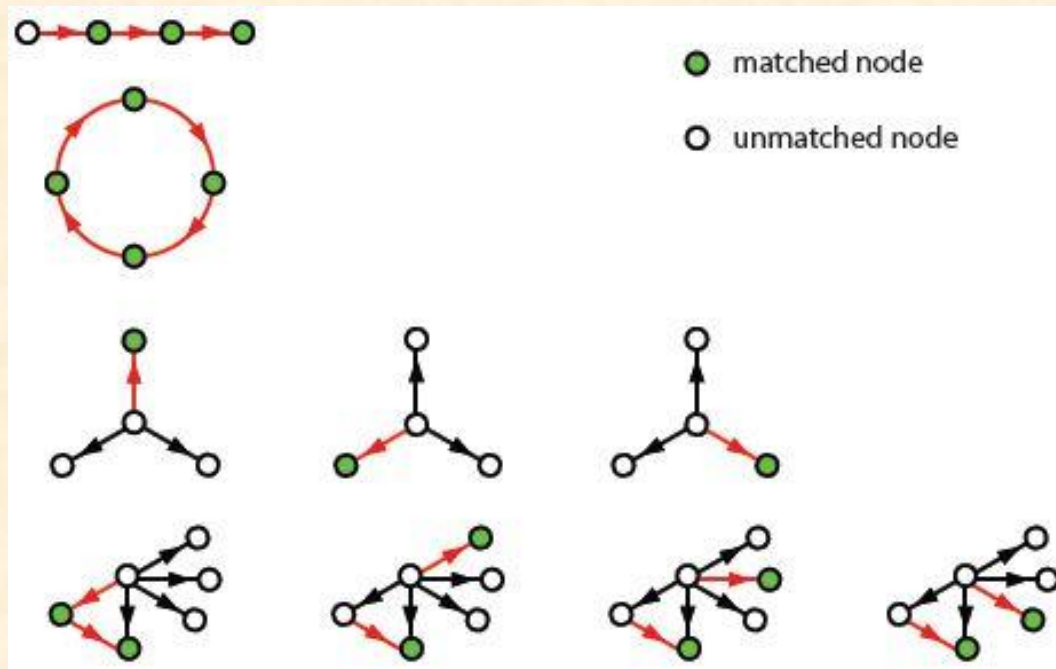
Unmatched
node



Matchings are not unique

Matching in Directed Networks

- **Matching:** a set of directed edges without common heads and tails
- **Matched node:** the tail node of a matching edge



Minimum Inputs Theorem

- The minimum number of inputs, or equivalently the minimum number of driver nodes, N_D , needed to fully control a network G , is **1** --- if there is a perfect matching in G .
- In this case, any single node can be chosen as the driver node.
- If there are no perfect matching in G , then N_D equals the number of unmatched nodes with respect to any maximum matching.
- In this case, the driver nodes = the unmatched nodes.

$$N_D = \max \{1, N_{\text{unmatched}}\}$$

Driver nodes = unmatched nodes

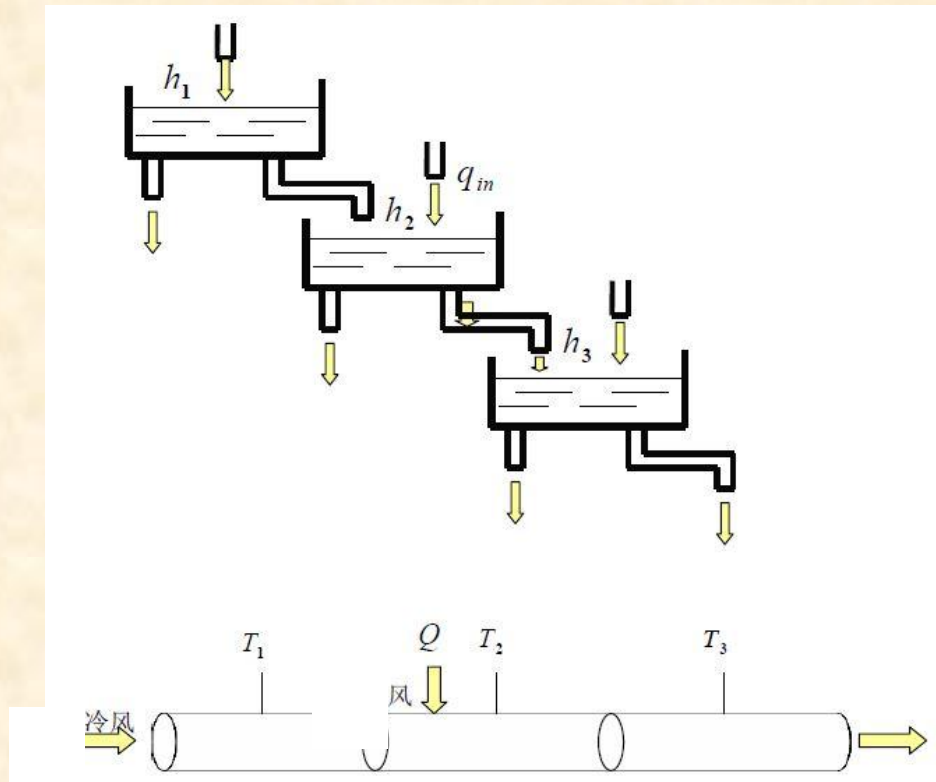
In retrospect: large-scale systems theory

Structural Controllability (and Structural Observability)

1. C.T. Lin, **Structural Controllability**, IEEE Trans. Auto Contr., 19(3): 201-208, 1974
2. R.W. Shields, J.B. Pearson, **Structural Controllability** of Multiinput Linear Systems, IEEE Trans. Auto. Contr., 21(2): 203-212, 1976
3. K. Glover, L.M. Silverman, Characterization of **Structural Controllability**, IEEE Trans. Auto. Contr., 21(4): 534-537, 1976
4. C.T. Lin, System Structure and Minimal **Structure Controllability**, IEEE Trans. Auto. Contr., 22(5): 855-862, 1977
5. S. Hosoe, K. Matsumoto, On the Irreducibility Condition in the **Structural Controllability** Theorem, IEEE Trans. Auto. Contr., 24(6): 963-966, 1979
6. H. Mayeda, On **Structural Controllability** Theorem, IEEE Trans. Auto. Contr., 26(3): 795-798, 1981
7. A. Linnemann, A Further Simplification in the Proof of the **Structural Controllability** Theorem, IEEE Trans. Auto. Contr., 31(7): 638-639, 1986
8. J. Willems, **Structural Controllability** and Observability, Syst. Contr. Lett., 8(1): 5-12, 1986

In retrospect: large-scale systems theory

Structural Analysis of Dynamical Systems



Q:
Is this kind of
structures
controllable?

(not get into details here)

Courtesy of Prof Yugeng Xi

Structural Controllability

Corresponding linearized system has the following general form:

