

Tutorial 1

Vector and Fundamental Forces in Nature



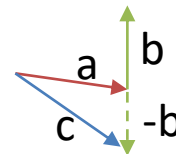
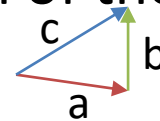
Lecture Outline

- **Chapter 3**, D Halliday, R Resnick, and J Walker, “Fundamentals of Physics” 9th Edition, Wiley (2005).
- Review of Vectors
 - Vector in Cartesian Coordinate
 - Position and unit vectors
 - Vector algebra
 - Vector product
 - Vectors in Cylindrical and Spherical coordinates.
- Fundamental interactions in Nature.
 - Forces in Nature.



Lecture 01 – Review

- In Lecture 1 we introduced the properties of vector
- Unlike the scalar which can be fully represented by its magnitude, a vector consists of both magnitude and direction.
- We can represent a vector geometrically using an arrow, where the length of the arrow represents its magnitude and the head of the arrow signifies its direction.
- To add two vectors together geometrically, we place the second vector **b** with its tail touching the head of the first vector **a**. The vector sum is the vector **c** = **a** + **b** joining the tail of the first vector **a** and the head of the second vector **b**.
- To subtract vector **b** from vector **a**, where the resulting vector, we can simply reverse the direction of **b** to **-b** and add the two vectors accordingly, **c** = **a** + (**-b**).



Lecture 01 – Review

- The addition and subtraction of vectors are both commutative and associative.
- We can express the components of the vector in a coordinate system, such as the Cartesian coordinate system, by projecting the vectors onto its x-, y- and z-axes. This is called the resolution of the vector.
- The resolution of the vector $\mathbf{a}$ in the Cartesian coordinate system results in three vectors $\mathbf{a}_x$, $\mathbf{a}_y$ and $\mathbf{a}_z$ pointing at the x-, y- and z- axes, where $\mathbf{a} = \mathbf{a}_x + \mathbf{a}_y + \mathbf{a}_z$.
- We define that a unit vector is a vector of unit magnitude, pointing in a particular direction. In the Cartesian coordinate system, the unit vectors pointing in the x-, y- and z- axes are usually designated by $\hat{i}$, $\hat{j}$, $\hat{k}$, where the hat on top of the letter represent it is a unit vector.

Lecture 01 – Review

- With the unit vector notation, one can now separates the magnitude and direction of the vector and $\mathbf{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$
- By separating the vector into its components, we can add vectors by simply adding their components that shares the same unit vector (direction) together.
- If $\mathbf{c} = \mathbf{a} + \mathbf{b} \rightarrow \mathbf{c} = (a_x+b_x)\hat{i} + (a_y+b_y)\hat{j} + (a_z+b_z)\hat{k}$
- Note that the relations among vectors do not depend on the origin or the orientation of the axes.
- Besides addition and subtraction, one can also multiply vectors.
- There are two kinds of vector product, the scalar (dot) product and the vector (cross) product.



Lecture 01 – Review

- The scalar or dot product between two vectors relates to the projection of the two vectors to each other.
- The scalar product between two vectors is written as $\vec{a} \cdot \vec{b}$ and it is defined as $\vec{a} \cdot \vec{b} = ab \cos \phi$ where ϕ is the angle between the two vectors.
- The vector or cross product between two vectors is written as $\vec{c} = \vec{a} \times \vec{b}$.
- To find the direction of the cross product, we will need to use the right-hand rule.
- A more straight forward way to determine the vector product is to use the unit-vector notation, where

$$\vec{a} \times \vec{b} = (a_x \hat{i} + a_y \hat{j} + a_z \hat{k}) \times (b_x \hat{i} + b_y \hat{j} + b_z \hat{k})$$

$$= (a_y b_z - b_y a_z) \hat{i} + (a_z b_x - b_z a_x) \hat{j} + (a_x b_y - b_x a_y) \hat{k}.$$



Lecture 01 – Review

- One can express the vectors in coordinate systems other than Cartesian. This is important in many situations, the geometries consists of symmetries that is not necessary along the planes in the Cartesian system.
- The coordinates of the cylindrical system are (r, θ, z) corresponds to the (radial, azimuthal, vertical) axes.
- The coordinates of the spherical system are (r, θ, ϕ) corresponds to the (radial, polar, azimuthal) axes.
- We have also introduced the four fundamental force of nature and they are:
 - Strong Nuclear Force
 - Weak Nuclear Force
 - Gravitational Force
 - Electromagnetic Force.



