

EE3122: Analogue Circuit Fundamentals

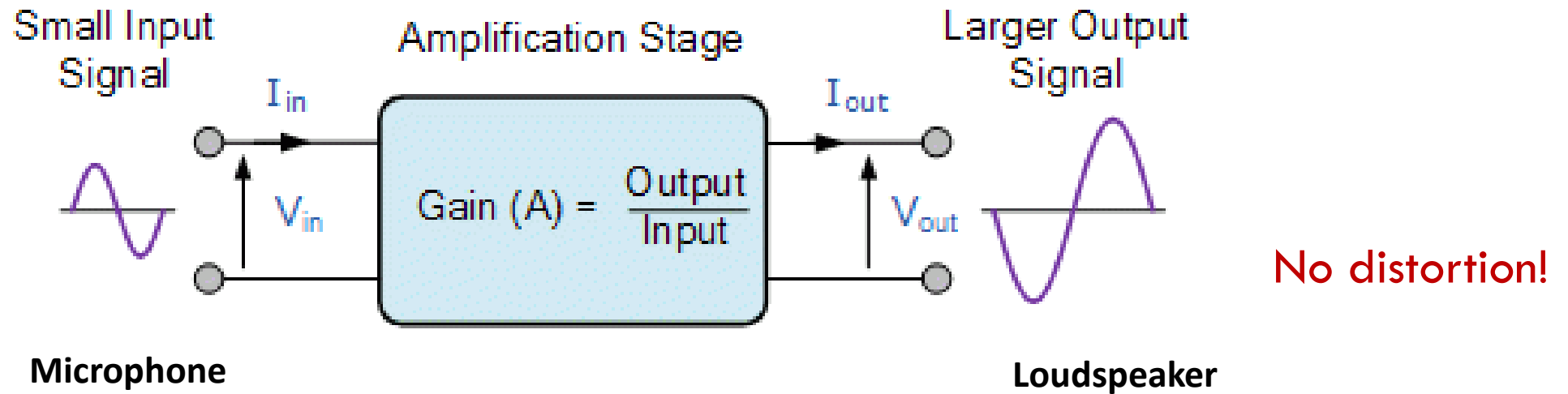
Single transistor Amplifiers

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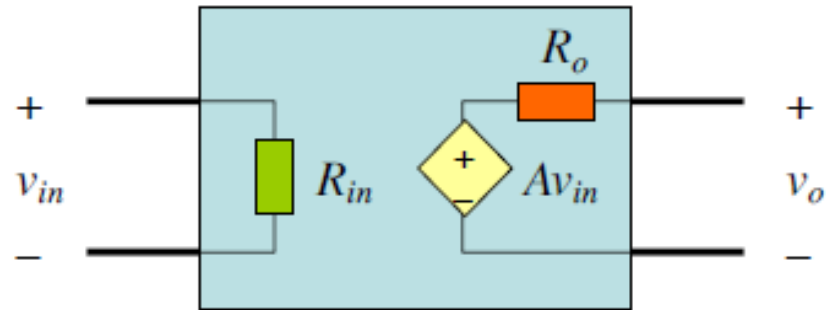
Amplifier



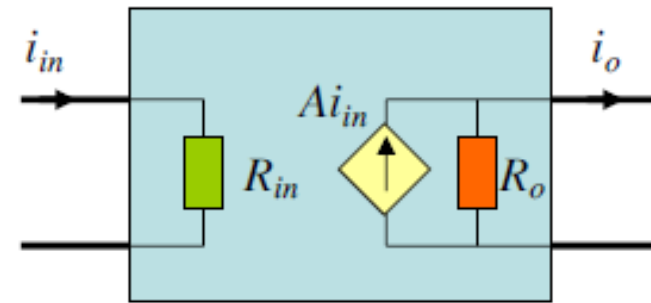
Increase the magnitude of the signal applied to its input.

What about Conservation of Energy??

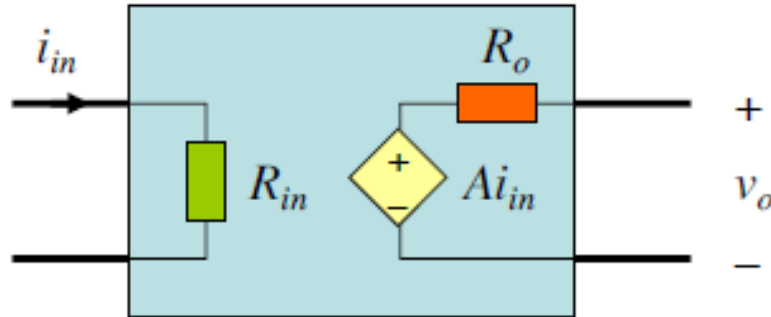
Two-port Models of amplifiers (circuit level)



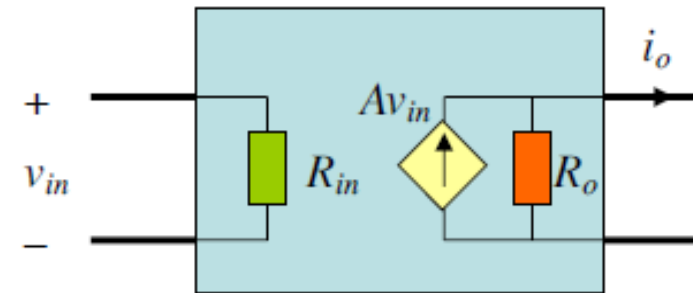
Voltage amplifier
Input: voltage Output: voltage