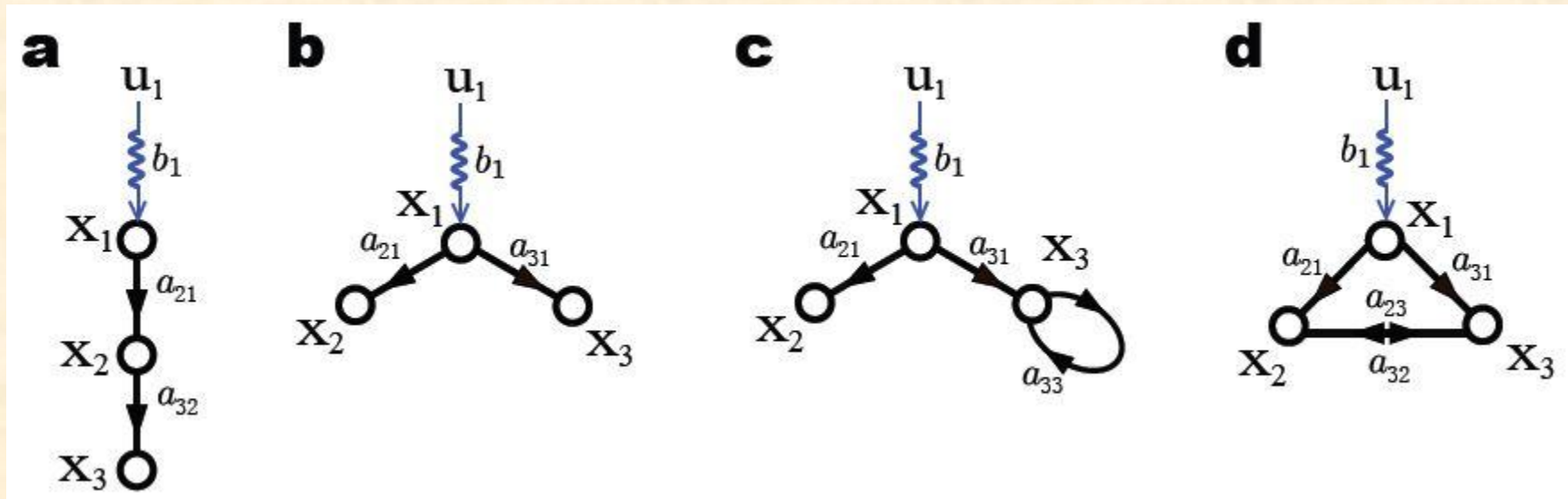
$$\begin{aligned}\dot{x}_1 &= a_{11}x_1 \\ \dot{x}_2 &= a_{21}x_1 + a_{22}x_2 + bu, \\ \dot{x}_3 &= a_{32}x_2 + a_{33}x_3\end{aligned}\quad \mathbf{A} = \begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ 0 & a_{32} & a_{33} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix}$$

$$\text{rank} \begin{bmatrix} \mathbf{B}, \mathbf{AB}, \mathbf{A}^2\mathbf{B} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ b & a_{22}b & a_{22}^2b \\ 0 & a_{32}b & a_{32}(a_{22} + a_{33})b \end{bmatrix} \leq 2$$



Uncontrollable

Examples: Structure matters



$$\mathbf{C} = [\mathbf{B}, \mathbf{A} \cdot \mathbf{B}, \mathbf{A}^2 \cdot \mathbf{B}]$$

$$b_1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & a_{21} & 0 \\ 0 & 0 & a_{32}a_{21} \end{bmatrix},$$

rank $\mathbf{C} = 3 = N$
controllable

$$b_1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & a_{21} & 0 \\ 0 & a_{31} & 0 \end{bmatrix},$$

rank $\mathbf{C} = 2 < N = 3$
uncontrollable

$$b_1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & a_{21} & 0 \\ 0 & a_{31} & a_{33}a_{31} \end{bmatrix},$$

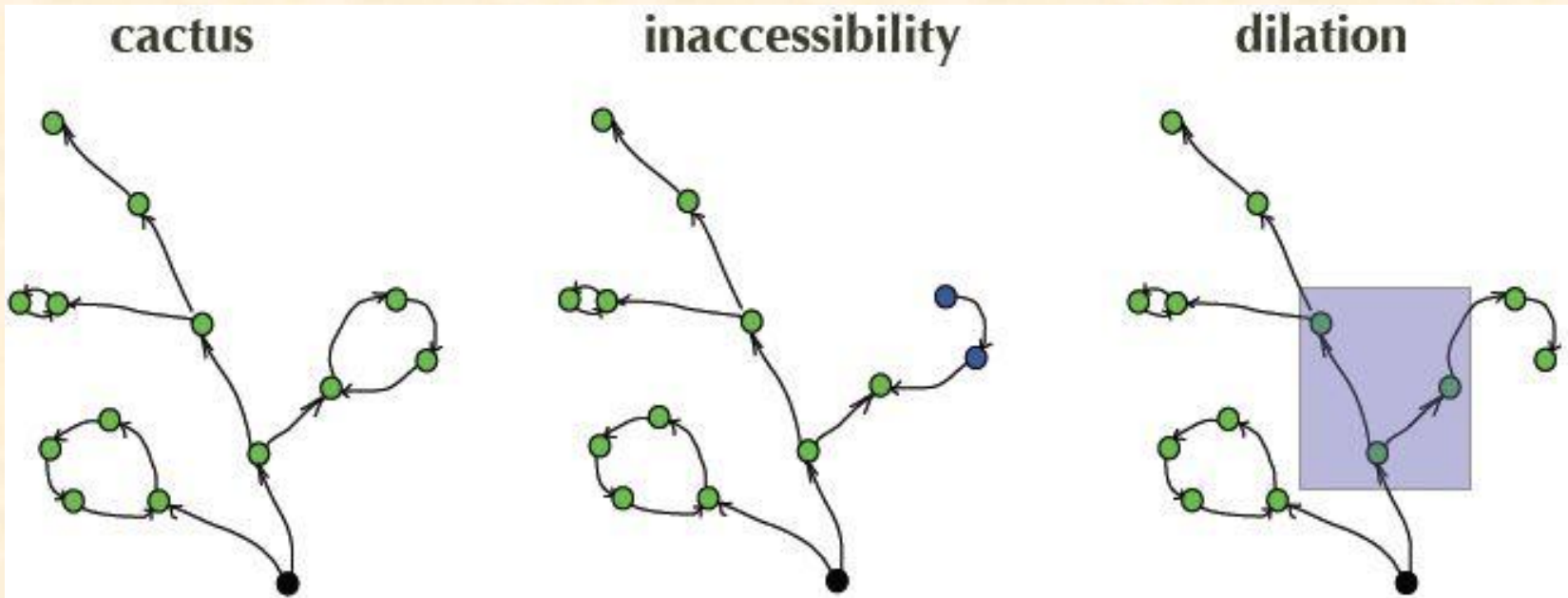
rank $\mathbf{C} = 3 = N$
controllable

$$b_1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & a_{21} & a_{23}a_{31} \\ 0 & a_{31} & a_{32}a_{21} \end{bmatrix}$$

rank $\mathbf{C} = ?$
controllable?

See also: Y.Y. Liu, J.J. Slotine, and A.L. Barabási, *Nature*, 2011

Building Blocks



Cactus is the minimum structure which contains no inaccessible nodes and no dilations

Structural Controllability Theorem

The following three statements are equivalent:

1. A LTI control system (A,B) is structurally controllable
2. The digraph $G(A,B)$ contains neither inaccessible nodes nor dilations
3. The digraph $G(A,B)$ is spanned by a cactus

Leader-Followers Network (case study)

$$\dot{x}_i = c_i \Gamma x_i + \sum_{j=1}^N L_{ij} \Gamma [x_i - x_j] + \delta_i B u_i \quad i = 1, \dots, q, q+1, \dots, N$$

$$\Rightarrow \dot{X}(t) = [(C - L) \otimes \Gamma] X(t) + (\Delta \otimes B) U(t)$$

$$X(t) = [x_1^T(t), \dots, x_N^T(t)]^T \in R^{nN}$$

$$U(t) = [u_1^T(t), \dots, u_q^T(t), 0^T, \dots, 0^T]^T \in R^{nN}$$

$$C = \text{diag}\{c_1, \dots, c_N\}$$

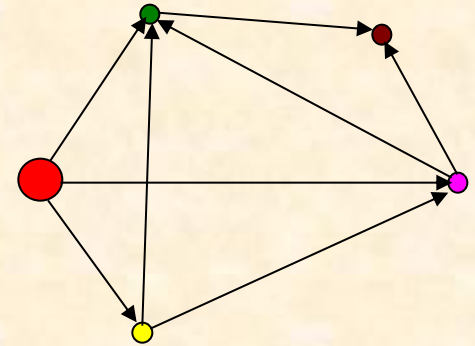
$$\Delta = \text{diag}\{1, \dots, 1, \overbrace{0, \dots, 0}^{N-q}\}$$

leaders: $V_l = \{1, \dots, q\}$

followers: $V_f = \{q+1, \dots, N\}$

leader graph: G_l

follower graph: G_f



Leader-Followers Network (case study)

$$\dot{X}(t) = [(C - L) \otimes \Gamma]X(t) + (\Delta \otimes B)U(t)$$

$$\bar{L} = L - C = \begin{bmatrix} L_l - C_l & L_{lf} \\ L_{fl} & L_f - C_f \end{bmatrix}$$

Two independent problems:

- (i) Controllability of the **isolated leader subsystem** $\dot{x} = \Gamma x + Bu$
- (ii) Controllability of the **extended graph** $\bar{G}$ (leaders + followers)

Note: (i) is classical; (ii) network controllability

Main Result

$$\dot{X}(t) = [(C - L) \otimes \Gamma]X(t) + (\Delta \otimes B)U(t) \quad (*)$$

Theorem: System $(*)$ is controllable if and only if the following two conditions are satisfied simultaneously:

- (i) (Γ, B) is a controllable matrix pair;
- (ii) there exists no left eigenvector of $\bar{L}$ with the first q entries being all zeroes.