Practice Questions: Vectors

Chapter 3

Questions: 9,13,16,26,34,38,41

Q9

•9 Two vectors are given by

$$\vec{a} = (4.0 \text{ m})\hat{i} - (3.0 \text{ m})\hat{j} + (1.0 \text{ m})\hat{k}$$

and $\vec{b} = (-1.0 \text{ m})\hat{i} + (1.0 \text{ m})\hat{j} + (4.0 \text{ m})\hat{k}.$

In unit-vector notation, find (a) $\vec{a} + \vec{b}$, (b) $\vec{a} - \vec{b}$, and (c) a third vector $\vec{c}$ such that $\vec{a} - \vec{b} + \vec{c} = 0$.

Unit-Vector notation:

Express the vector as the component form of the coordinate system.

The vectors in the question are already in their unit-vector notations

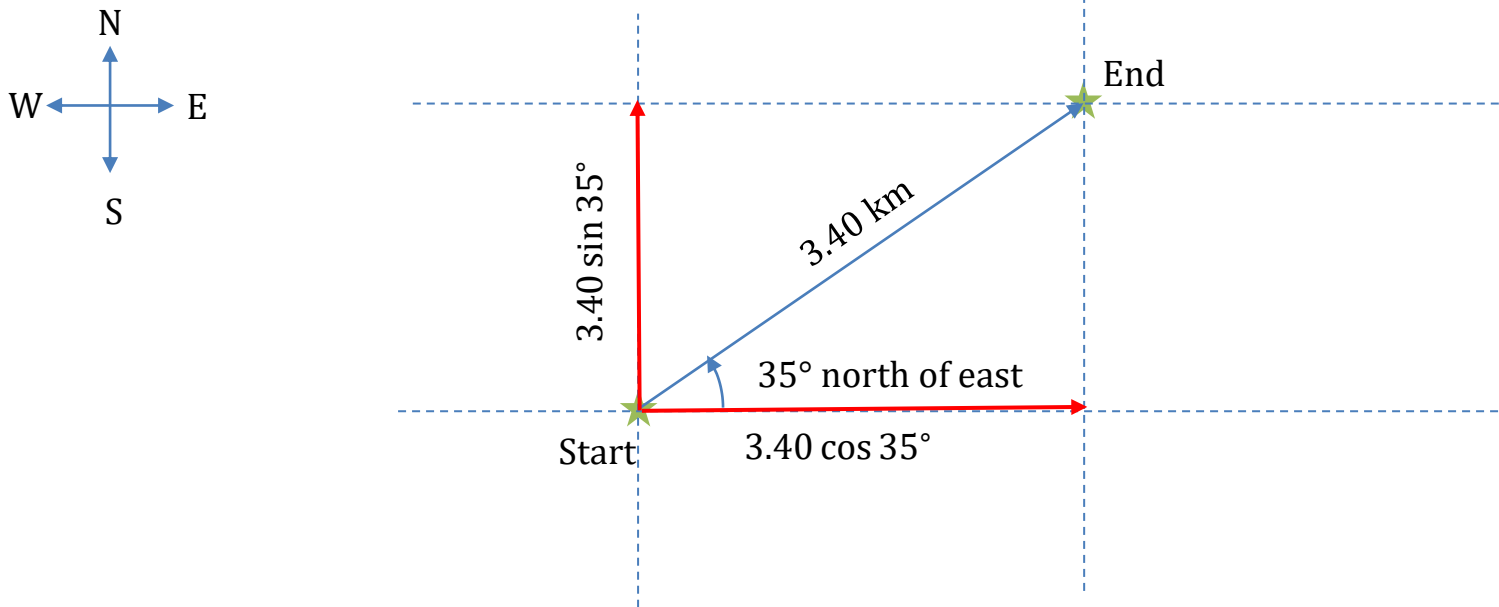
$$\text{a) } \vec{a} + \vec{b} = [4.0 + (-1.0)]\hat{i} + [(-3.0) + 1.0]\hat{j} + (1.0 + 4.0)\hat{k} = (3.0\hat{i} - 2.0\hat{j} + 5.0\hat{k}) \text{ m}$$

