EE3210 Signals & Systems

Due on Midnight, Feb 28, 2021

Homework #1

1. Total mark is 100 points (= 20 points per problem $\times$ 5 problems)
2. Solution will be posted on March 3rd on Canvas website
3. Submission due by Feb 28, 2021, midnight. We will accept late submission until March 2, 2021
4. Late submission penalty; -5 points per day
    - Full mark: 20 points (Feb 28), 15 points (March 01), 10 points (March 02), and 0 points for any late submission after March 3rd.
5. Online submission through Canvas
    - Scan or taking a photo of your anwser sheet, then upload to Canvas

Q1. Sol)

Even part $x_e(t) = \frac{1}{2}\left(x(t) + x(-t)\right)$

Odd part $x_o(t) = \frac{1}{2}\left(x(t) - x(-t)\right) = x(t) - x_e(t)$

a) $x_e(t) = \frac{1}{2}\left(\underbrace{(\cos t + \sin t + \cos t \sin t)}_{x(t)} + \underbrace{(\cos t - \sin t - \cos t \sin t)}_{x(-t)}\right)$

$= \cos t$

$x_o(t) = \sin(t) + \cos(t)\sin(t)$

b) $x_e(t) = \frac{1}{2}\left(\underbrace{(1 + t + 4t^2)}_{x(t)} + \underbrace{(1 - t + 4t^2)}_{x(-t)}\right) = 1 + 4t^2$

$x_o(t) = t$

c) $x_e(t) = \frac{1}{2}\left\{\underbrace{\cos(t) + \sin^2(2t)}_{x(t)} + \underbrace{\cos(t) + \sin^2(2t)}_{x(-t)}\right\} = \cos(t) + \sin^2(2t)$

$x_o(t) = \phi$

d) $x_e(t) = \frac{1}{2}\left\{t^2\cos t + t\sin(2t) + t^2\cos t + t\sin(2t)\right\} = t^2\cos(t) + t\sin(2t)$

$x_o(t) = \phi$

e) $x_e(t) = \frac{1}{2}\left\{\underbrace{1 + t\cos(t) + t^2\sin(t) + t^3\sin(t)\cos(t)}_{x(t)} + \underbrace{1 \cancel{+ t\cos(t)}\ (-t\cos(t)) - t^2\sin(t) + t^3\sin t\cos t}_{x(-t)}\right\}$

$= 1 \cancel{+ t\cos(t)} + t^3\sin(t)\cos(t) = 1 + t^3\sin(t)\cos(t)$

$x_o(t) = \cancel{t^2\sin(t)}\ t\cos(t) + t^2\sin(t)$

f) $x_e(t) = \frac{1}{2}\left\{\underbrace{(1 + t^3)\cos(9t)}_{x(t)} + \underbrace{(1 - t^3)\cos(9t)}_{x(-t)}\right\} = \cos(9t)$

$x_o(t) = t^3\cos(9t)$

g) $x_e(t) = \frac{1}{2}\left\{\underbrace{(t + t^3)\cos(3t)}_{x(t)} + \underbrace{(-t - t^3)\cos(3t)}_{x(-t)}\right\} = \phi$

$x_o(t) = (t + t^3)\cos(3t)$

Q 1. Sol)

h) $x_e(t) = \frac{1}{2}\{\underbrace{(t+t^3)\tan(3t)}_{x(t)} \quad \underbrace{-(-t-t^3)\tan(3t)}_{x(-t)}\} = (t+t^3)\tan(3t)$

$x_o(t) = \phi$

i) $x_e(t) = \frac{1}{2}\{\underbrace{t(1+t+t^2)}_{x(t)} \quad \underbrace{-t(1-t+t^2)}_{x(-t)}\} = t^2$

$x_o(t) = t(1+t^2)$

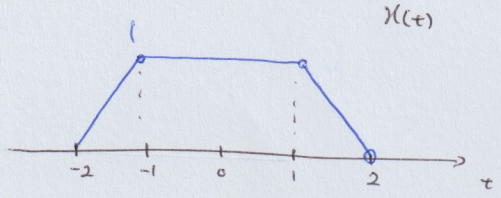
j) $x_e(t) = \frac{1}{2}\{\underbrace{\cos(2t)+\cos(3t)+\cos(t)\sin(2t)}_{x(t)} + \underbrace{\cos(2t)+\cos(3t)-\cos(t)\sin(2t)}_{x(-t)}\}$

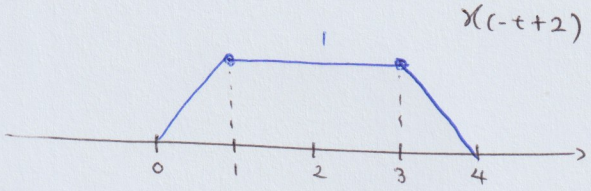
$= \cos(2t)+\cos(3t)$

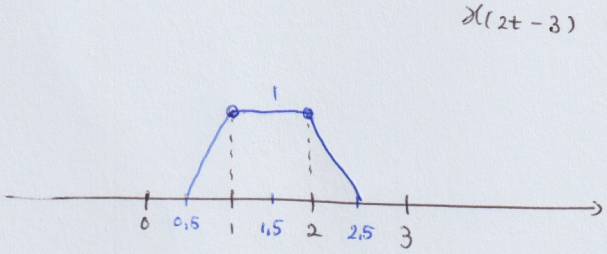
$x_o(t) = \cos(t)\sin(2t)$

Q2. Sol)