Current amplifier
Input: current Output: current

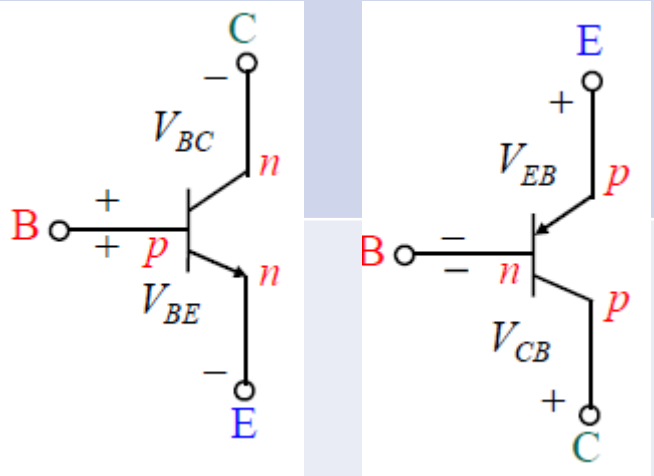


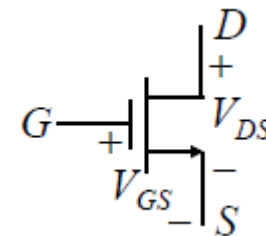
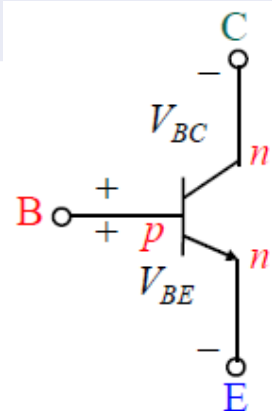
Transresistance amplifier
Input: current Output: voltage



Transconductance amplifier
Input: voltage Output: current

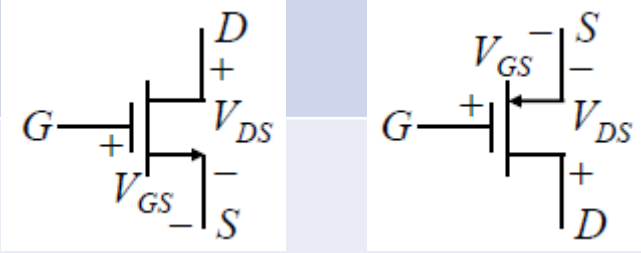
Single transistor amplifier – 3 types

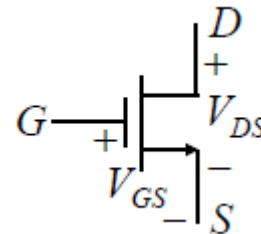
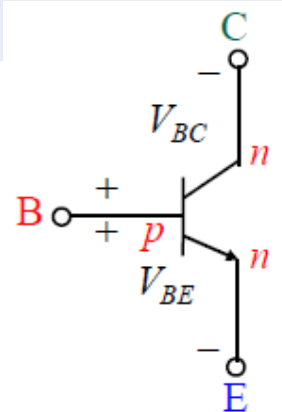
BJT	Common Emitter (CE)	Common Base (CB)	Common Collector (CC)
	Emitter connects to DC voltage (AC ground)	Base connects to DC voltage (AC ground)	Collector connects to DC voltage (AC ground)
	Negative gain, i.e. $A < -1$ e.g. $A = -10$	Positive gain, i.e. $A > 1$ e.g. $A = 10$	Voltage follower, i.e. $A \sim 1$



BJT		MOSFET
Emitter (E)	$\longleftrightarrow$	Source (S)
Base (B)	$\longleftrightarrow$	Gate (G)
Collector (C)	$\longleftrightarrow$	Drain (D)

Single transistor amplifier – 3 types

MOS	Common Source (CS)	Common Gate (CG)	Common Drain (CD)
	Source connects to DC voltage (AC ground)	Gate connects to DC voltage (AC ground)	Drain connects to DC voltage (AC ground)
	Negative gain, i.e. $A < -1$ e.g. $A = -10$	Positive gain, i.e. $A > 1$ e.g. $A = 10$	Voltage follower, i.e. $A \sim 1$



BJT		MOSFET
Emitter (E)	$\longleftrightarrow$	Source (S)
Base (B)	$\longleftrightarrow$	Gate (G)
Collector (C)	$\longleftrightarrow$	Drain (D)