$$\text{b) } \vec{a} - \vec{b} = [4.0 - (-1.0)]\hat{i} + [(-3.0) - 1.0]\hat{j} + (1.0 - 4.0)\hat{k} = (5.0\hat{i} - 4.0\hat{j} - 3.0\hat{k}) \text{ m}$$

$$\text{c) } \vec{a} - \vec{b} + \vec{c} = 0 \rightarrow \vec{c} = \vec{b} - \vec{a} = -(5.0\hat{i} - 4.0\hat{j} - 3.0\hat{k}) \text{ m} = (-5.0\hat{i} + 4.0\hat{j} + 3.0\hat{k})$$

Q13

•13 A person desires to reach a point that is 3.40 km from her present location and in a direction that is 35.0° north of east. However, she must travel along streets that are oriented either north–south or east–west. What is the minimum distance she could travel to reach her destination?



We find the components and then add them (as scalars, not vectors).

With $d = 3.40$ km and $q = 35.0^\circ$

we find $d \cos q + d \sin q = 3.4 * \cos 35 + 3.4 * \sin 35 = 4.74$ km.

Q16

•16 For the displacement vectors $\vec{a} = (3.0 \text{ m})\hat{i} + (4.0 \text{ m})\hat{j}$ and $\vec{b} = (5.0 \text{ m})\hat{i} + (-2.0 \text{ m})\hat{j}$, give $\vec{a} + \vec{b}$ in (a) unit-vector notation, and as (b) a magnitude and (c) an angle (relative to $\hat{i}$). Now give $\vec{b} - \vec{a}$ in (d) unit-vector notation, and as (e) a magnitude and (f) an angle.

(a) $\vec{a} + \vec{b} = (3.0\hat{i} + 4.0\hat{j}) \text{ m} + (5.0\hat{i} - 2.0\hat{j}) \text{ m} = (8.0 \text{ m})\hat{i} + (2.0 \text{ m})\hat{j}.$

(b) The magnitude of $\vec{a} + \vec{b}$ is

$$|\vec{a} + \vec{b}| = \sqrt{(8.0 \text{ m})^2 + (2.0 \text{ m})^2} = 8.2 \text{ m}.$$

(c) The angle between this vector and the $+x$ axis is

$$\tan^{-1}[(2.0 \text{ m})/(8.0 \text{ m})] = 14^\circ.$$

(d) $\vec{b} - \vec{a} = (5.0\hat{i} - 2.0\hat{j}) \text{ m} - (3.0\hat{i} + 4.0\hat{j}) \text{ m} = (2.0 \text{ m})\hat{i} - (6.0 \text{ m})\hat{j}.$

(e) The magnitude of the difference vector $\vec{b} - \vec{a}$ is

$$|\vec{b} - \vec{a}| = \sqrt{(2.0 \text{ m})^2 + (-6.0 \text{ m})^2} = 6.3 \text{ m}.$$

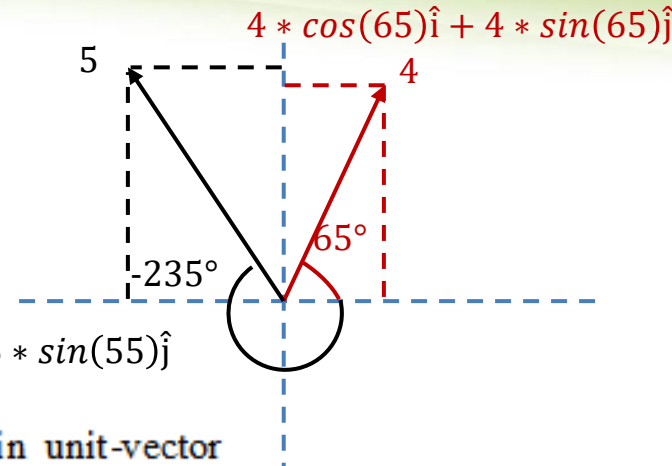
(f) The angle between this vector and the $+x$ axis is $\tan^{-1}[(-6.0 \text{ m})/(2.0 \text{ m})] = -72^\circ$. The vector is 72° clockwise from the axis defined by $\hat{i}$.

