

CITY UNIVERSITY OF HONG KONG

Course code & title : EE3101 Communication Engineering

Session : Semester B 04/05

Time allowed : Two hours

This paper has 7 pages. (including this page)

Instructions to candidates:

1. This paper consists of 4 questions.
 2. Answer any three questions.
 3. All questions carry equal marks.
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Materials, aids & instruments permitted to be used during examination:

Approved calculator.

Examination of EE3101 Communications Engineering

Problem 1

Consider a binary transmission system with the binary information

represented by two waveforms, $s_1(t) = \sqrt{\frac{2E}{T}} \cos(\omega_1 t)$ and

$s_2(t) = \sqrt{\frac{2E}{T}} \cos(\omega_2 t)$ for the bit duration of T . The prior probabilities of

these two waveforms are $P_1 = \text{Prob}\{s_1\} = 0.7$ and $P_2 = \text{Prob}\{s_2\} = 0.3$.

The channel noise is AWGN with two-sided noise power spectral density

$$\frac{N_0}{2}.$$

- (a) Draw the diagram of the matched filter receiver for the binary transmission system. (6 marks)
- (b) Find the relationship between ω_1 , ω_2 and T so that the two waveforms are orthogonal to each other. (6 marks)
- (c) Find the optimum threshold. (10 marks)
- (d) Find the minimum probability of bit error with the bit energy to noise ratio $\frac{E_b}{N_0} = 6\text{dB}$. (7 marks)
- (e) What is $\frac{E_b}{N_0}$ that gives the same bit error probability as found in part (d) for equally likely binary signals? (4 marks)

Problem 2

Consider the linear block code with the generator defined by

$$\mathbf{G} = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

- (a) Find all the codewords. (8 marks)
- (b) What is the error-correcting capability of the code? (5 marks)
- (c) Find the parity check matrix. (5 marks)
- (d) Compute the syndrome table for all the error patterns of less than two errors. (8 marks)
- (e) Is the received vector 01001010 a codeword? If not, under what assumption you can correct it? (4 marks)
- (f) Is the received vector 00101000 a codeword? If not, can you correct it? (3 marks)

Problem 3

A differential encoding system QPSK is used to transmit data in AWGN channel with single-sided noise power spectral density N_0 . The system transfer function is assumed to be a raised cosine filter with a roll-off characteristic of $\alpha = 0.5$. Assume also that a Gray code is used for the symbol to bit assignment.

- (a) What is the purpose of differential encoding for QPSK? (4 marks)
(b) Find the data phase of DQPSK for the bit stream

$$\{1\ 1\ 0\ 0\ 1\ 0\ 1\ 0\ 1\ 1\ 1\ 0\} \quad (7 \text{ marks})$$

- (c) Show that the probability of bit error for QPSK is

$$P_{B,QPSK} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

where E_b is the bit energy. (10 marks)

- (d) The probability of bit error for coherently detected DQPSK can be expressed as

$$P_{B,DQPSK} \cong 2Q\left(\sqrt{\frac{2E_b}{N_0}}\right) - 4\left[Q\left(\sqrt{\frac{2E_b}{N_0}}\right)\right]^2$$

[8 extra points will be given for a correct proof of the above bit error probability formula].

What $\frac{E_b}{N_0}$ is required for $P_{B,DQPSK} = 10^{-3}$. (8 marks)

[Approximation is allowed if necessary.]

- (e) What is the data rate for the system bandwidth be equal to 60 kHz? (4 marks)

Problem 4

Coherent BPSK system is sensitive to phase error and frequency error.

Consider that the local reference signal of the receiver using matched filter has a fixed phase error ϕ in comparing with the received signal.

- (a) Find the bit error probability of the matched filter for equally likely binary signals for the fixed phase error ϕ . (8 marks)
- (b) Derive the closed transfer function of a phase locked loop (PLL) and show how the PLL can track the constant phase error. (15 marks)
- (c) Find the loop filter of PLL that gives zero phase error for a constant frequency error in the reference signal. (10 marks)

TABLE B.1 Complementary Error Function $Q(x) = \int_x^\infty (1/\sqrt{2\pi}) \exp(-u^2/2) du$

x	$Q(x)$									
	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2168	0.2148
0.8	0.2169	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002

$$\operatorname{erfc}(x) = 2Q(x\sqrt{2}) \quad (\text{B.20})$$

$$Q(x) = \frac{1}{2} \operatorname{erfc}\left(\frac{x}{\sqrt{2}}\right) \quad (\text{B.21})$$

$$Q(x) \approx \frac{1}{x\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right)$$

Often-Used Identities

$$\cos x \cos y = \frac{1}{2} \cos (x + y) + \frac{1}{2} \cos (x - y) \quad (\text{D.1})$$

$$\sin x \sin y = -\frac{1}{2} \cos (x + y) + \frac{1}{2} \cos (x - y) \quad (\text{D.2})$$

$$\sin x \cos y = \frac{1}{2} \sin (x + y) + \frac{1}{2} \sin (x - y) \quad (\text{D.3})$$

$$\cos x \sin y = \frac{1}{2} \sin (x + y) - \frac{1}{2} \sin (x - y) \quad (\text{D.4})$$

$$\sin (x \pm y) = \sin x \cos y \pm \cos x \sin y \quad (\text{D.5})$$

$$\cos (x \pm y) = \cos x \cos y \mp \sin x \sin y \quad (\text{D.6})$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x) \quad (\text{D.7})$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x) \quad (\text{D.8})$$

$$\sin x \cos x = \frac{1}{2} \sin 2x \quad (\text{D.9})$$

$$\sin x + \sin y = 2 \sin \frac{1}{2}(x + y) \cos \frac{1}{2}(x - y) \quad (\text{D.10})$$

$$\sin x - \sin y = 2 \cos \frac{1}{2}(x + y) \sin \frac{1}{2}(x - y) \quad (\text{D.11})$$

$$\cos x + \cos y = 2 \cos \frac{1}{2}(x + y) \cos \frac{1}{2}(x - y) \quad (\text{D.12})$$

$$\cos x - \cos y = -2 \sin \frac{1}{2}(x + y) \sin \frac{1}{2}(x - y) \quad (\text{D.13})$$

$$\sin x = \frac{e^{jx} - e^{-jx}}{2j} \quad (\text{D.14})$$

$$\cos x = \frac{e^{jx} + e^{-jx}}{2} \quad (\text{D.15})$$

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