Notation for DC & AC signals

Total voltage = DC voltage + AC voltage

$$V_{BE} = V_{BE} + v_{be}$$

Total current = DC current + AC current

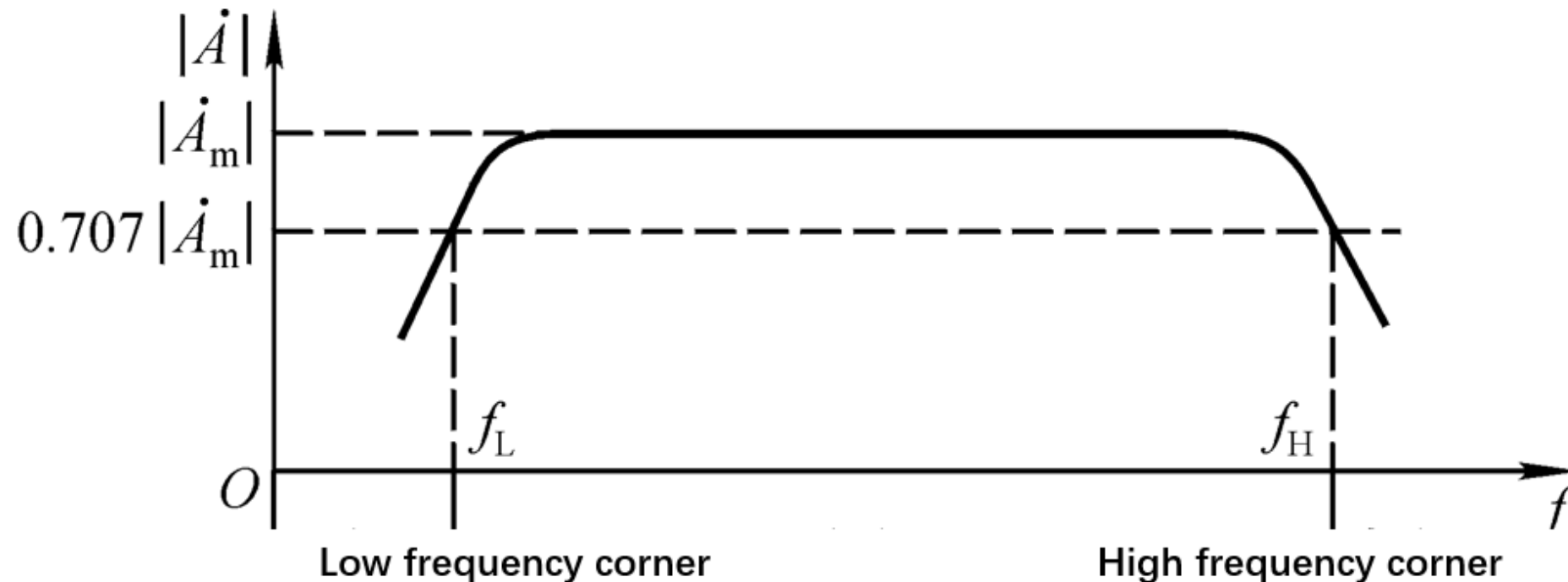
$$i_{BE} = I_{BE} + i_{be}$$

Other notation

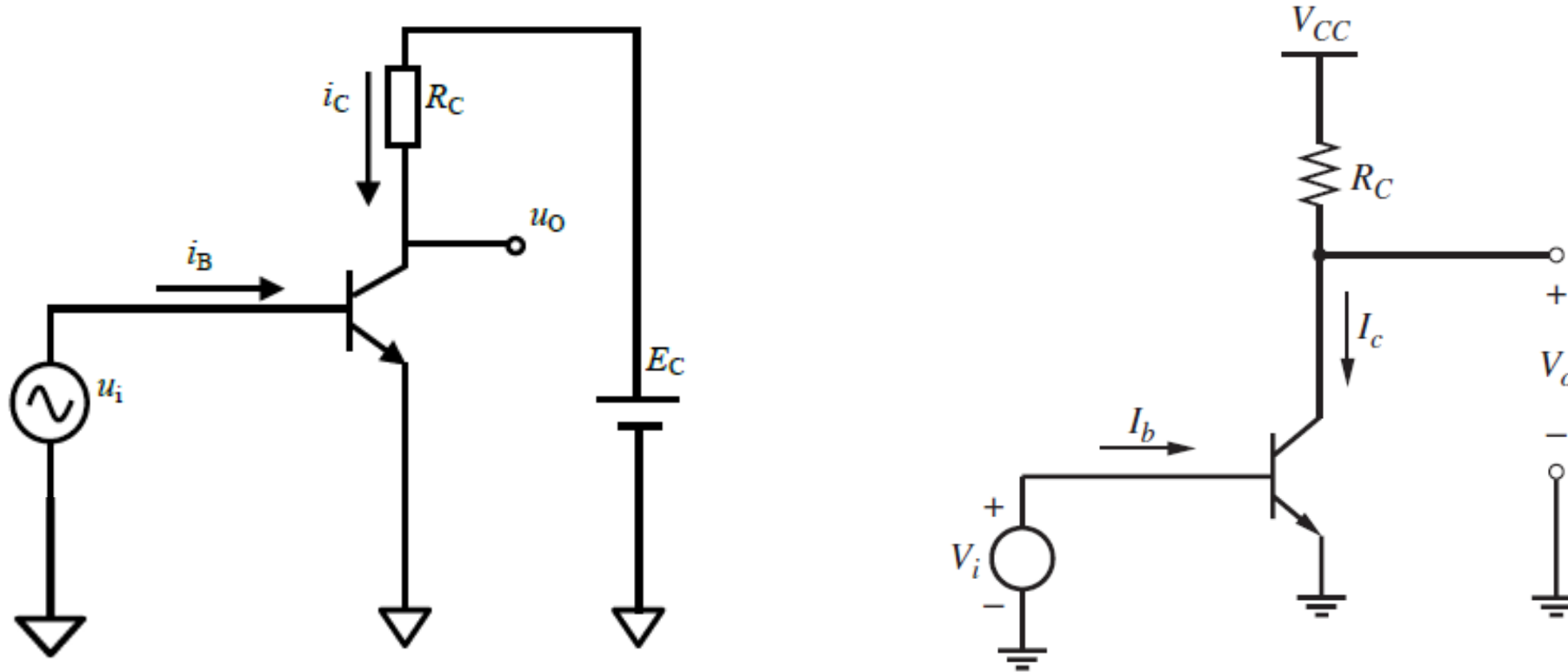
DC voltage: V_{BB} AC voltage: u_i

Frequency Response

- Amplifiers and filters use resistors, inductors, or capacitor networks (RLC) within their design.
- Internal capacitors in transistors.

