

EF2452: Mathematics for Economics and Finance

Lecture 1

Part II of II

Department of Economics and Finance
City University of Hong Kong
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Main Topics

- Sets, subsets, unions, intersections
- Real numbers and integers and important subsets of these
- Graphs and the solutions of equations

What is a set?

- A set is the mathematical term for a collection of objects defined precisely so that any given object is either in the set or not in the set.
- Examples: $X = \{2, 3, 5, 7, 8, 9\}$, $Y = \{1, 4, 5, 7, 9\}$. The members or elements of set X and set Y are the numbers listed within the brackets.
- When x is an object in a set S , we write $x \in S$ meaning 'x belongs to S ' or 'x is a member of S '.
- $x \notin S$: 'x does not belong to S '

Set

- Another way of specifying a set is by means of a property that its members must possess; for example

$$Z = \{n \mid n \text{ is a positive whole number less than } 5\}$$

is read as Z is the set of n such that n is a positive whole number less than 5. So, this particular set could also be written as $\{1, 2, 3, 4\}$.

- The set which has no members is called the empty set and is denoted by ϕ .

Subset, Union, and Intersection

- We say that the set U is a subset of the set V and write $U \subseteq V$ if every member of U is a member of V , that is, $U \subseteq V$ if and only if $x \in U$ implies $x \in V$.
- Given two sets A and B , the union $A \cup B$ is defined as the set whose members belong to A or B (or both A and B):

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

- The intersection $A \cap B$ is defined as the set whose members belong to both A and B :

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

- Set minus ($A \setminus B$) is defined as the set whose members belong to A but are not in B :

$$A \setminus B = \{x \mid x \in A \text{ and } x \notin B\}$$

Subset, Union, and Intersection: An Example

- $X = \{2, 3, 5, 7, 8, 9\}$, $Y = \{1, 4, 5, 7, 9\}$, $Z = \{1, 2, 3, 4\}$

$$X \cup Y = \{1, 2, 3, 4, 5, 7, 8, 9\}$$

$$X \cap Y = \{5, 7, 9\}$$

$$Y \cup Z = \{1, 2, 3, 4, 5, 7, 9\}$$

Also, note that

$$Z \subseteq X \cup Y$$

$$X \cap (Y \cap Z) = (X \cap Y) \cap Z = \phi$$

Some Important Sets

- $\mathbb{R}$: the set of real numbers; a real number can be described by a decimal representation and can be thought of as points on a line.
- $\mathbb{R}_+$: the set of non-negative real numbers $\{x|x \geq 0\}$
- $\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$: the set of integers
- The positive integers are also known as natural numbers: $\mathbb{N} = \{1, 2, 3, \dots\}$.

Ordered Pairs

- We use the notation $\mathbb{R}^2$ for the set of ordered pairs (x, y) of real numbers.
- $\mathbb{R}^2$ is the set usually depicted as the set of points in a plane, x and y being the coordinates of a point with respect to a pair of axes.
- $\mathbb{R}_+^2$ consists of those points for which x and y are both non-negative. For example, think of the (q, p) pair corresponding to the price and quantity of a good (both of which must be non-negative).

Solving A Quadratic Equation

- How to find the set of solutions of a quadratic equation

$$ax^2 + bx + c = 0$$

- First note that $a \neq 0$; because if $a = 0$ the equation reduces to a linear one with $x = \frac{-c}{b}$.
- Method of factorization: for example, suppose we have $x^2 - 5x + 6 = 0$, we can rewrite it as:

$$x^2 - 2x - 3x + 6 = 0$$

$$\implies x(x - 2) - 3(x - 2) = 0$$

$$\implies (x - 2)(x - 3) = 0$$

Therefore, the set $\{x \mid x^2 - 5x + 6 = 0\}$ has two members, 2 and 3.

