

Block C Unit 2 Outline

1) Analysis with single source

- Methodology [Section 10.1]
- Two examples

Alexander & Sadiku,
“Fundamentals of Electric Circuits”
5th Edition Chapters 9 &10

2) Analysis with multiple sources (single frequency)

- Nodal analysis [Section 10.2] (1 example)
- Mesh analysis [Section 10.3] (1 example)

3) Analysis by superposition (multiple frequencies)

- Circuits containing DC sources and AC sources of a single frequency (1 example)
- Circuits containing multiple AC sources with different frequencies (1 example)

4) AC Power: Instantaneous vs. Average

General methodology

- 1) Transform the circuit to the frequency domain
 - Transform AC sources from time domain sinusoids to phasor form
 - Work out impedances for a given frequency ω
- 2) Solve the problem using circuit techniques
- 3) Transform phasor form solutions back to time domain

$$v(t) = V_m \cos(\omega t + \phi) \Leftrightarrow V = V_m \angle \phi$$

(Time-domain
representation)

(Phasor-domain
representation)

TABLE 9.3

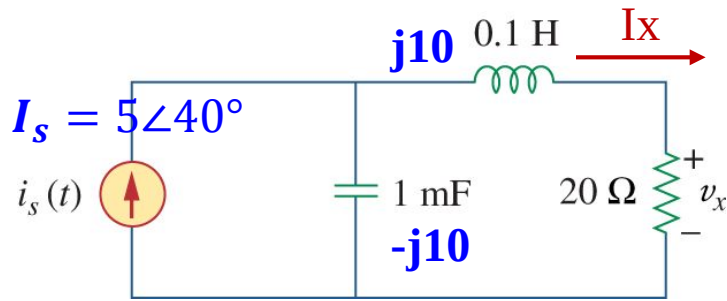
Impedances and admittances of passive elements.

Element	Impedance	Admittance
R	$Z = R$	$Y = \frac{1}{R}$
L	$Z = j\omega L$	$Y = \frac{1}{j\omega L}$
C	$Z = \frac{1}{j\omega C}$	$Y = j\omega C$

1A) Single source analysis example 1

Alexander 9.50

Given that $i_s(t) = 5 \cos(100t + 40^\circ)$, determine $v_x(t)$



Step 1a: Transform source to phasor

$$i_s(t) = 5 \cos(100t + 40^\circ) \rightarrow I_s = 5 \angle 40^\circ, \omega = 100 \frac{\text{rad}}{\text{s}}$$

Step 1b: Work out the impedance values

$$1 \text{ mF} \rightarrow -j/(100 \cdot 0.001) = -j10$$

$$0.1 \text{ H} \rightarrow j100 \cdot 0.1 = j10$$

Step 2: Solve using circuit analysis methods

Using current divider rule,

$$I_x = \left(\frac{-j10}{-j10 + j10 + 20} \right) (5 \angle 40^\circ) = (-j0.5)(5 \angle 40^\circ) = 2.5 \angle -50^\circ$$

$$V_x = 20 I_x = 50 \angle -50^\circ$$

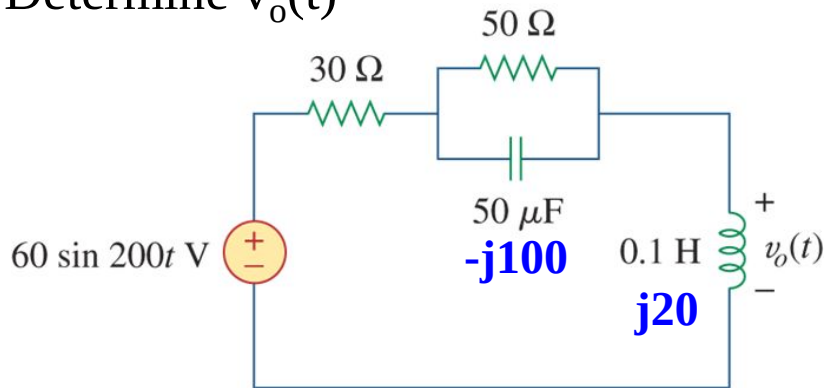
Step 3: Transform back to time domain

$$V_x = 50 \angle -50^\circ, \omega = 100 \frac{\text{rad}}{\text{s}} \rightarrow v_x(t) = 50 \cos(100t - 50^\circ) \text{ V}$$