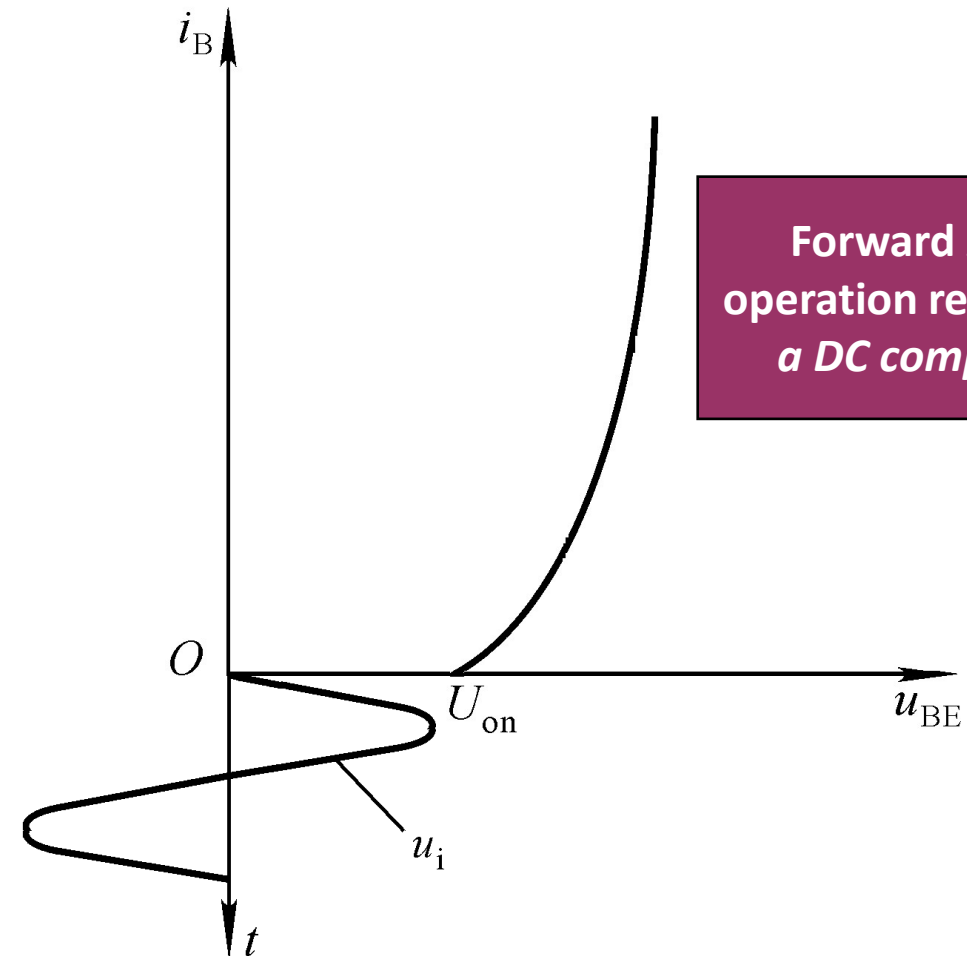
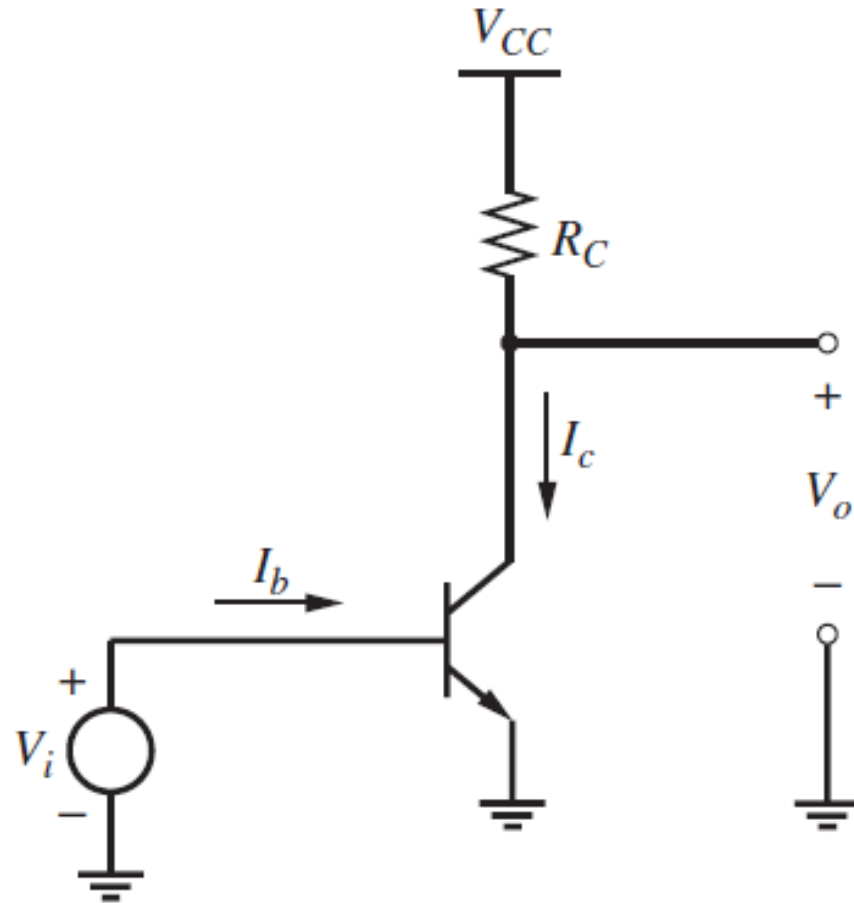


