

Due date: Sunday, 3 July 2022 before 11:59 p.m.

Contribution of this assignment to the final mark: 5%

- Answer **all** questions. Show **all** steps.
- Please submit the softcopy of your work (as a **single pdf file**) to Canvas on or before the due date.

Questions

1. Suppose that $U = [0, \infty)$ is the universal set. Let $A = [3, 7]$ and $B = (5, 9]$ be two intervals; $D = \{1, 2, 3, 4, 5, 6\}$ and $E = \{5, 6, 7, 8, 9, 10\}$ be two sets. Find the following sets and write your answers in set/interval notations:

- (a) $(A \cup E) \cap B^c$
- (b) $(A^c \cap B)^c \cup E$
- (c) $(A \setminus D) \cap (B \setminus E)$

2. Find the largest possible domain and largest possible range for each of the following real-valued functions:

- (a) $F(x) = \sqrt{\frac{4x+3}{2x-1}}$
- (b) $G(x) = \frac{2}{x^2 - 6x + 8}$

Write your answers in set/interval notations.

3. Let $f: [0, \infty) \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ be two functions defined by

$$f(x) = \sqrt{x} - 1 \quad \text{and} \quad g(x) = \begin{cases} x + 2 & \text{for } x < 1 \\ x^2 & \text{for } x \geq 1 \end{cases}.$$

Find the expressions for the following composite functions and state their largest possible domains:

- (a) $(f \circ f)(x)$
- (b) $(g \circ f)(x)$
- (c) $(g \circ g)(x)$

4. Given that $\sin \alpha = -\frac{3}{5}$ and $\tan \beta = -\frac{8}{15}$, where $\pi < \alpha < \frac{3\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$. Find the exact values of $\csc(\alpha - \beta)$ and $\cos\left(\frac{\alpha}{2} + \beta\right)$.

(Questions 5 to 7 are on the next page.)

5. (a) Prove the identity

$$\sin x + 2 \sin 3x + \sin 5x = 4 \cos^2 x \sin 3x.$$

- (b) Using (a), or otherwise, find the general solution (in radians) of the equation

$$\sin x + \sin 3x + \sin 5x = 0.$$

6. Find the sum

$$S = \cos(2x) + \cos(4x) + \cos(6x) + \cdots + \cos(2nx),$$

where n is a positive integer. Express your answer as a product and quotient of trigonometric functions.

7. (a) Determine whether $x - 1$, $x + 2$ and $x + 3$ are factors of $q(x) = x^4 + 4x^3 + 4x^2 - 9$.

- (b) Express $\frac{2x^3 + 8x^2 - 4x + 18}{x^4 + 4x^3 + 4x^2 - 9}$ into partial fractions.

- End -