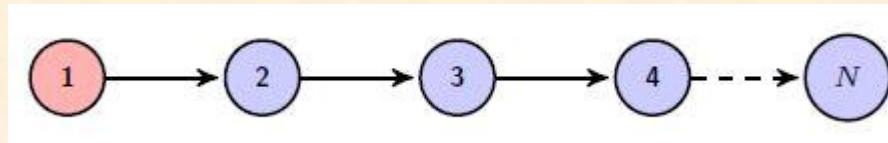
$$\bar{L} = L - C = \begin{bmatrix} L_l - C_l & L_{lf} \\ L_{fl} & L_f - C_f \end{bmatrix}$$

Two Trivial Corollaries

- **Assume:** $c_1 = c_2 = \dots = c_N = c$

Corollary 1 A directed path is **controllable** if the beginning node is selected to be the only leader

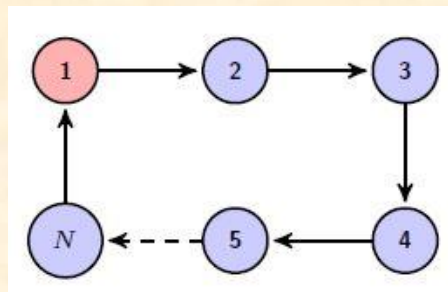
$$\bar{L} = \begin{bmatrix} -c & 0 & 0 & \dots & 0 \\ -1 & 1-c & 0 & \dots & 0 \\ 0 & -1 & 1-c & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \dots & -1 & 1-c \end{bmatrix}$$



Two Trivial Corollaries

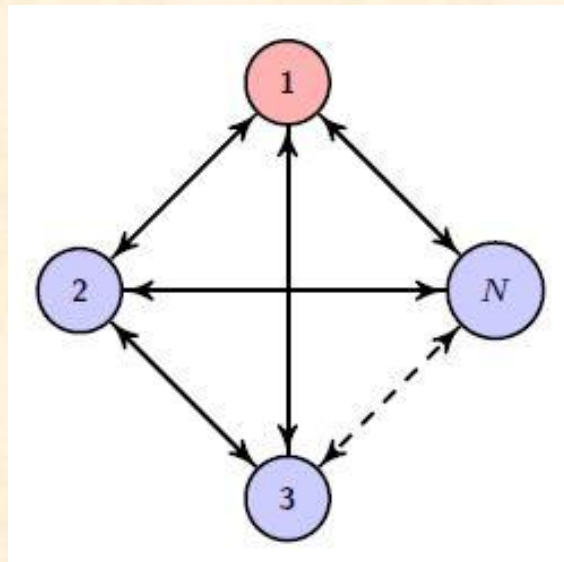
Corollary 2 A directed cycle with a single leader is controllable

$$\bar{L} = \begin{bmatrix} 1-c & 0 & 0 & \cdots & -1 \\ -1 & 1-c & 0 & \cdots & 0 \\ 0 & -1 & 1-c & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & -1 & 1-c \end{bmatrix}$$



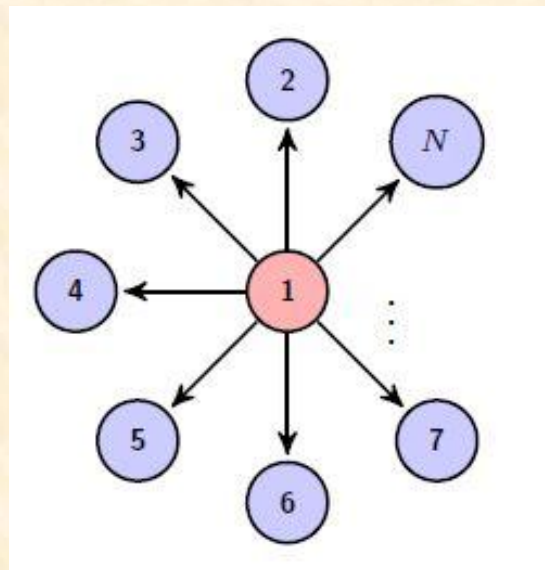
Two Surprising Corollaries

Corollary 3 A complete digraph with a single leader is uncontrollable.

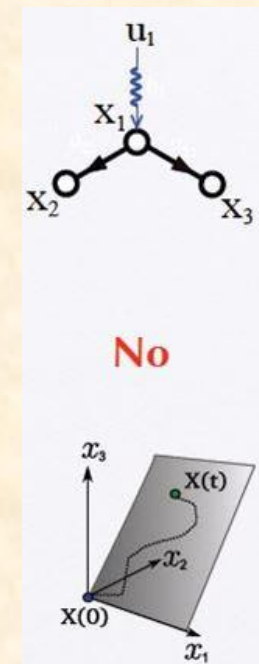


Two Surprising Corollaries

Corollary 4 A star digraph ($N > 2$) is uncontrollable with the center node being the leader



Recall:



References

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- Yan G, Ren J, Lai Y C, et al. Controlling complex networks: How much energy is needed?. Physical Review Letters, 2012, 108(21): 218703.

BREAK

10 minutes

Complex Dynamical Networks:

Lecture 9b: Flocking Control

EE 6605

Instructor: G Ron Chen



Consensus and Control over Complex Networks

- Swarm Dynamics / Modeling
- Consensus Protocols / Analysis
- Flocking Algorithms / Control
- DEMO



Fish Schooling



Birds Flocking

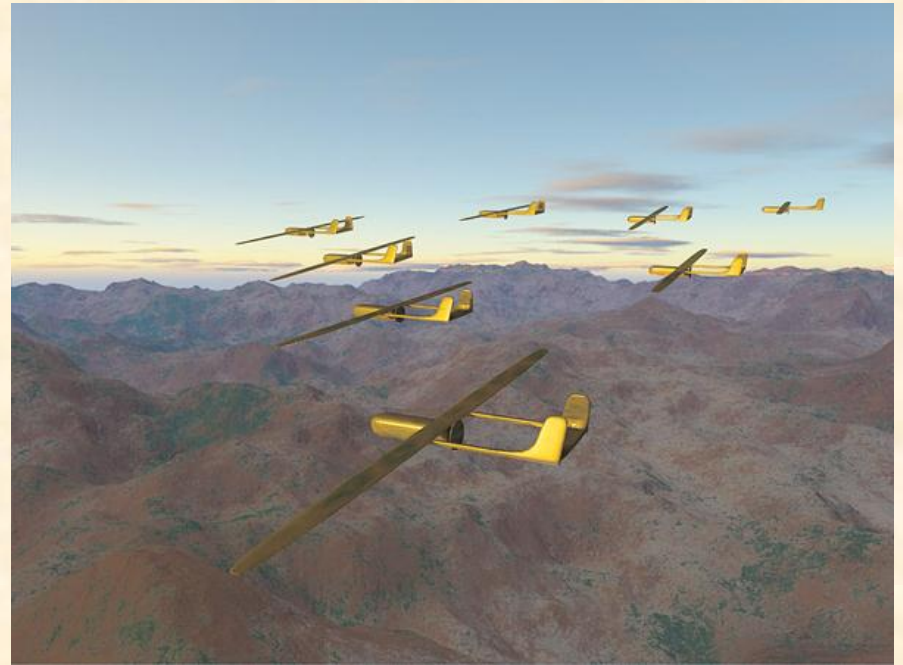
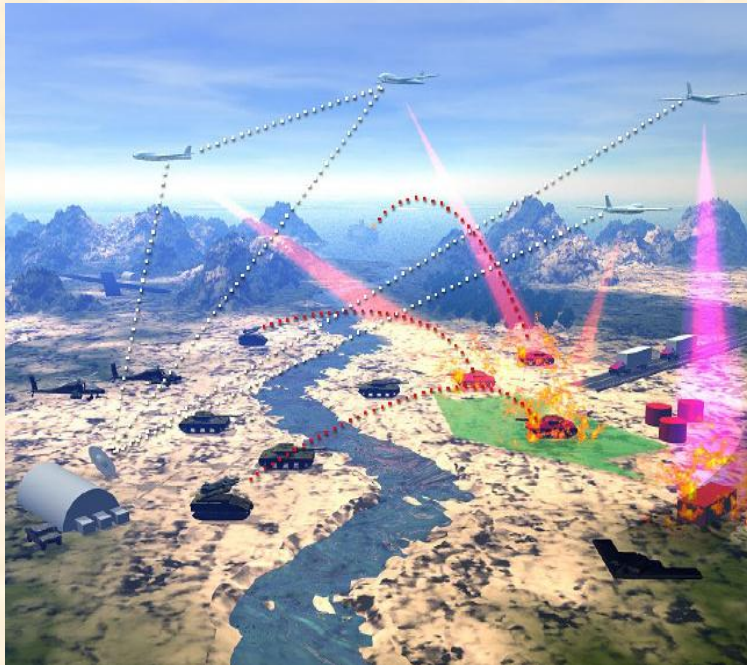