1B) Single source analysis example 2

Alexander 9.42

Determine $v_o(t)$



Step 1a: Transform source to phasor

$$60 \sin(200t) \rightarrow 60 \angle -90^\circ, \omega = 200 \frac{\text{rad}}{\text{s}}$$

$$60 \cos(200t - 90^\circ)$$

Step 1b: Work out the impedance values

$$50 \mu\text{F} \rightarrow -j/(200 \cdot 50 \cdot 10^{-6}) = -j100$$

$$0.1 \text{ H} \rightarrow j200 \cdot 0.1 = j20$$

Step 2: Solve using circuit analysis methods

$$50 \parallel -j100 = \frac{(50)(-j100)}{(50 - j100)} = 40 - j20$$

Using voltage divider rule,

$$V_o = \left(\frac{j20}{40 - j20 + 30 + j20} \right) (60 \angle -90^\circ) = 17.14 \angle 0^\circ$$

Step 3: Transform back to time domain

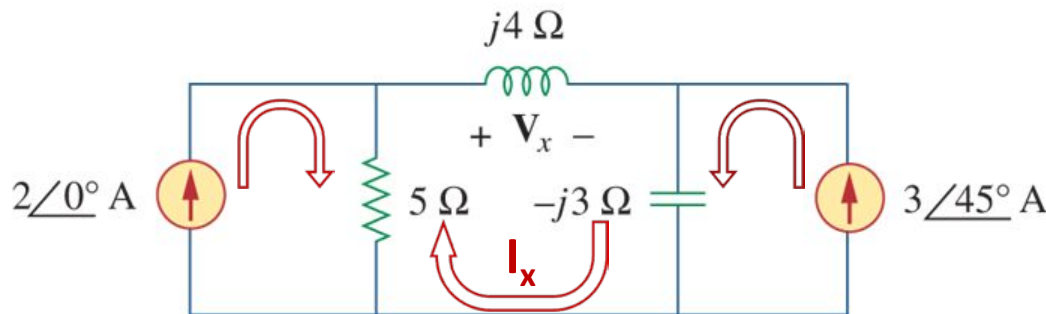
$$V_o = 17.14 \angle 0^\circ, \omega = 200 \frac{\text{rad}}{\text{s}} \rightarrow v_x(t) = 17.14 \cos(200t) \text{ V}$$

2A) Single freq multiple sources example 1

Focus only on STEP 2 (solving by circuit techniques) in this example

Alexander 10.16

Use **MESH CURRENT ANALYSIS** to find V_x



Applying KVL around mesh I_x :

$$I_x(j4 - j3 + 5) - (2\angle 0^\circ)(5) + (3\angle 45^\circ)(-j3) = 0$$

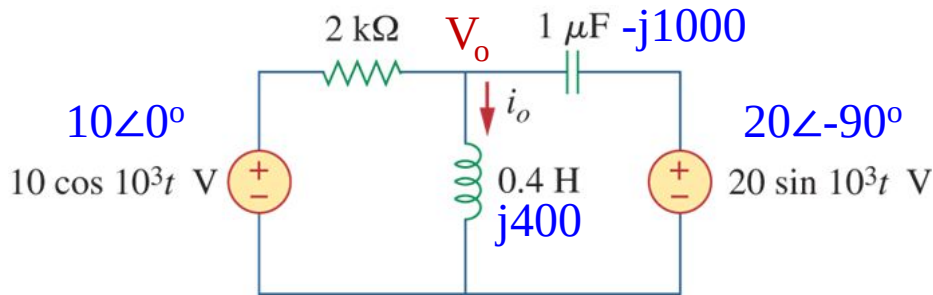
$$\Rightarrow I_x = 1.437\angle 48.94^\circ \text{ A}$$

$$V_x = (1.437\angle 48.94^\circ)(j4) = 5.749\angle 138.94^\circ \text{ V}$$

2B) Single freq multiple sources example 2

Alexander 10.26

Use nodal voltage analysis to find $i_o(t)$



Step 1a: Transform source to phasor

$$10 \cos(10^3 t) \rightarrow 10 \angle 0^\circ, \omega = 10^3 \text{ rad/s}$$

$$20 \sin(10^3 t) \rightarrow 20 \angle -90^\circ, \omega = 10^3 \text{ rad/s}$$

Step 1b: Work out the impedance values

$$1 \mu\text{F} \rightarrow -j/(10^3 * 10^{-6}) = -j1000$$

$$0.4 \text{ H} \rightarrow j10^3 * 0.4 = j400$$

Step 2: Solve using circuit analysis methods

Applying KCL at node V_o :

$$\frac{(10 \angle 0^\circ) - V_o}{2000} + \frac{(20 \angle -90^\circ) - V_o}{-j1000} = \frac{V_o}{j400}$$