Q26

••26 What is the sum of the following four vectors in (a) unit-vector notation, and as (b) a magnitude and (c) an angle?

$$\vec{A} = (2.00 \text{ m})\hat{i} + (3.00 \text{ m})\hat{j} \quad \vec{B}: 4.00 \text{ m, at } +65.0^\circ$$

$$\vec{C} = (-4.00 \text{ m})\hat{i} + (-6.00 \text{ m})\hat{j} \quad \vec{D}: 5.00 \text{ m, at } -235^\circ$$



The vector equation is $\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$. Expressing $\vec{B}$ and $\vec{D}$ in unit-vector notation, we have $(1.69\hat{i} + 3.63\hat{j}) \text{ m}$ and $(-2.87\hat{i} + 4.10\hat{j}) \text{ m}$, respectively. Where the length unit is not displayed in the solution below, the unit meter should be understood.

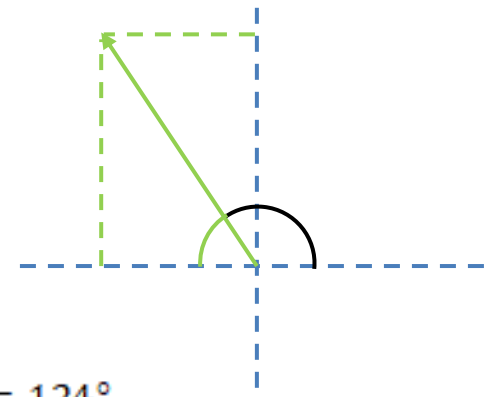
(a) Adding corresponding components, we obtain $\vec{R} = (-3.18 \text{ m})\hat{i} + (4.72 \text{ m})\hat{j}$.

(b) Using Eq. 3-6, the magnitude is

$$|\vec{R}| = \sqrt{(-3.18 \text{ m})^2 + (4.72 \text{ m})^2} = 5.69 \text{ m.}$$

(c) The angle is

$$\theta = \tan^{-1} \left(\frac{4.72 \text{ m}}{-3.18 \text{ m}} \right) = -56.0^\circ \text{ (with } -x \text{ axis).}$$



If measured counterclockwise from $+x$ -axis, the angle is then $180^\circ - 56.0^\circ = 124^\circ$.

Q34

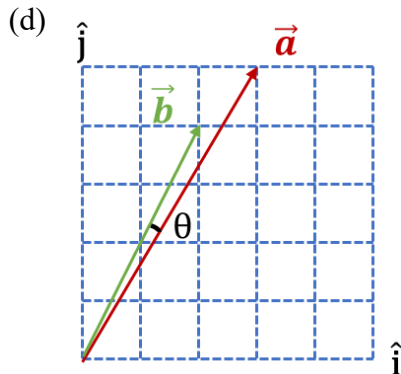
Q34 Two vectors are presented as $\vec{a} = 3.0\hat{i} + 5.0\hat{j}$ and $\vec{b} = 2.0\hat{i} + 4.0\hat{j}$. Find (a) $\vec{a} \times \vec{b}$, (b) $\vec{a} \cdot \vec{b}$, (c) $(\vec{a} + \vec{b}) \cdot \vec{b}$, and (d) the component of $\vec{a}$ along the direction of $\vec{b}$. (Hint: For (d), consider Eq. 3-20 and Fig. 3-18.)

(a) $\vec{a} \times \vec{b} = (a_x b_y - a_y b_x) \hat{k}$ since all other terms vanish, due to the fact that neither $\vec{a}$ nor $\vec{b}$ have any z components. Consequently, we obtain $[(3.0)(4.0) - (5.0)(2.0)]\hat{k} = 2.0\hat{k}$.

(b) $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y$ yields $(3.0)(2.0) + (5.0)(4.0) = 26$.