Flocking: to congregate or travel in flock

DEMO

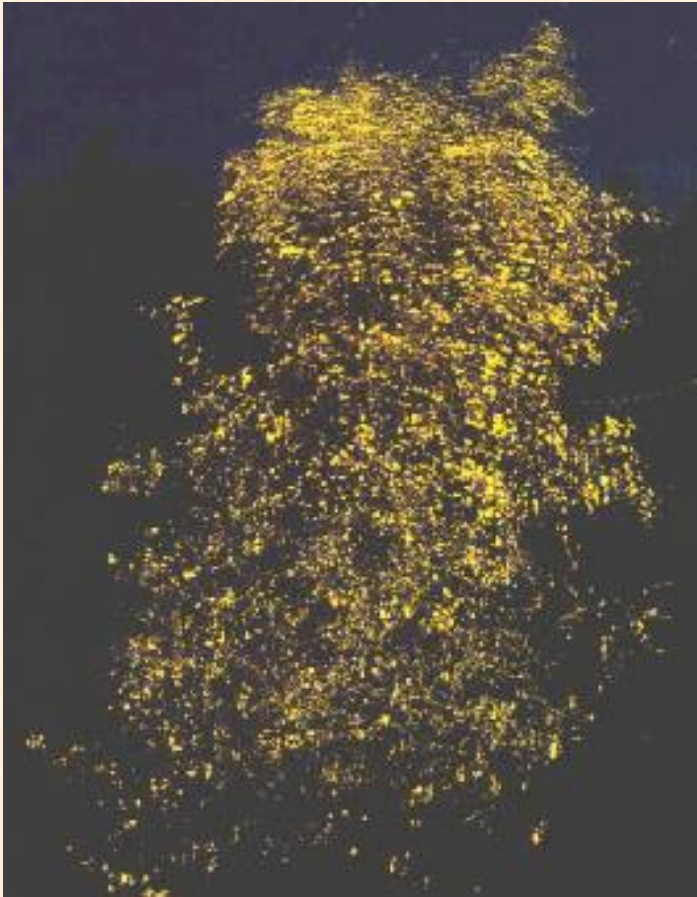
Consensus

A position reached by a group as a whole

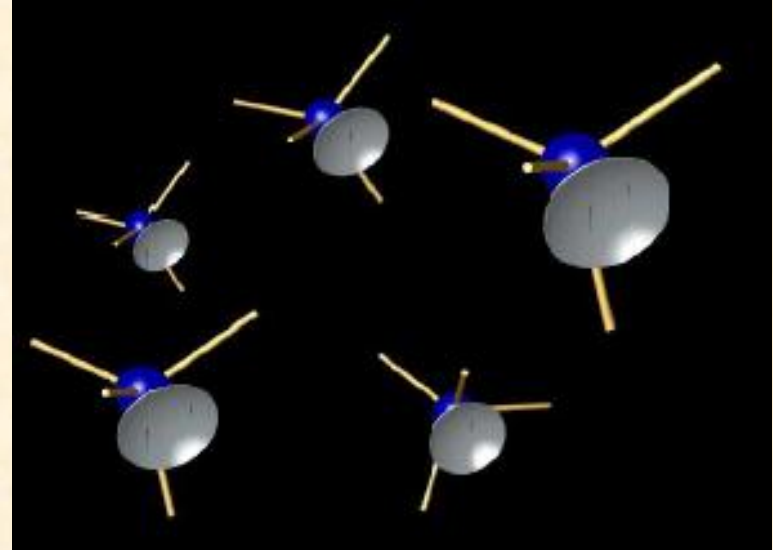


Battle space management scenario

Fireflies Synchronization



Attitude Alignment



The attitude of each spacecraft is synchronized with its two adjacent neighbors via a bi-directional communication channel

What are in common ?

- ❖ Swarming
- ❖ Flocking
- ❖ Rendezvous
- ❖ Consensus
- ❖ Synchrony
- ❖ Cooperation
- ❖



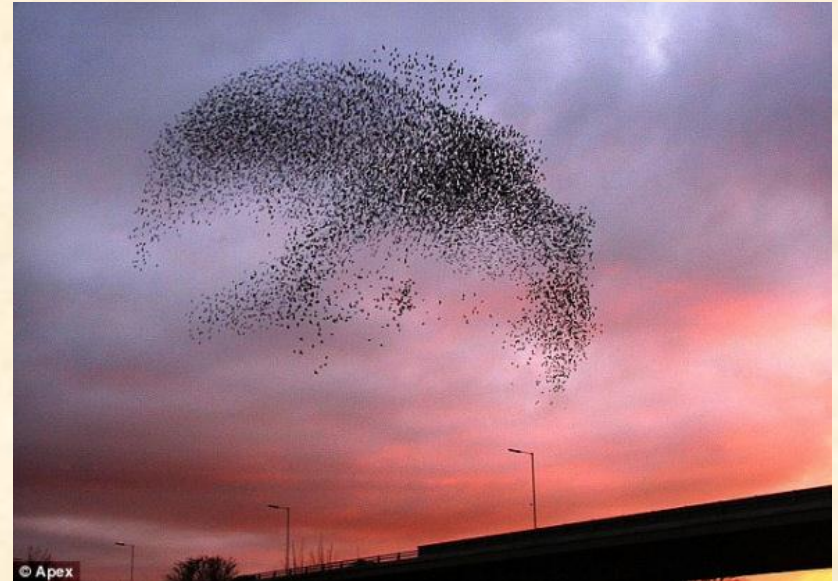
Distributed coordination of a network of agents:

- ✓ Agents
- ✓ Network
- ✓ Distributed local control
- ✓ Global consensus

Flocking



Flocking (Some Real Photos)



DEMO

Vicsek Model

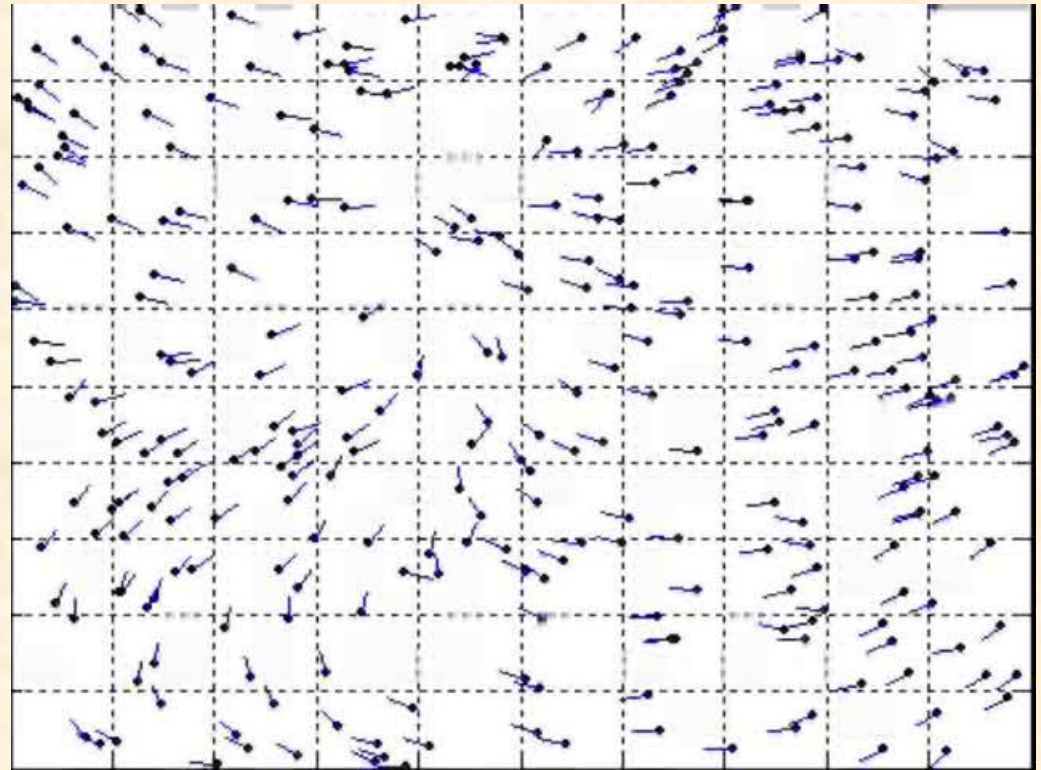
Position:

$$x_i(t+1) = x_i(t) + v\Delta t$$

Heading:

$$\theta(t+1) = \langle \theta(t) \rangle_R + \Delta\theta$$

→ **coordinated motion**



Finally converging to average direction of initial headings

Vicsek, Czirok, Jacob, Cohen, Shochet, “Novel type of phase transition in a system of self-driven particles,” Phys. Rev. Lett., 1995, 75 (6): 1226

Vicsek Model

DEMO

Convergence Analysis

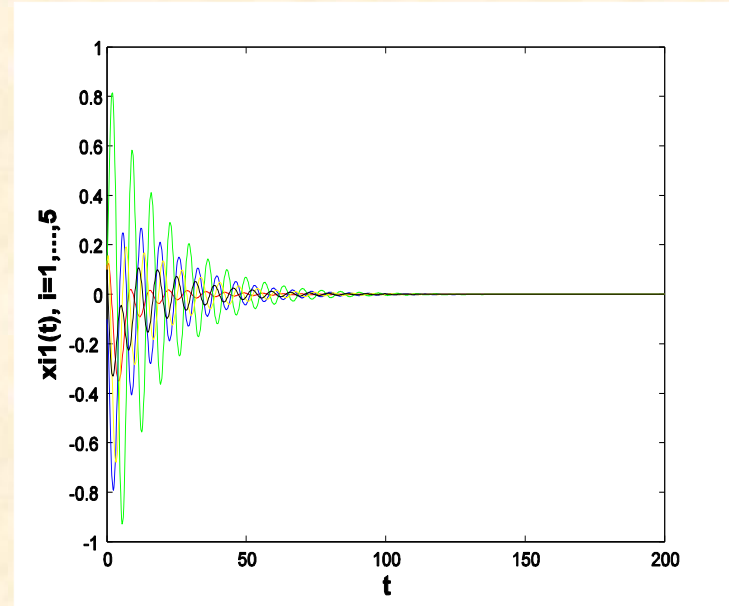
$$\theta(t+1) = F\theta(t)$$

$$\lim_{t \rightarrow \infty} \theta(t) = \theta_{ss} \mathbf{1}$$

F – random matrix

θ_{ss} – steady state

$$\mathbf{1} = [1, 1, \dots, 1]^T$$



Jadbabaie, Lin, Morse, “Coordination of groups of mobile autonomous agents using nearest neighbor rules,” IEEE Trans. Auto. Control, 2003, 48(6): 988-1001

Moreau, “Stability of multiagent systems with time-dependent communication links,” IEEE Trans. Auto. Control, 2005, 50(2): 169-182

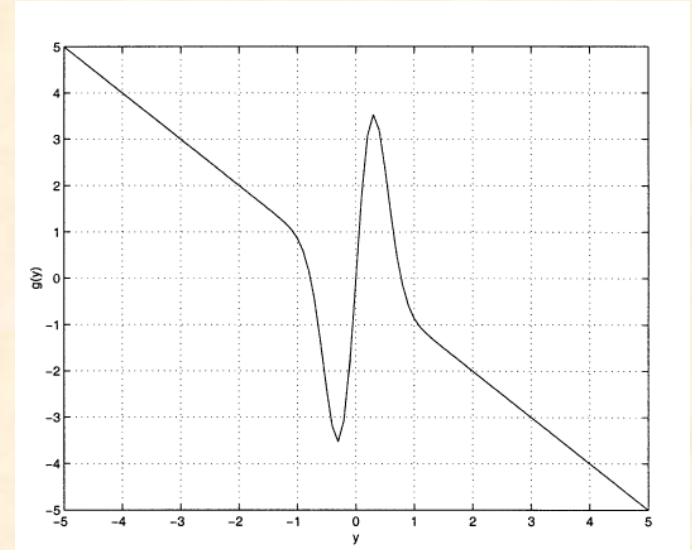
Stability Analysis

- ❖ A group of M globally nonlinearly coupled individuals:

$$\frac{dx_i}{dt} = \sum_{j=1}^M g(x_i - x_j), \quad i = 1, 2, \dots, M$$

Attraction/Repulsion function:

$$g(y) = -y \left[a - b \exp \left(-\frac{\|y\|^2}{c} \right) \right]$$



- ❖ All the agents will converge to a spherical region:

$$B_\varepsilon = \{x : \|x - \bar{x}\| \leq \varepsilon\} \quad \bar{x} = \frac{1}{M} \sum_{i=1}^M x_i \quad \varepsilon = \frac{b}{a} \sqrt{c/2} \exp(-1/2)$$

Gazi, Passino, IEEE Trans. Auto. Control, 2003, 48: 692–697

Stability Analysis of Foraging Swarms

$$\dot{x}^i = -\nabla_{x^i} \sigma(x^i) + \sum_{j=1, j \neq i}^M g(x^i - x^j), i = 1, \dots, M$$

A/R function: $g(y) = -y[g_a(\|y\|) - g_r(\|y\|)]$

Linear attraction

$$g_a(\|x^i - x^j\|) = a,$$

bounded repulsion

$$g_r(\|x^i - x^j\|) \|x^i - x^j\| \leq b,$$

$$\|\nabla_y \sigma(y)\| \leq \bar{\sigma} \quad \rightarrow \quad x^i(t) \rightarrow B_\varepsilon(\bar{x}(t))$$

$$M \rightarrow \infty$$



$$\varepsilon = b/a$$

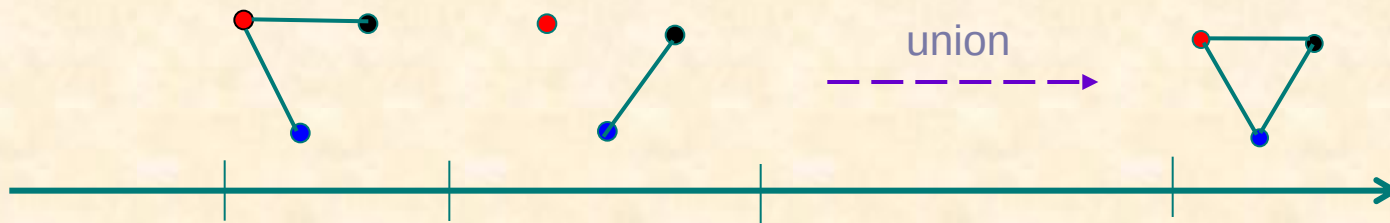
$$\varepsilon = \frac{M-1}{aM} \left[b + \frac{2\bar{\sigma}}{M} \right]$$

Gazi, Passino, IEEE Trans. Sys. Man Cybern., B: 2004, 34(1): 539-557

Consensus

Design a network connection topology, or design local control law, so that $\|x_i - x_j\| \rightarrow 0$ (consensus = synchronization)

Consensus is achieved asymptotically if there exists an infinite sequence of bounded intervals such that the union of the graphs over such intervals is totally connected.