Common-Emitter Configuration of BJT

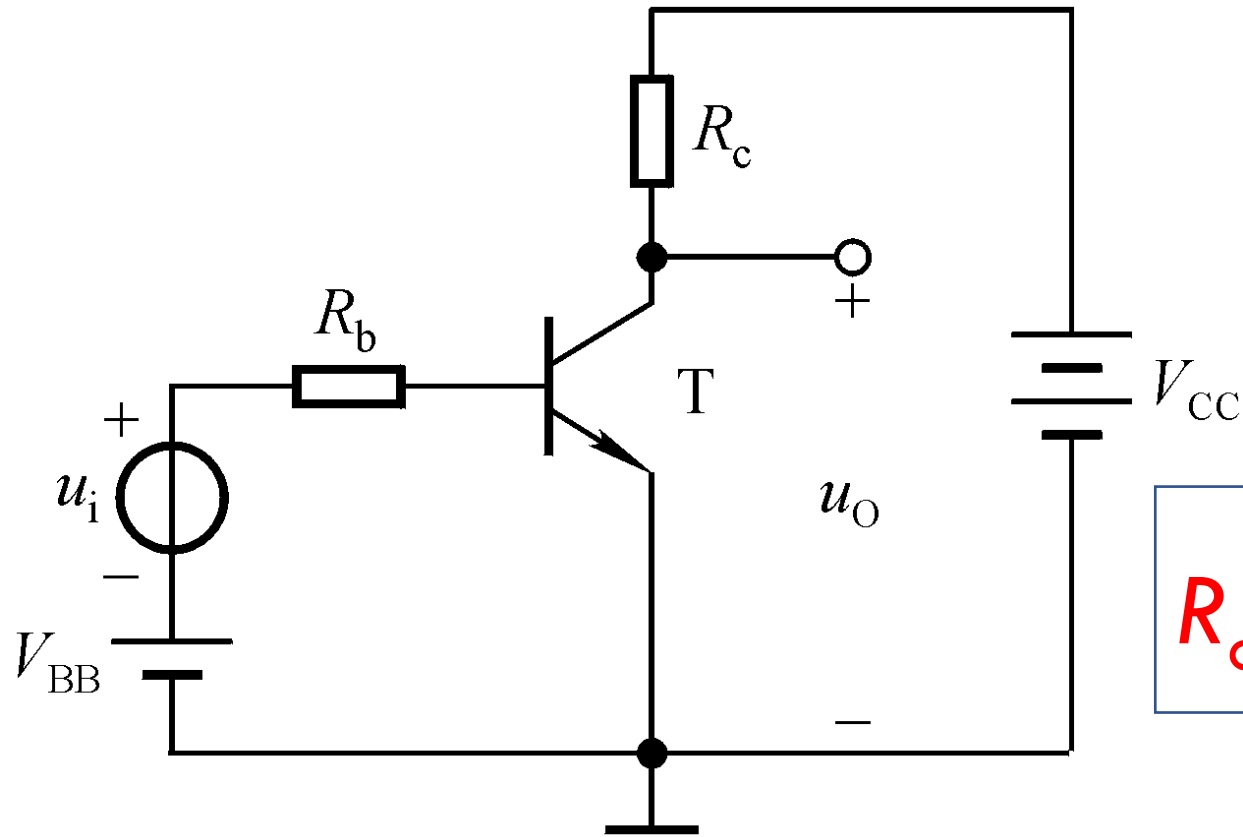


Input: Base of BJT, Output: Collector of BJT

Common Emitter (C-E): Need for DC Biasing



C-E configuration: DC Biasing

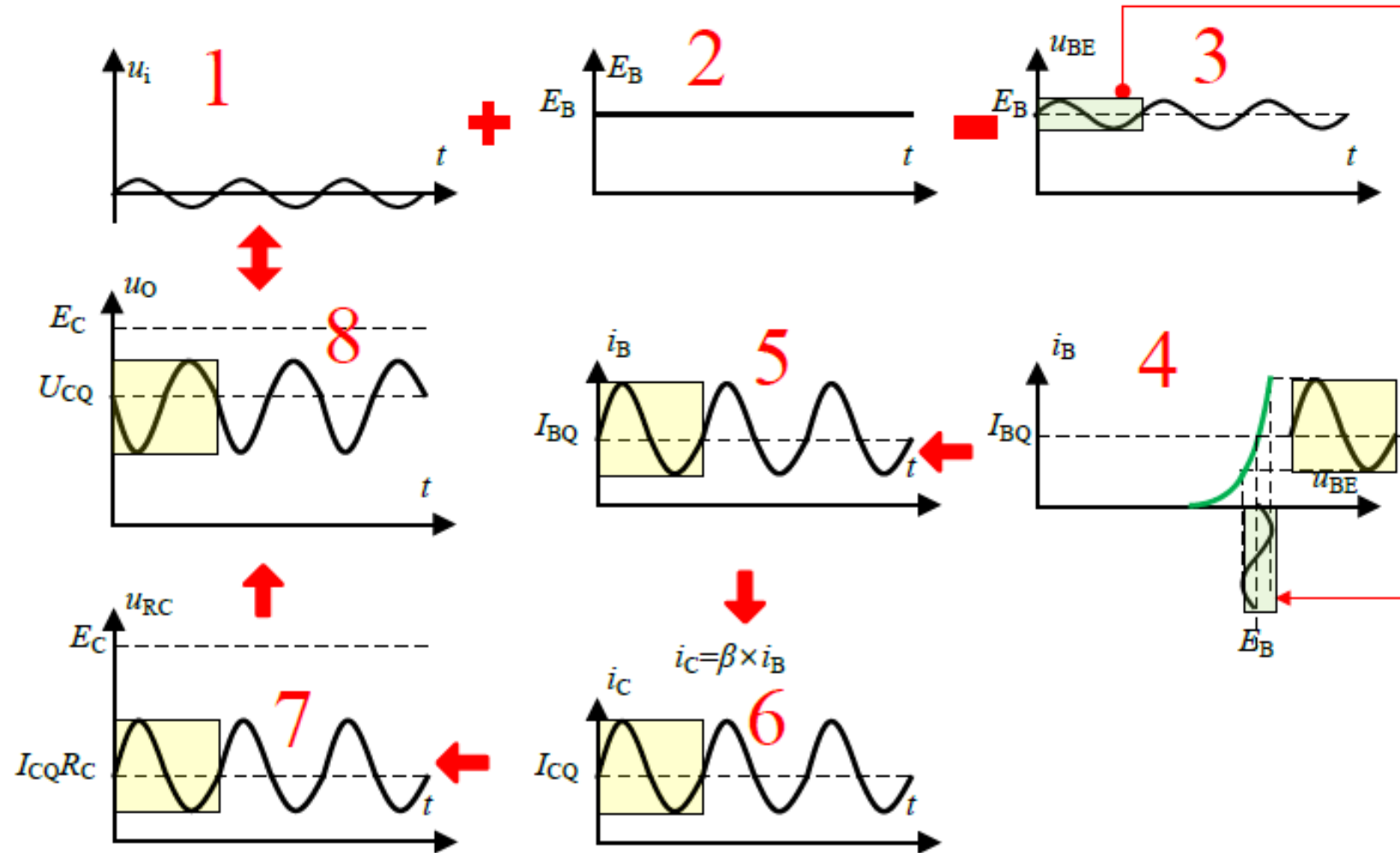


**Forward
Bias**

$$V_{BB} - I_b R_b = V_{BE} > V_{on}$$

R_c : suitable V_{CE}

C-E configuration

