

EE3114 HW-4 Solutions

4-1 Use Ruth criterion to determine the stability of the transfer function with following:

a) $D(s) = s^3 + 2s^2 + s + 2$ (2 points)

b) $D(s) = s^4 + s^3 - 3s^2 - s + 2$ (2 points)

SOLUTION:

a) Using the formulas for Routh array, we have:

s^3	1	1
s^2	2	2
s^1	4	0
s^0	2	

Because there is no sign change in the first column, the transfer function is stable.

b) Using the formulas for Routh array, we have:

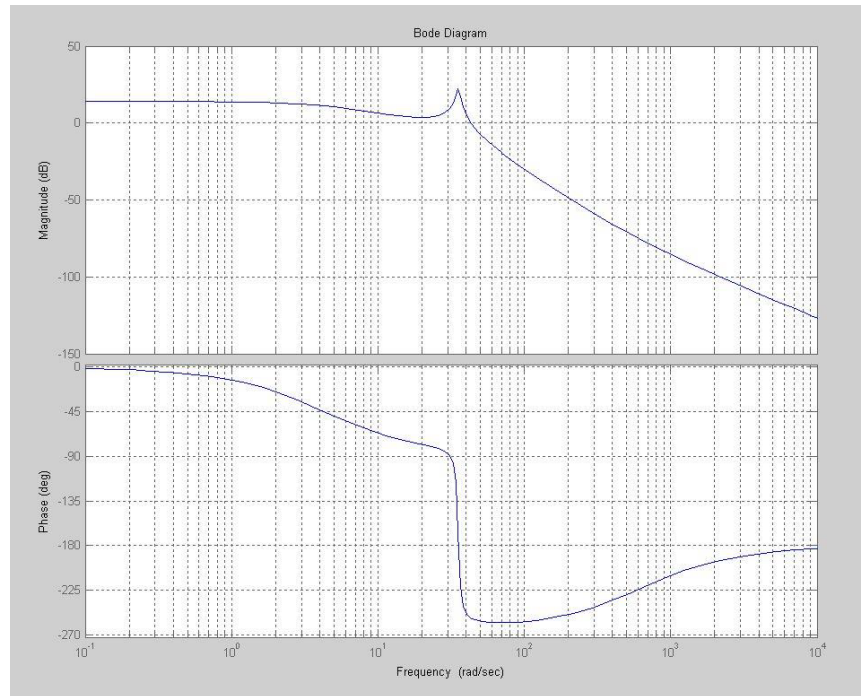
s^4	1	-3	2
s^3	1	-1	0
s^2	-2	2	0
s^1	-4	0	0
s^0	2		

Because there are two sign changes in the first column, the transfer function is unstable.

REMARKS:

For a), all roots have negative real parts except for a pair on the imaginary axis, i.e., $s = \pm j$, thus calling the system semi-stable (marginally stable) is also acceptable.

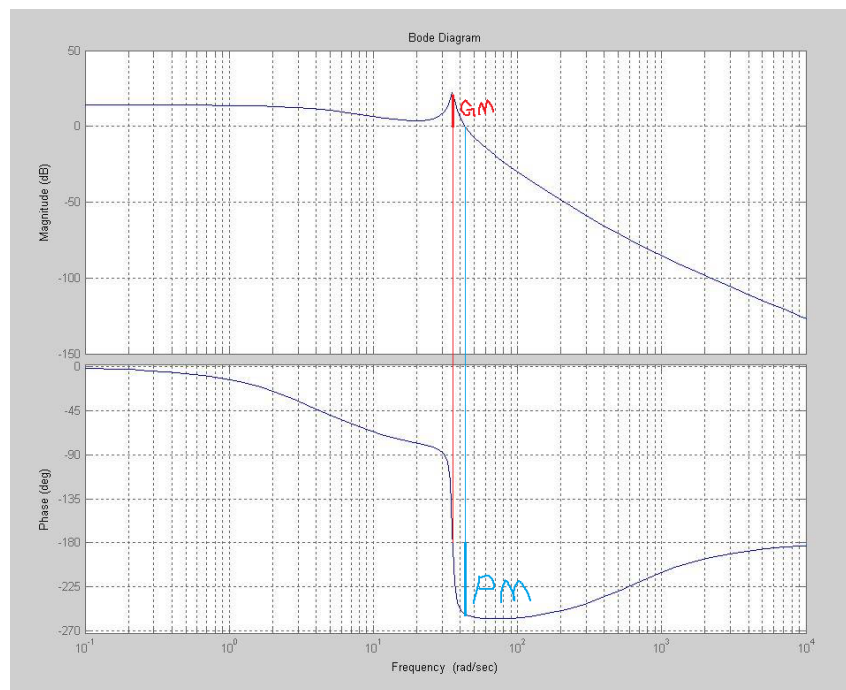
4-2 Given a Bode plot from MatLab as follows, determine the system Magnitude Margin and Phase Margin: (3 points)



SOLUTION:

Magnitude Margin: $GM \cong -20dB$

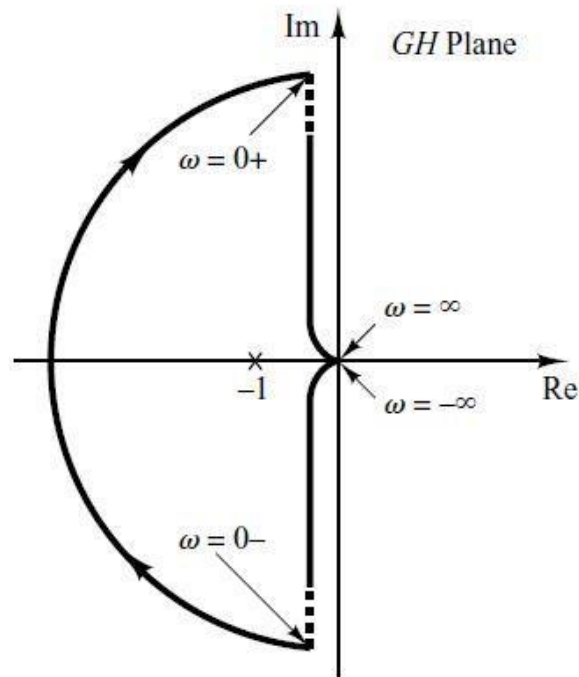
Phase Margin: $PM \cong -255^\circ - (-180^\circ) = -75^\circ$



REMARKS:

Acceptable thresholds of GM and PM are $(-15dB \sim -23dB)$ and $(-70^\circ \sim -85^\circ)$

4-3 The system with $G(s)H(s) = \frac{2}{s(2s-1)}$ has a Nyquist plot as follows. Determine whether the corresponding closed-loop system is stable or not. (3 points)



SOLUTION 1:

The transfer function $G(s)H(s)$ has one pole ($s = 1/2$) in RHP ($P = 1$). But the Nyquist plot does not encircle the point $-1 + j0$ counterclockwise once, so the closed-loop system is unstable.

SOLUTION 2:

(i) Transfer function $G(s)H(s)$ has one pole ($s = 1/2$) in RHP ($P = 1$).

(ii) $\lim_{s \rightarrow \infty} G(s)H(s) = 0$ (a constant).

For the closed-loop system to be stable, one must have $N + P = 0$, or $N = -P = -1$, i.e., the Nyquist plot must encircle the point $-1 + 0j$ counterclockwise once. But now the Nyquist plot encircle the point $-1 + 0j$ clockwise, i.e., $N = 1$ ($\neq -1$), therefore the closed-loop system is unstable.