$$\Rightarrow V_o = 15.8 \angle 71.57^\circ \text{ V}$$

$$I_o = (15.8 \angle 71.57^\circ) / j400 = 39.5 \angle -18.43^\circ \text{ mA}$$

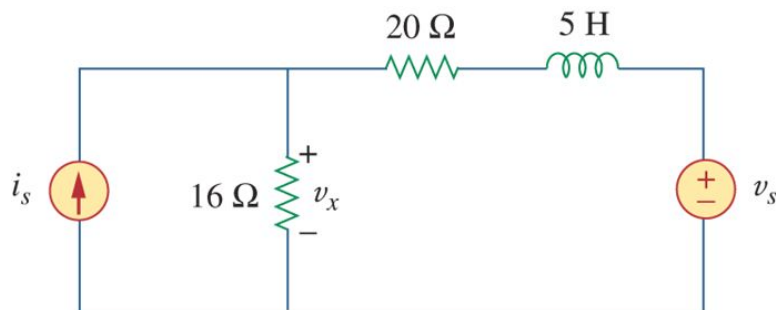
Step 3: Transform back to time domain

$$I_o = 39.5 \angle -18.43^\circ \text{ mA}, \omega = 10^3 \text{ rad/s} \rightarrow i_o(t) = 39.5 \cos(10^3 t - 18.43^\circ) \text{ mA}$$

3A) Superposition (multiple freq) example 1

Alexander 10.44 (Modified)

Given $v_s(t) = 50\cos(2t)$ and $i_s = 0.9$ A, find $v_x(t)$



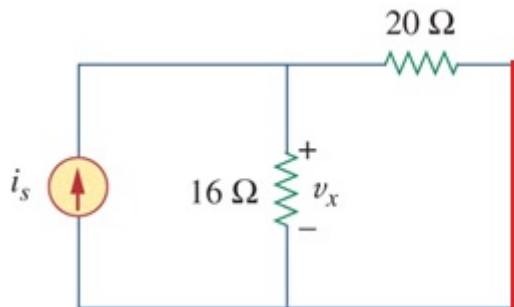
Method: Analyze circuit one frequency at a time

In the case, first analyze at DC then at 2 rad/s and then add up the two solutions (**superposition**)

At DC, redraw circuit without $v_s(t)$:

Replace voltage source with short circuit

Replace inductor with short circuit



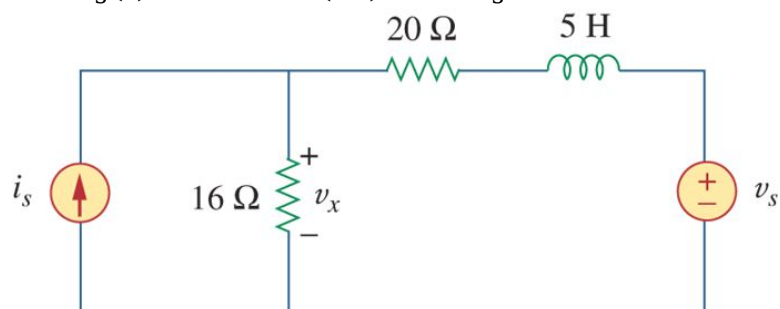
Resistance across current source = $20 \parallel 16 = 80/9\ \Omega$

$V_x = 0.9 * 80/9 = 8\text{ V}$ (DC)

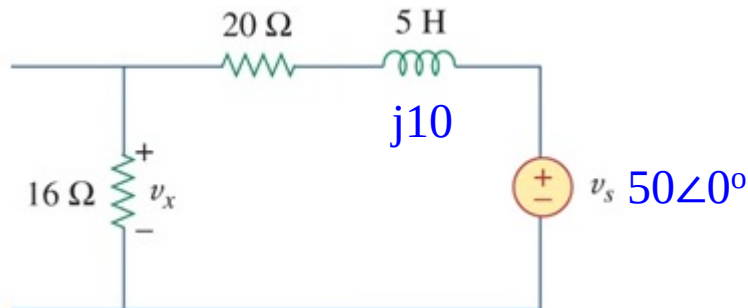
3A) Superposition (multiple freq) example 1

Alexander 10.44 (Modified)

Given $v_s(t) = 50\cos(2t)$ and $i_s = 0.9$ A, find $v_x(t)$



At 2 rad/s, redraw circuit without i_s :
Replace current source with open circuit