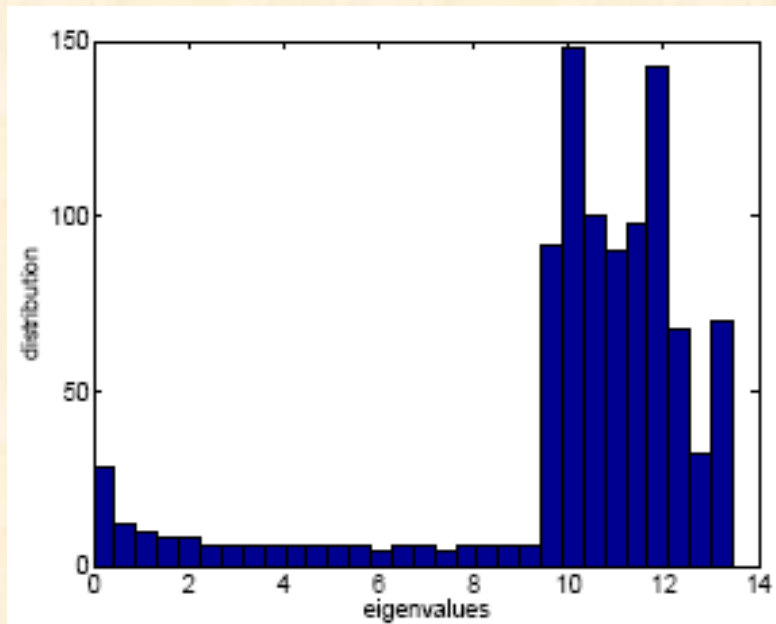
Olfati-Saber, Murray, "Consensus problems in networks of agents with switching topology and time-delays," IEEE Trans. Auto. Control, 2004, 49(9): 1520-1533

Small-World Networks are better for Consensus

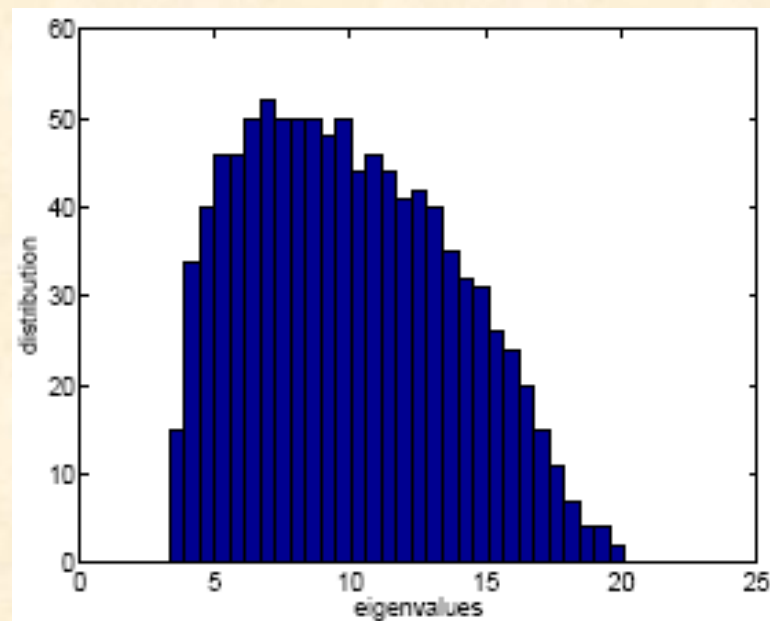
$$\lambda_N / \lambda_2$$

Condition number -- the smaller, the better

Regular networks



Small-world networks



1000 times average

Olfati-Saber, "Ultrafast consensus in small-world networks," Amer. Control Conf., 2005

Flocking





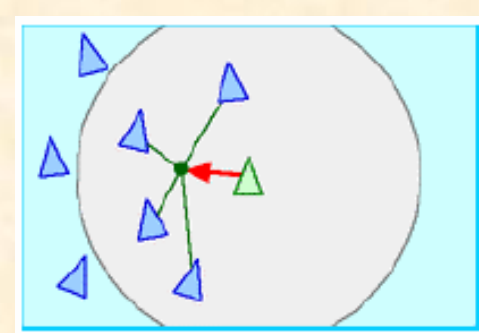
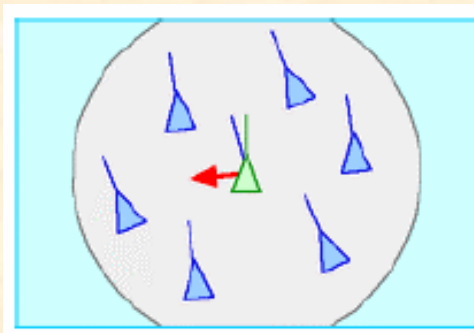
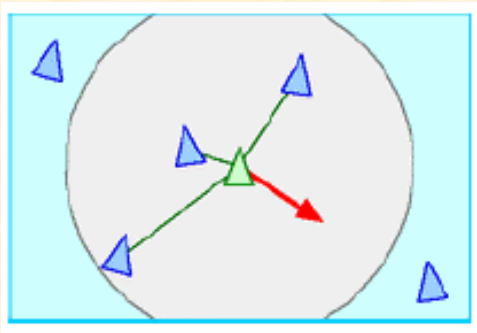
Boids Flocking Model

Three Rules:

Separation: Steer to avoid crowding local flockmates

Alignment: Steer to move toward the average heading of local flockmates

Cohesion: Steer to move toward the average position of local flockmates

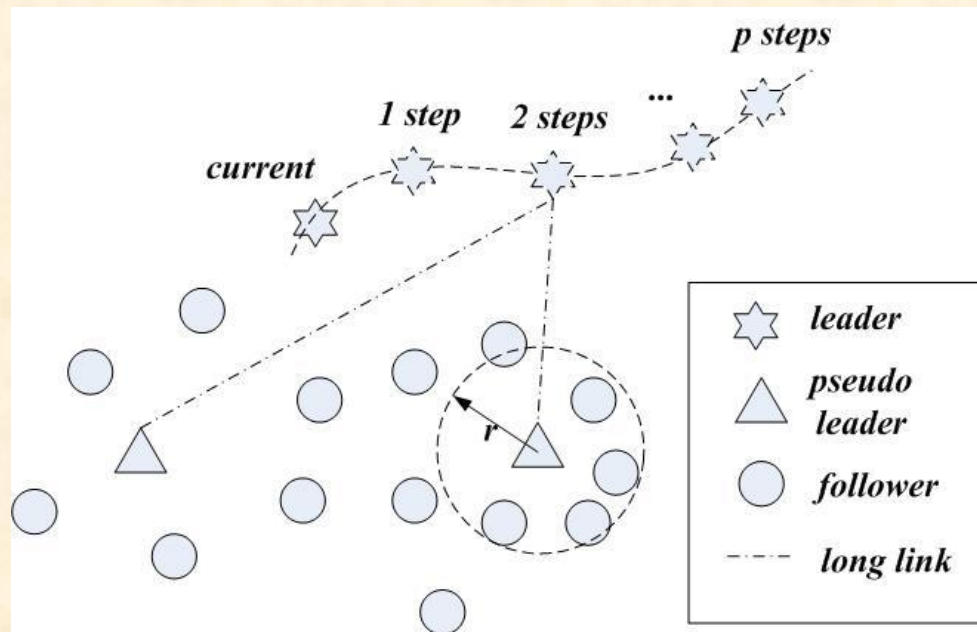


Reynolds, “Flocks, herd, and schools: A distributed behavioral model,”