(c) $\vec{a} + \vec{b} = (3.0 + 2.0)\hat{i} + (5.0 + 4.0)\hat{j} \Rightarrow (\vec{a} + \vec{b}) \cdot \vec{b} = (5.0)(2.0) + (9.0)(4.0) = 46$.

The component of one vector (1st) to the other vector (2nd), is the Scalar (Dot) Product of the two vectors divided by the magnitude of the 2nd vector



The component of $|\vec{a}|$ along the direction of $|\vec{b}|$ is: $|\vec{a}|\cos\theta$

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos\theta \Rightarrow |\vec{a}|\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

$$|\vec{a}|\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{(3.0\hat{i} + 5.0\hat{j}) \cdot (2.0\hat{i} + 4.0\hat{j})}{\sqrt{2.0^2 + 4.0^2}} = \frac{6 + 20}{\sqrt{20}} \approx 5.8$$

Q38

•38  For the following three vectors, what is $3\vec{C} \cdot (2\vec{A} \times \vec{B})$?

$$\vec{A} = 2.00\hat{i} + 3.00\hat{j} - 4.00\hat{k}$$

$$\vec{B} = -3.00\hat{i} + 4.00\hat{j} + 2.00\hat{k} \quad \vec{C} = 7.00\hat{i} - 8.00\hat{j}$$

$$\hat{i} \times \hat{j} = \hat{k}, \hat{j} \times \hat{k} = \hat{i}, \hat{k} \times \hat{i} = \hat{j}$$

we obtain

$$\begin{aligned} 2\vec{A} \times \vec{B} &= 2(2.00\hat{i} + 3.00\hat{j} - 4.00\hat{k}) \times (-3.00\hat{i} + 4.00\hat{j} + 2.00\hat{k}) \\ &= 44.0\hat{i} + 16.0\hat{j} + 34.0\hat{k}. \end{aligned}$$

Next, making use of

$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$

$$\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0$$

$$3\vec{C} \cdot (2\vec{A} \times \vec{B}) = 3(7\hat{i} - 8\hat{j}) \cdot (44\hat{i} + 16\hat{j} + 34\hat{k}) = 3 \times (7 \times 44 - 8 \times 16) = 540$$

Q41

••41 SSM ILW WWW Use the definition of scalar product, $\vec{a} \cdot \vec{b} = ab \cos \theta$, and the fact that $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$ to calculate the angle between the two vectors given by $\vec{a} = 3.0\hat{i} + 3.0\hat{j} + 3.0\hat{k}$ and $\vec{b} = 2.0\hat{i} + 1.0\hat{j} + 3.0\hat{k}$.

. Since $ab \cos \phi = a_x b_x + a_y b_y + a_z b_z$,

$$\cos \phi = \frac{a_x b_x + a_y b_y + a_z b_z}{ab}.$$

The magnitudes of the vectors given in the problem are

$$a = |\vec{a}| = \sqrt{(3.00)^2 + (3.00)^2 + (3.00)^2} = 5.20$$

$$b = |\vec{b}| = \sqrt{(2.00)^2 + (1.00)^2 + (3.00)^2} = 3.74.$$

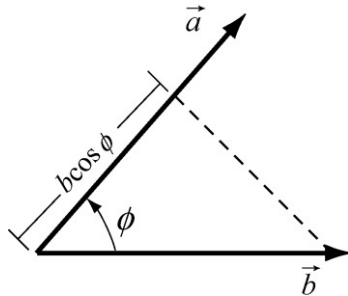
Q41

The angle between them is found from

$$\cos \varphi = \frac{(3.00)(2.00) + (3.00)(1.00) + (3.00)(3.00)}{(5.20)(3.74)} = 0.926.$$

The angle is $\phi = 22^\circ$.

As the name implies, the scalar product (or dot product) between two vectors is a scalar quantity. It can be regarded as the product between the magnitude of one of the vectors and the scalar component of the second vector along the direction of the first one, as illustrated below (see also in Fig. 3-18 of the text):



$$\vec{a} \cdot \vec{b} = ab \cos \varphi = (a)(b \cos \varphi)$$