Final step: Add up both solutions
 $v_x(t) = 21.41\cos(2t - 15.52^\circ) + 8\text{ V}$

Step 1a: Transform source to phasor

$50\cos(2t) \rightarrow 50\angle 0^\circ$, $\omega = 2\text{ rad/s}$

Step 1b: Work out the impedance values

$5\text{ H} \rightarrow j2 \cdot 5 = j10$

Step 2: Solve using circuit analysis methods

Using voltage divider rule,

$$\mathbf{V}_x = \left(\frac{16}{16 + 20 + j10} \right) (50\angle 0^\circ) = 21.41\angle -15.52^\circ$$

Step 3: Transform back to time domain

$\mathbf{V}_x = 21.41\angle -15.52^\circ$, $\omega = 2\text{ rad/s}$

$\rightarrow v_x(t) = 21.41\cos(2t - 15.52^\circ)\text{ V}$

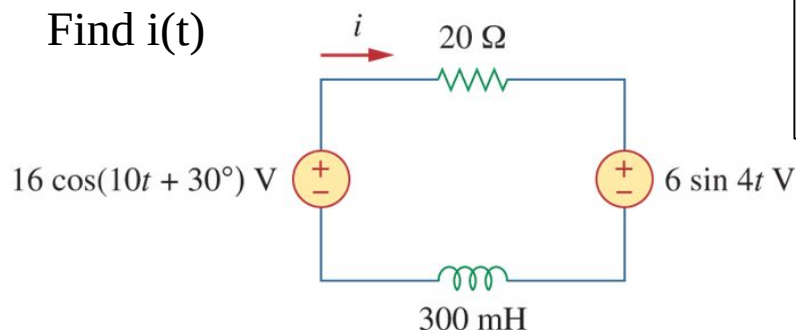
(NEVER SKIP THIS STEP)

Note that there are 2 terms in the final form (one at DC and another at 2 rad/s)

3B) Superposition (multiple freq) example 2

Alexander 10.45

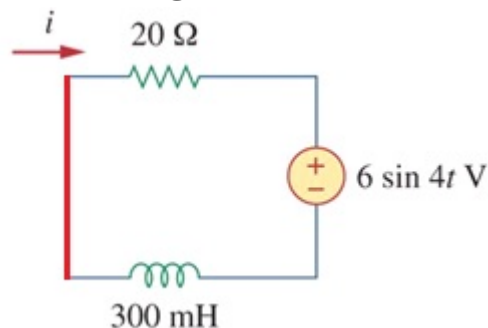
Find $i(t)$



Method: Analyze circuit one frequency at a time
In the case, first analyze at 4 rad/s then 10 rad/s.
Finally, add up the two solutions (**superposition**)

At 4 rad/s, redraw circuit without 10 rad/s source:

Replace 10 rad/s voltage source with short circuit



Step 1a: Transform source to phasor

$6 \sin(4t) \rightarrow 6 \angle -90^\circ$, $\omega = 4 \text{ rad/s}$

Step 1b: Work out the impedance values

$300 \text{ mH} \rightarrow j4 \cdot 0.3 = j1.2$

Step 2: Solve using circuit analysis methods

$$\mathbf{I}^*(20 + j1.2) + 6 \angle -90^\circ = 0$$

$$\mathbf{I} = 0.2995 \angle 86.57^\circ \text{ A}$$

Step 3: Transform back to time domain

$$\mathbf{I} = 0.2995 \angle 86.57^\circ, \omega = 4 \text{ rad/s}$$

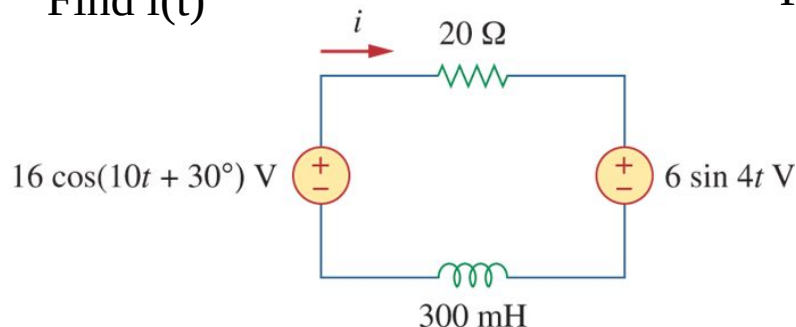
$$\rightarrow i(t) = 0.2995 \cos(4t + 86.57^\circ) \text{ A}$$