Computer Graphics, 1987, 21(4): 15-24 <http://www.red3d.com/cwr/boids/>

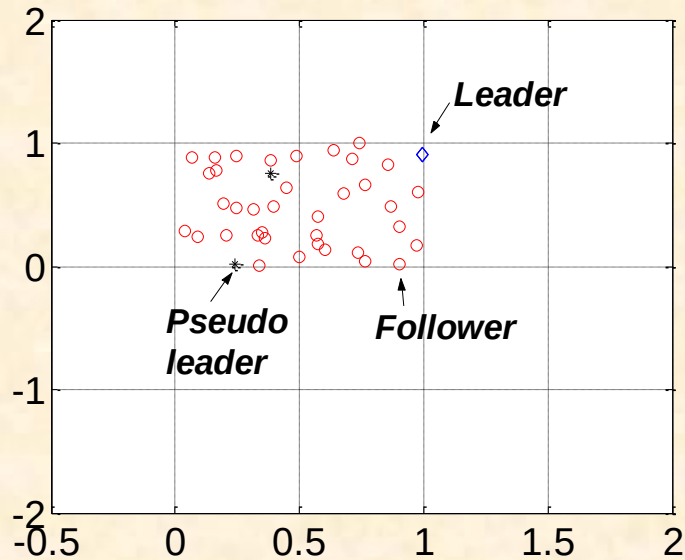
Flocking: Small-World Organization

Small-world communication generates “pseudo-leaders” who control their neighbors

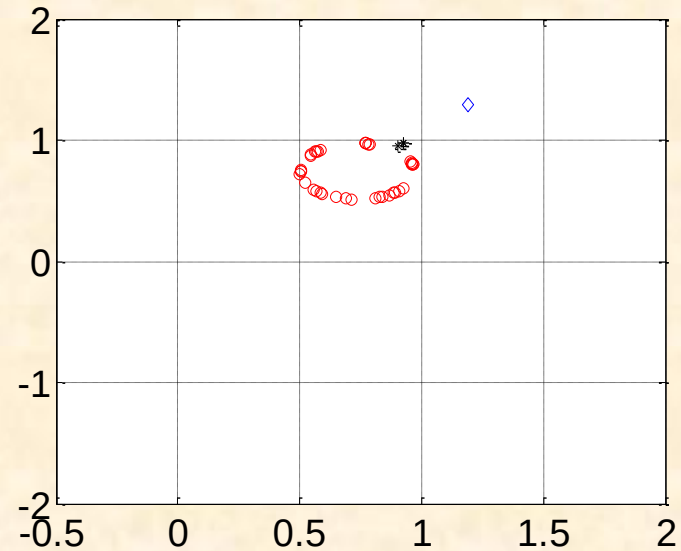


Zhang, Chen, “Small-world network-based coordinate predictive control of flocks,”
PhysCon, Germany, 2007