Solving A Quadratic Equation: A General Technique

- There is a more general technique for determining the solutions to a quadratic equation.
- Since $a \neq 0$, we may divide the equation $ax^2 + bx + c = 0$ by a to obtain

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Complete the square to rewrite the above equation as

$$\left(x + \frac{b}{2a}\right)^2 + \frac{c}{a} - \frac{b^2}{4a^2} = 0$$

- This follows from the identity

$$\left(x + \frac{b}{2a}\right)^2 = x^2 + 2x\left(\frac{b}{2a}\right) + \left(\frac{b}{2a}\right)^2 = x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}$$

Solving A Quadratic Equation: A General Technique

- The equation we now have to solve is

$$\left(x + \frac{b}{2a}\right)^2 + \frac{c}{a} - \frac{b^2}{4a^2} = 0$$

$$\implies \left(x + \frac{b}{2a}\right)^2 = -\frac{c}{a} + \frac{b^2}{4a^2} = \frac{b^2 - 4ac}{4a^2}$$

- Then, if $b^2 - 4ac < 0$, there is no solution as $\left(x + \frac{b}{2a}\right)^2 > 0$.
- If $b^2 - 4ac = 0$, then the equation is

$$\left(x + \frac{b}{2a}\right)^2 = 0$$

$$\implies x = \frac{-b}{2a}$$

Solving A Quadratic Equation: A General Technique

- If $b^2 - 4ac > 0$, then the equation is

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

- So, there are two possibilities:

$$x + \frac{b}{2a} = \frac{\sqrt{b^2 - 4ac}}{2a} \quad \text{or} \quad x + \frac{b}{2a} = -\frac{\sqrt{b^2 - 4ac}}{2a}$$

Therefore, there are two solutions for x :

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

In other words,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solving A Quadratic Equation: A General Technique

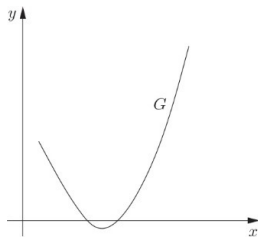
- Often, the word ‘root’ is used to describe a solution to a quadratic equation and we may speak, for example, of a quadratic equation having two roots.
- The quantity $b^2 - 4ac$ is known as the discriminant of the quadratic equation.

Solving Equations Simultaneously

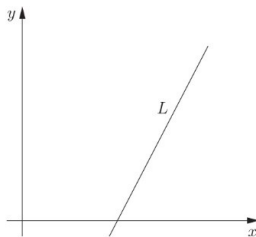
- An important problem in economics is to solve equations simultaneously. We saw earlier that a market equilibrium can be obtained by simultaneously solving two equations where both demand and supply equations were linear.
- Now, we consider a case where one of the equations is non-linear.
- Suppose we wish to determine the set $G \cap L$, where

$$G = \{(x, y) \mid y = x^2 - 5x + 6\} \quad L = \{(x, y) \mid y = 2x - 6\}$$

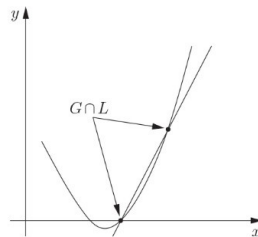
Solving Equations Simultaneously



(a)



(b)



(c)

Solving Equations Simultaneously

- We look for the points of intersection of the graph G with the line L .
- We must have:

$$\begin{aligned}x^2 - 5x + 6 &= 2x - 6 \\ \implies x^2 - 7x + 12 &= 0\end{aligned}$$

- The solutions are

$$x_1 = \frac{7 + \sqrt{49 - (4)(12)}}{2} = 4$$

$$x_2 = \frac{7 - \sqrt{49 - (4)(12)}}{2} = 3$$

- Then, substituting the value of x in any of the two equations gives $y_1 = 2$ and $y_2 = 0$.
- $G \cap L$ consists of the two points $(4, 2)$ and $(3, 0)$.

References

Following chapter of the textbook:

2 Mathematical Terms and Notations