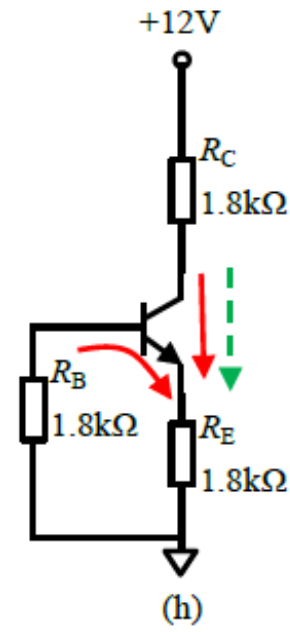
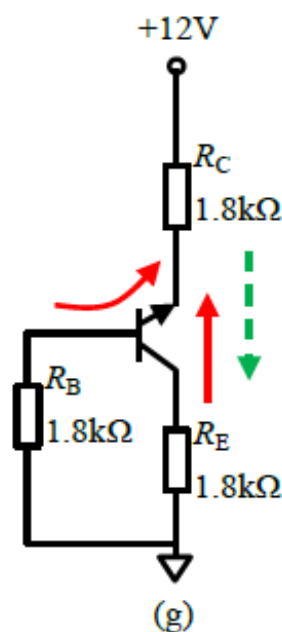
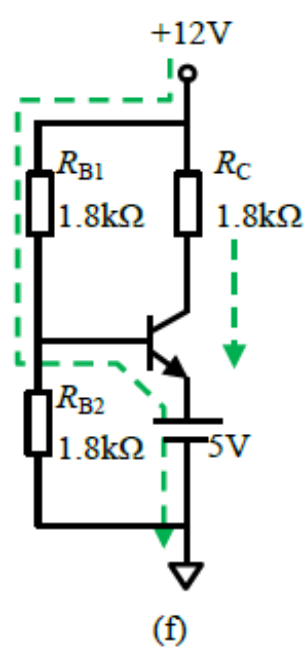
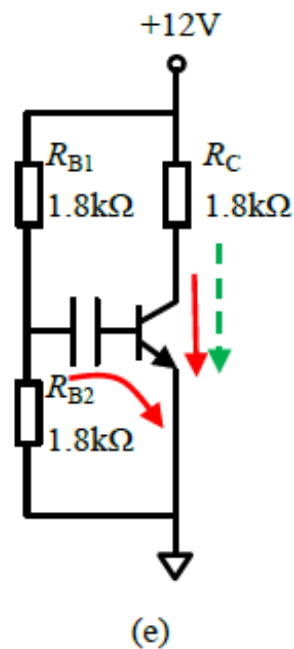
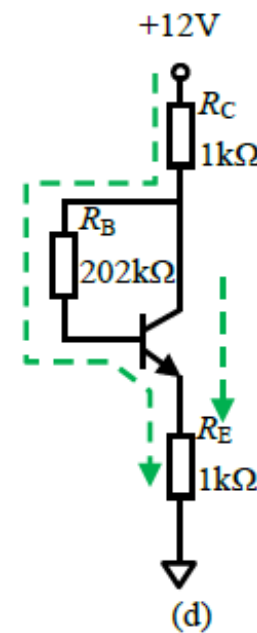
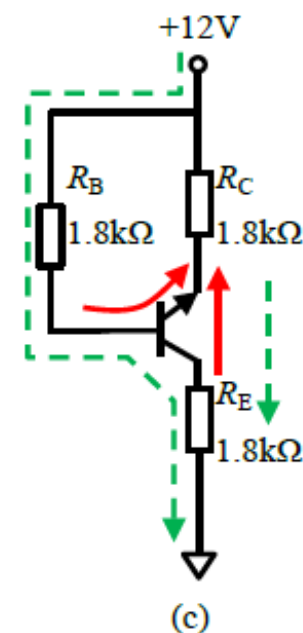
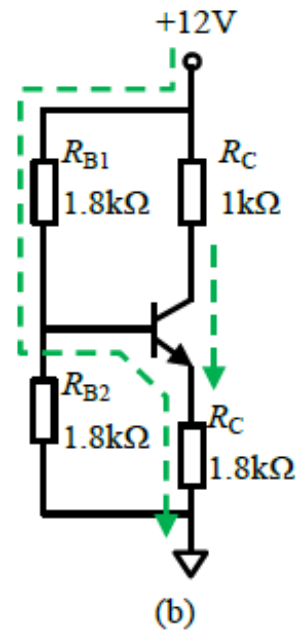
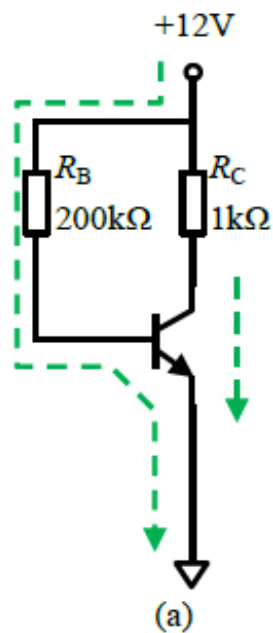