NEVER SKIP THIS STEP

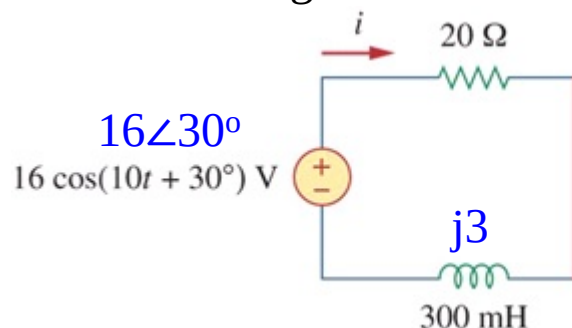
3B) Superposition (multiple freq) example 2

Alexander 10.45

Find $i(t)$



At **10** rad/s, redraw circuit without 4 rad/s source:
Replace 4 rad/s voltage source with short circuit



Step 2: Solve using circuit analysis methods

$$\mathbf{I}^*(20 + j3) = 16 \angle 30^\circ$$

$$\mathbf{I} = 0.7911 \angle 21.47^\circ \text{ A}$$

Step 3: Transform back to time domain

$$\mathbf{I} = 0.7911 \angle 21.47^\circ, \omega = 10 \text{ rad/s}$$

$$\rightarrow i(t) = 0.7911 \cos(10t + 21.47^\circ) \text{ A}$$

Step 1a: Transform source to phasor

$$16 \cos(10t + 30^\circ) \rightarrow 16 \angle 30^\circ, \omega = 10 \text{ rad/s}$$

Step 1b: Work out the impedance values

$$300 \text{ mH} \rightarrow j10 * 0.3 = j3$$

NEVER SKIP THIS STEP

Final step: Add up both solutions

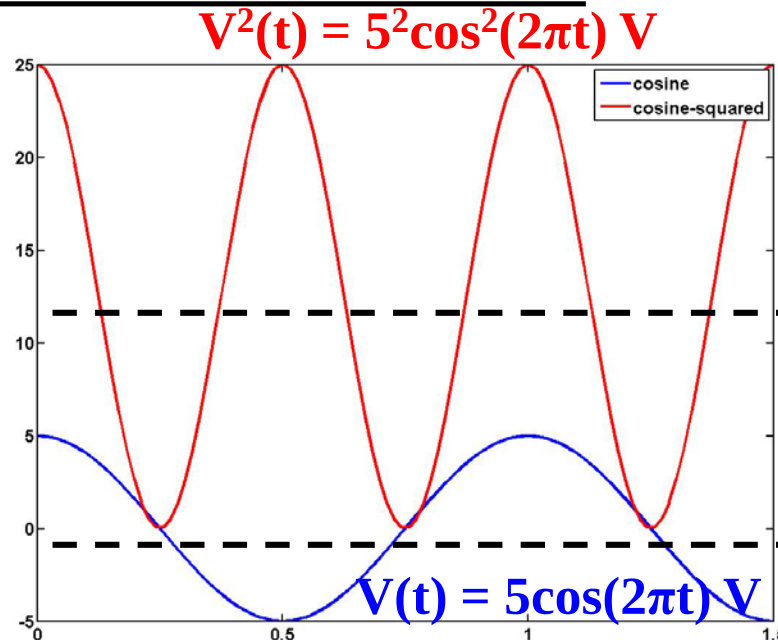
$$i(t) = 0.2995 \cos(4t + 86.57^\circ) + 0.7911 \cos(10t + 21.47^\circ) \text{ A}$$

Note that there are 2 terms in the final form
(one at 4 rad/s and another at 10 rad/s)

AC Power: Instantaneous vs. Average

Although the average voltage across a resistor with a sinusoidal AC voltage across it is zero, note that the average power dissipated is not zero. Since $P = V^2/R$, we should consider the **square of the voltage**.

The following graph shows the voltage across a 1Ω resistor in blue and the corresponding square of this voltage in red. The red curve therefore shows the **INSTANTANEOUS POWER**.



The instantaneous power is clearly sinusoidal with a DC offset:

$$\begin{aligned} V^2(t) &= 5^2 * 0.5 * (1 + \cos(4\pi t)) \\ &= V_{\text{peak}}^2 * 0.5 * \cos(4\pi t) + \\ &\quad V_{\text{peak}}^2 * 0.5 \\ &= 12.5\cos(4\pi t) + 12.5 \end{aligned}$$

Mean of $V^2 = 12.5$ (DC offset)

Average power of $V^2(t)$ is 12.5 W
Average power of $V^2(t) = 0.5 * V_{\text{peak}}^2$

Mean of $V = 0$