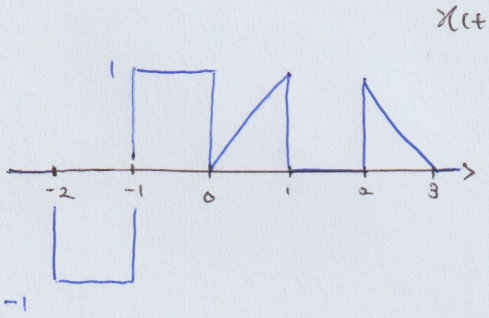
a)

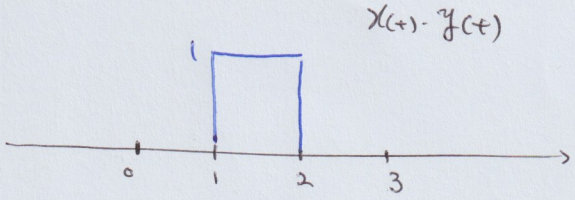

x(t)
1
-2
-1
0
1
2
t

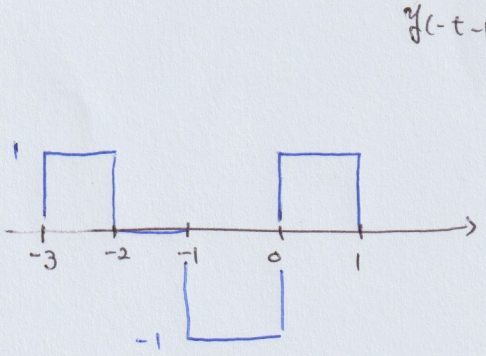

x(-t+2)
1
0
1
2
3
4

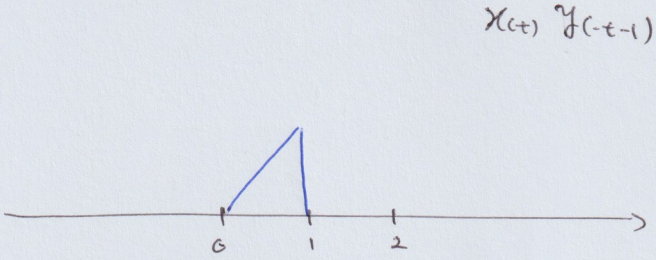

x(2t-3)
1
0
0,5
1
1,5
2
2,5
3

b)


x(t) - y(t)
1
-2
-1
0
1
2
3
-1

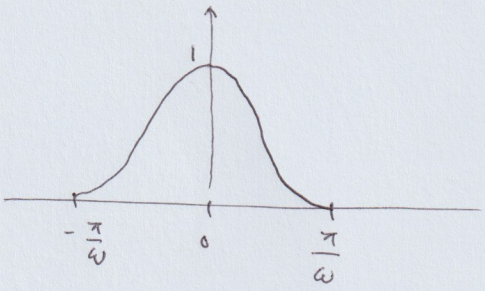

x(t) · y(t)
1
0
1
2
3


y(-t-1)
1
-3
-2
-1
0
1
-1


x(t) y(-t-1)
0
1
2

(Q 3. Sol)

a-1) $x(t) = \frac{1}{2}[\cos(\omega t)+1]$ for $-\frac{\pi}{\omega} \le t \le \frac{\pi}{\omega}$, $\Rightarrow 0$ otherwise



$$\text{Energy} = \int_{-\frac{\pi}{\omega}}^{\frac{\pi}{\omega}} |x(t)|^2 dt$$

$$= \frac{1}{4}\int_{-\frac{\pi}{\omega}}^{\frac{\pi}{\omega}} (\cos(\omega t)+1)^2 dt$$

use change of variable '$\omega t = \zeta$', then $dt = \frac{1}{\omega} d\zeta$

$\Rightarrow$

$$E = \frac{1}{4\omega}\int_{-\pi}^{\pi} (\cos\zeta+1)^2 d\zeta = \frac{1}{4\omega}\int_{-\pi}^{\pi}(\cos^2\zeta + 2\cos\zeta + 1)\, d\zeta$$

$$= \frac{1}{4\omega}\int_{-\pi}^{\pi}\left\{\frac{1}{2} + \frac{1}{2}\cos(2\zeta) + 2\cos\zeta + 1\right\} d\zeta$$

$$= \frac{1}{4\omega}\int_{-\pi}^{\pi}\left(\frac{3}{2} + \frac{1}{2}\cos 2\zeta + 2\cos\zeta\right) d\zeta$$

$$= \frac{1}{4\omega}\left[3\pi + \left\{\frac{1}{4}\sin(2\zeta) + 2\sin\zeta\Big|_{-\pi}^{\pi}\right\}\right] = \frac{3\pi}{4\omega}$$ → Energy Signal