Exercise

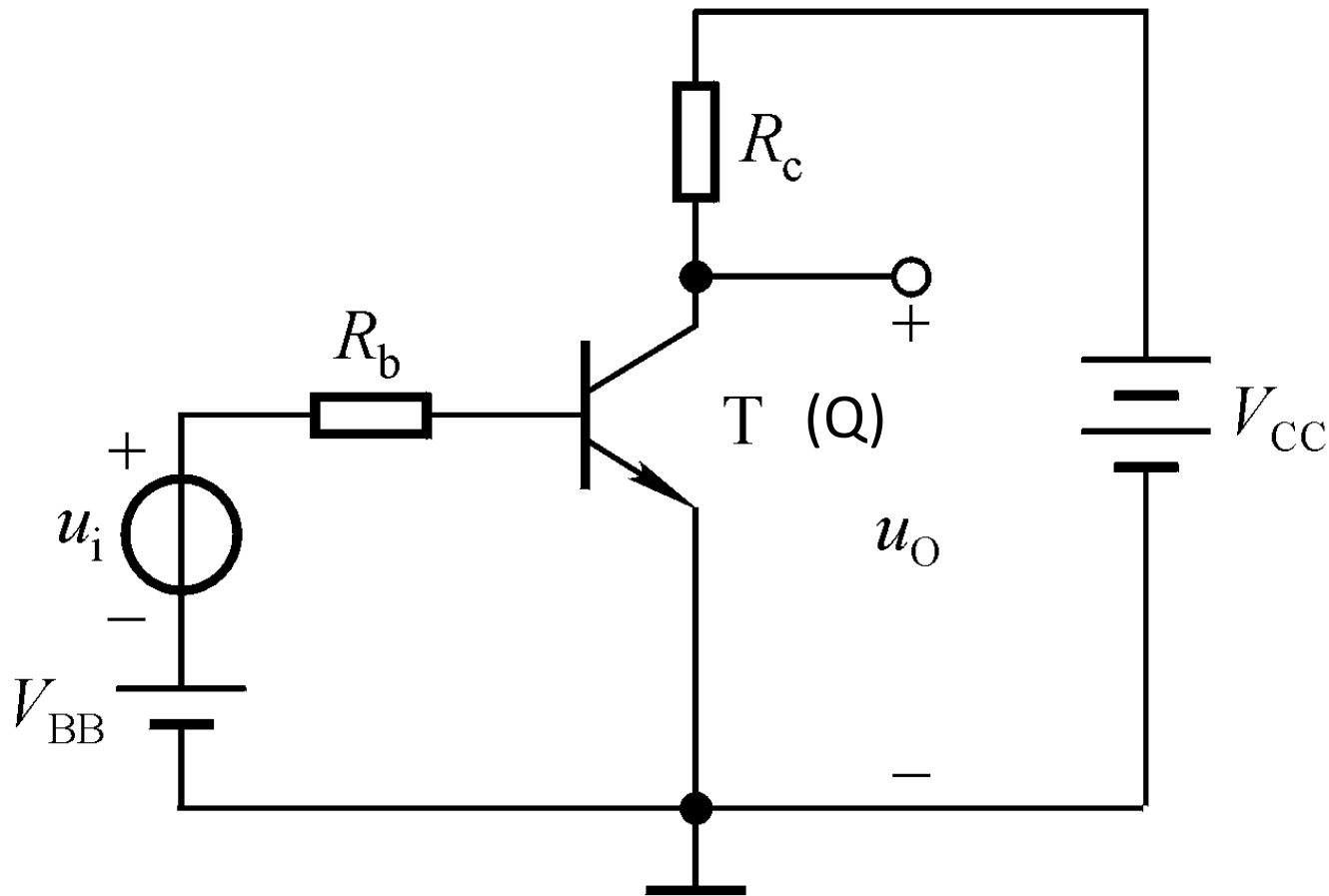
Can these configurations work properly?

**A,B,D,F:
Yes!**

- (c) Current cannot flow from emitter to collector and emitter to base.
- (e) Capacitor blocks base current.
- (g) Current cannot flow from emitter to collector.
- (h) No base current.



DC biasing: Quiescent point (Q-point)



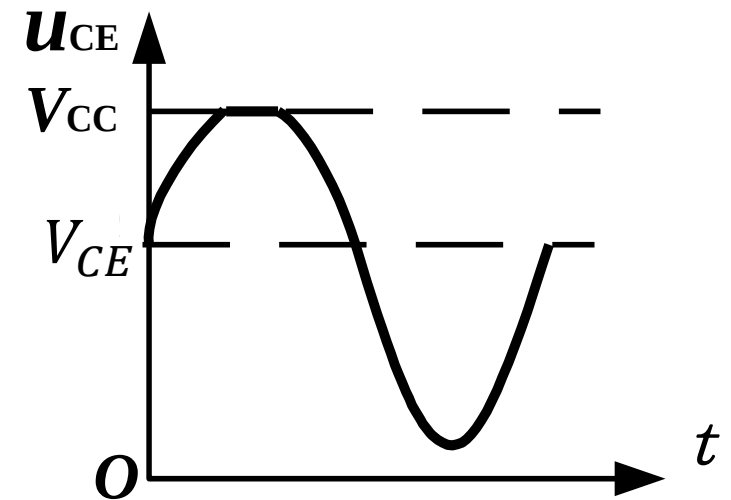
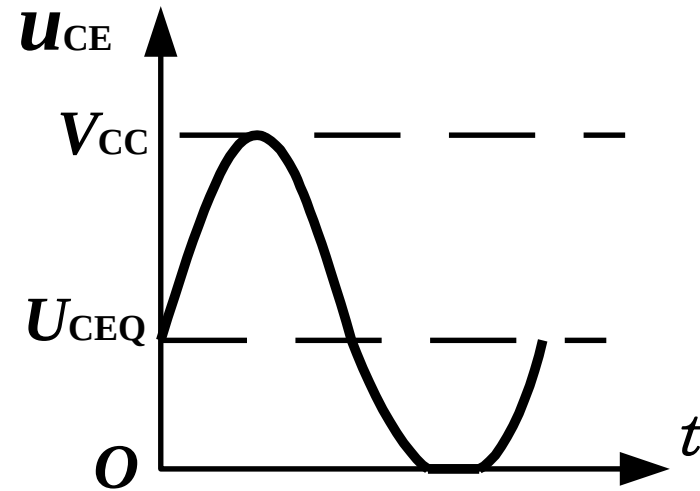
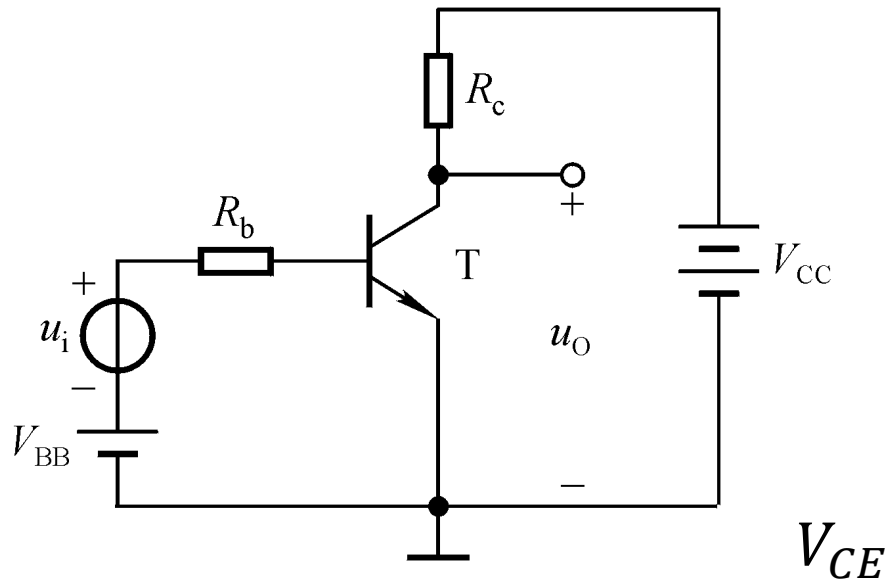
$$U_i = 0$$