Flocking: Small-World Organization

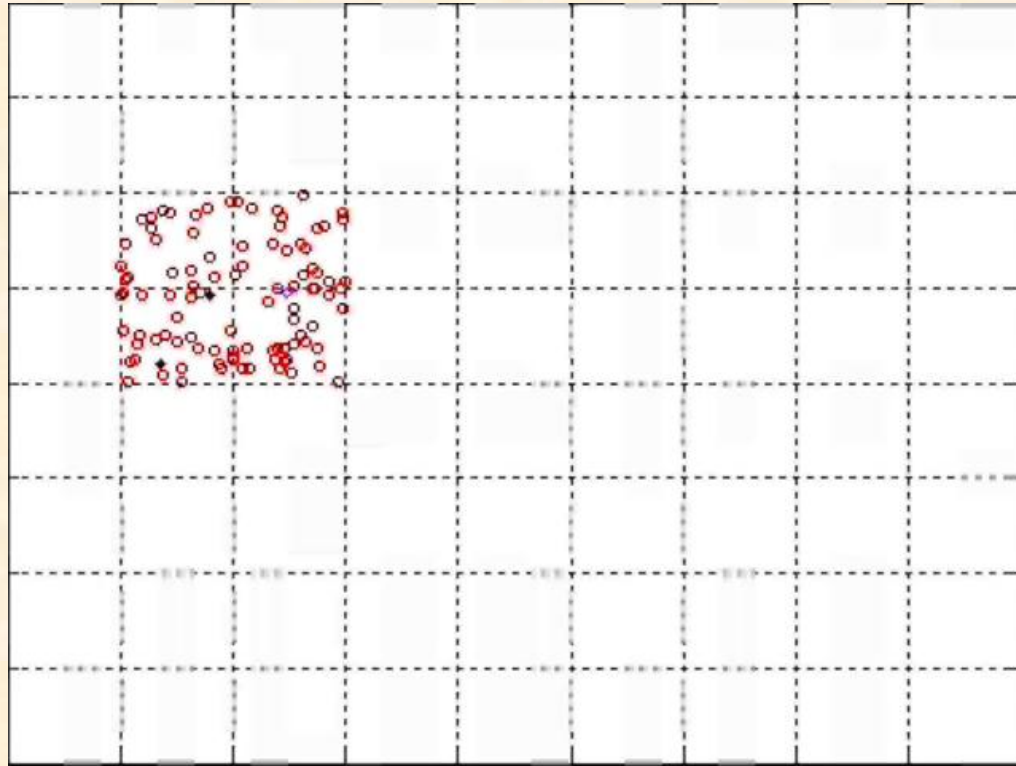


Initial position of the flock



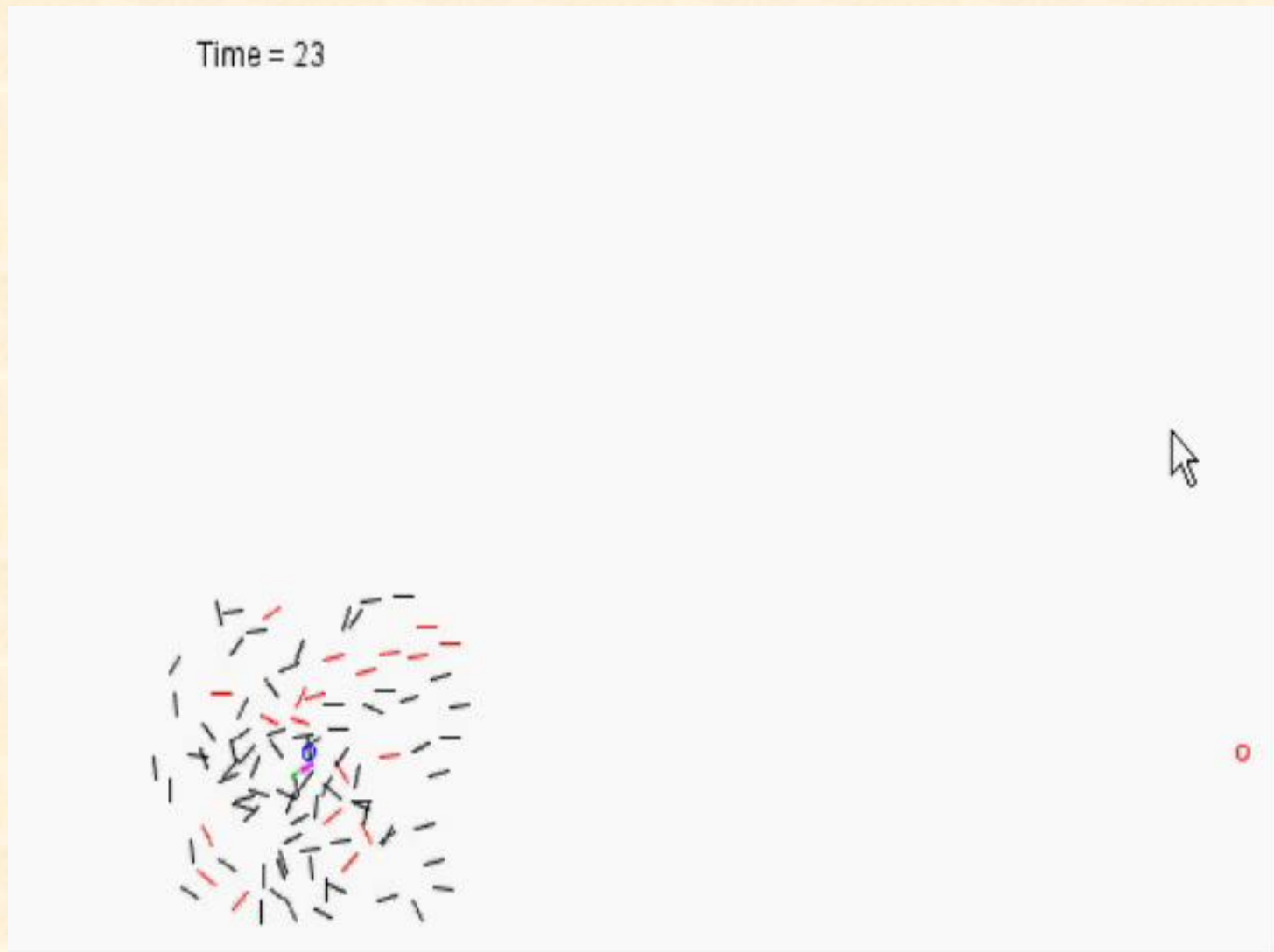
Flock position after 40 iterations

Flocking: DEMO



H. Zhang, G. Chen, "Small-world network-based coordinate predictive control of flocks," PhysCon, Germany, 2007

Another Flocking Algorithm: DEMO

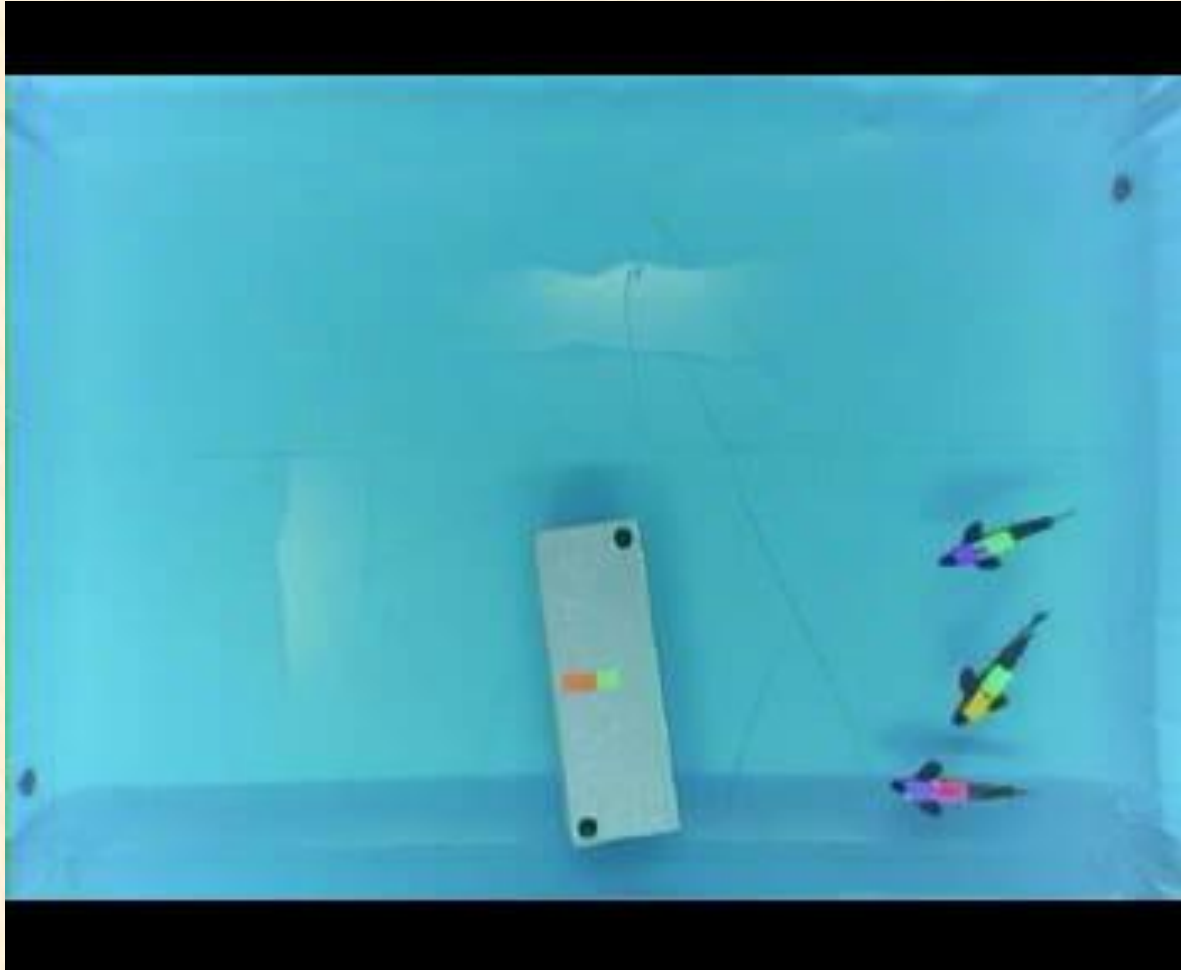


Movies: Robot Fish



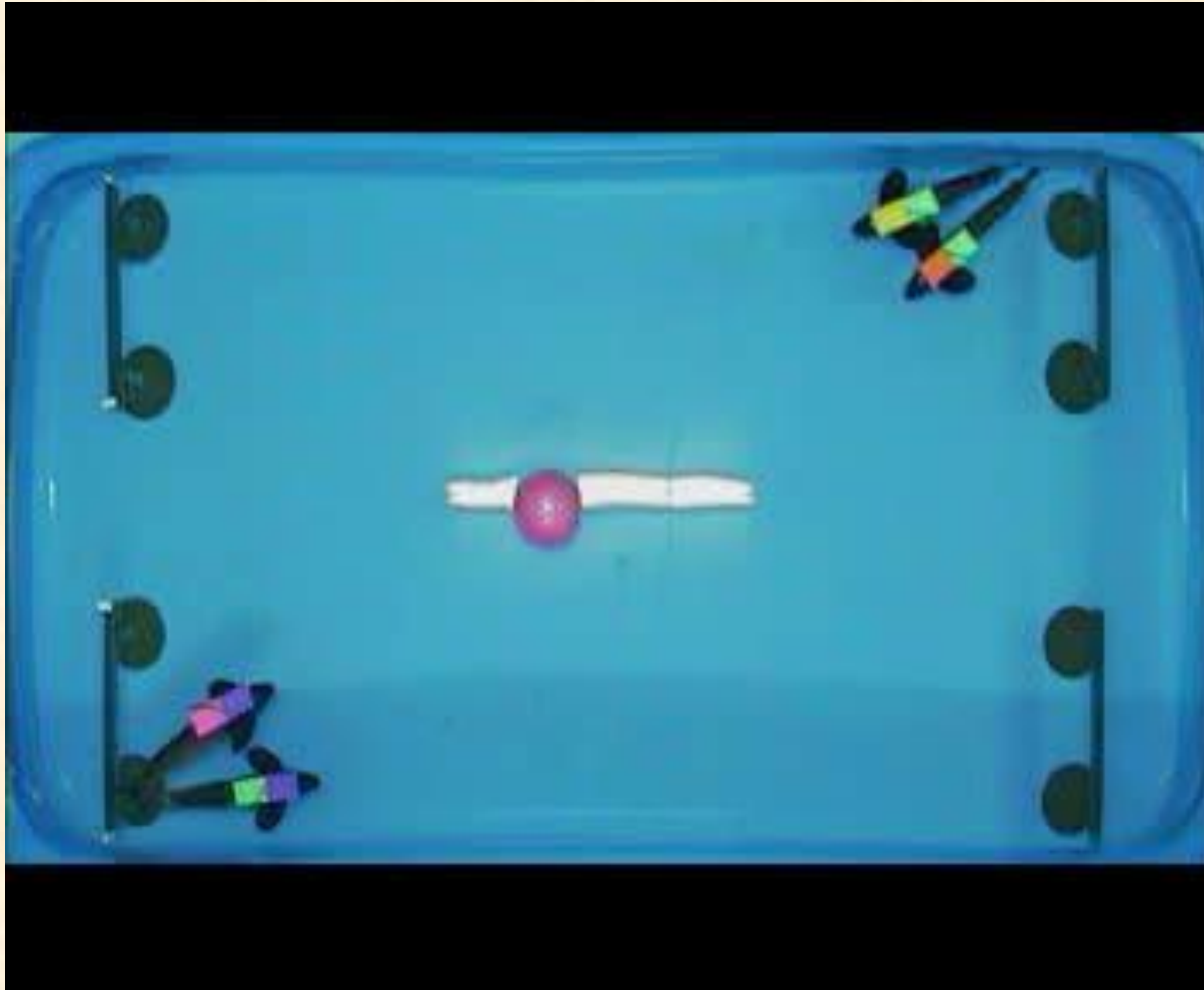
Acknowledgement: College of Engineering, Peking University

Movie 1: Coordination



Acknowledgement: College of Engineering, Peking University

Movie 2: Cooperation



Acknowledgement: College of Engineering, Peking University

End