($\frac{1}{4}\sin(2\zeta) + 2\sin\zeta$ evaluated from $-\pi$ to π = 0)

a-2)

$$E = \int_{-\infty}^{\infty}|x(t)|^2 dt = \int_1^2 x(t)^2 dt + \int_3^4 x(t)^2 dt.$$ → Energy Signal

$$= \underbrace{\int_1^{\frac{3}{2}}\{6(t-1)\}^2 dt}_{①} + \underbrace{\int_{\frac{3}{2}}^2\{-6(t-2)\}^2 dt}_{②} + \int_3^4 1\cdot dt$$

$$= \underbrace{36\int_0^{\frac{1}{2}}\zeta^2 d\zeta}_{=1/24} + \underbrace{36\int_{-\frac{1}{2}}^0 l^2 dl}_{=1/24} + 1 = 4$$

① Use change of variable $t-1=\zeta$

② $t-2=l$

Q 3. Sol)

b-1) $x(t) = \underbrace{3\cos(4\pi t)}_{T_1} + \underbrace{\sin(5\pi t)}_{T_2}$

Period for the first term is $T_1 = \frac{1}{2}$

Period for the second term is $T_2 = \frac{2}{5}$

Since the ratio $\frac{T_1}{T_2} = \frac{1}{2} \cdot \frac{5}{2} = \frac{5}{4}$ is rational,

$x(t)$ is periodic and the fundamental period of $x(t)$ is $lcm\left(\frac{1}{2}, \frac{2}{5}\right) = 2$

b-2) $x(t) = |\cos(4\pi t)|$

→ Periodic signal, $T_0 = \frac{1}{4}$

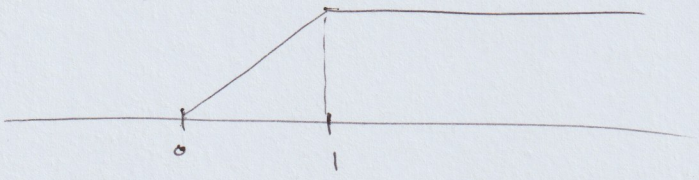
b-3) Non periodic.

Q.4. Sol)

a-1) $S(t) = h(t) * u(t)$.

$= (u(t) - u(t-1)) * u(t) = u(t) * u(t) - u(t-1) * u(t)$

$= t u(t) - (t-1) u(t-1)$



a-2)

$S(t) = h(t) * u(t)$

$= (\delta(t) - \delta(t-3)) * u(t) = u(t) - u(t-3)$

b-1) $h(t) = e^{2t} u(t-2)$

memory, causal, Non-stable

b-2) $h(t) = u(t+4) - 2u(t-2)$, Non-stable

memory, Non-causal,

b-3) $h(t) = e^{15t} [u(t-1) - u(t-100)]$, Stable.

memory, causal,

Q 5. Sol)

a)

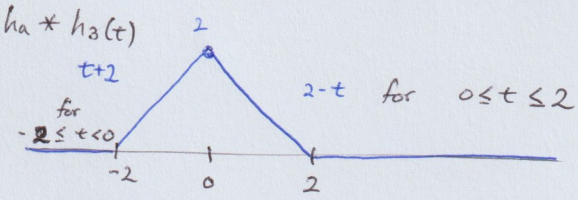
$$h(t) = \underbrace{(h_1(t) - h_2(t))}_{h_a} * h_3(t) * h_4(t)$$

$$h_1(t) - h_2(t) = u(t+1) - u(t-1)$$

$$h_a * h_3(t) = (u(t+1) - u(t-1)) * (u(t+1) - u(t-1))$$

$$= (t+2)u(t+2) - t\,u(t) - t\,u(t) + (t-2)u(t-2)$$

$$= (t+2)u(t+2) - 2t\,u(t) + (t-2)u(t-2)$$



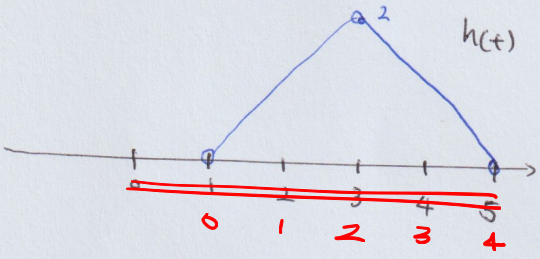
Denote $f(t) = h_a(t) * h_3(t)$

$$h(t) = (h_a * h_3(t)) * h_4(t)$$

$$= (h_a(t) * h_3(t)) * \delta(t-\cancel{3}\ 2)$$

$$= f(t-\cancel{3}\ 2)$$

$$= \begin{cases} \cancel{t-1}\ t & \text{for } \cancel{1 \le t < 3}\ 0 \le t < 2 \\ \cancel{5-t}\ 4-t & \text{for } \cancel{3 \le t < 5}\ 2 \le t < 4 \\ 0 & \text{otherwise} \end{cases}$$



b). Memory, Causal, Stable system