Quiescent Point (Q Point)

$$I_{BQ}, I_{CQ} \text{ (} I_{EQ} \text{)}$$

$$V_{BE}, V_{CE}$$

Why Q-point is important?



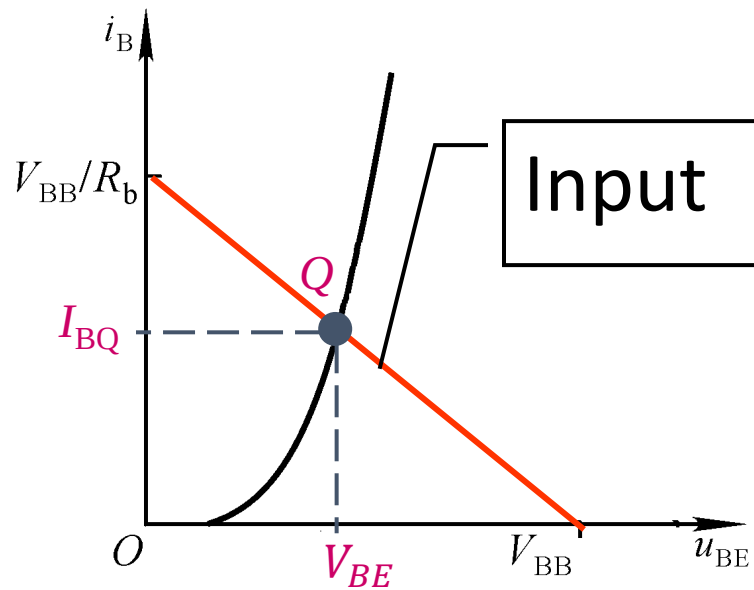
Suitable Q Point: No (Less) distortion

Suitable I_{BQ} , I_{CQ} (I_{EQ}), V_{BE} , V_{CE}

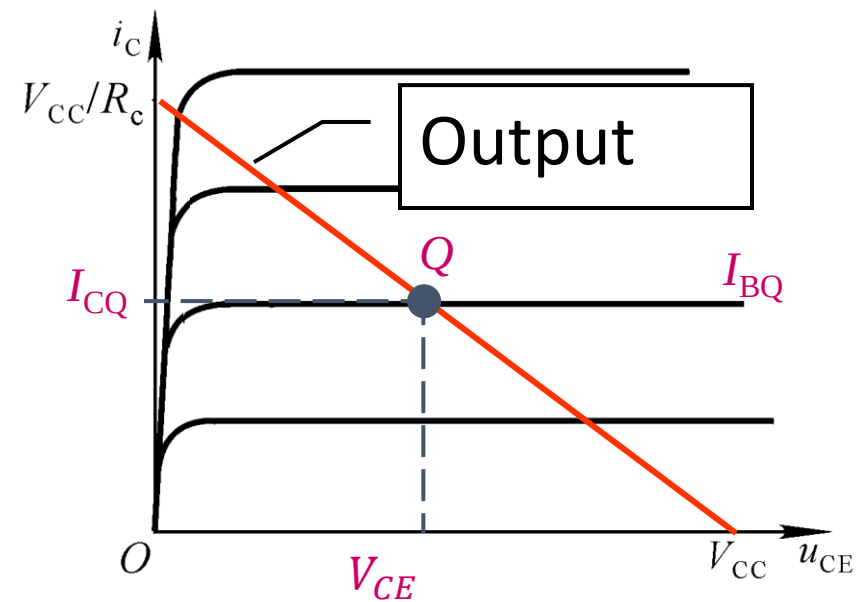
Why Q-point is important?

- Amplification may not be taking place over the whole signal cycle due to incorrect biasing levels.
- The input signal may be too large, causing the amplifiers transistors to be limited by the supply voltage.
- The amplification may not be a linear signal over the entire frequency range of inputs.
- Careful choice of the transistor and biasing components (Q Point) can help minimize the effect of amplifier distortion!

C-E configuration

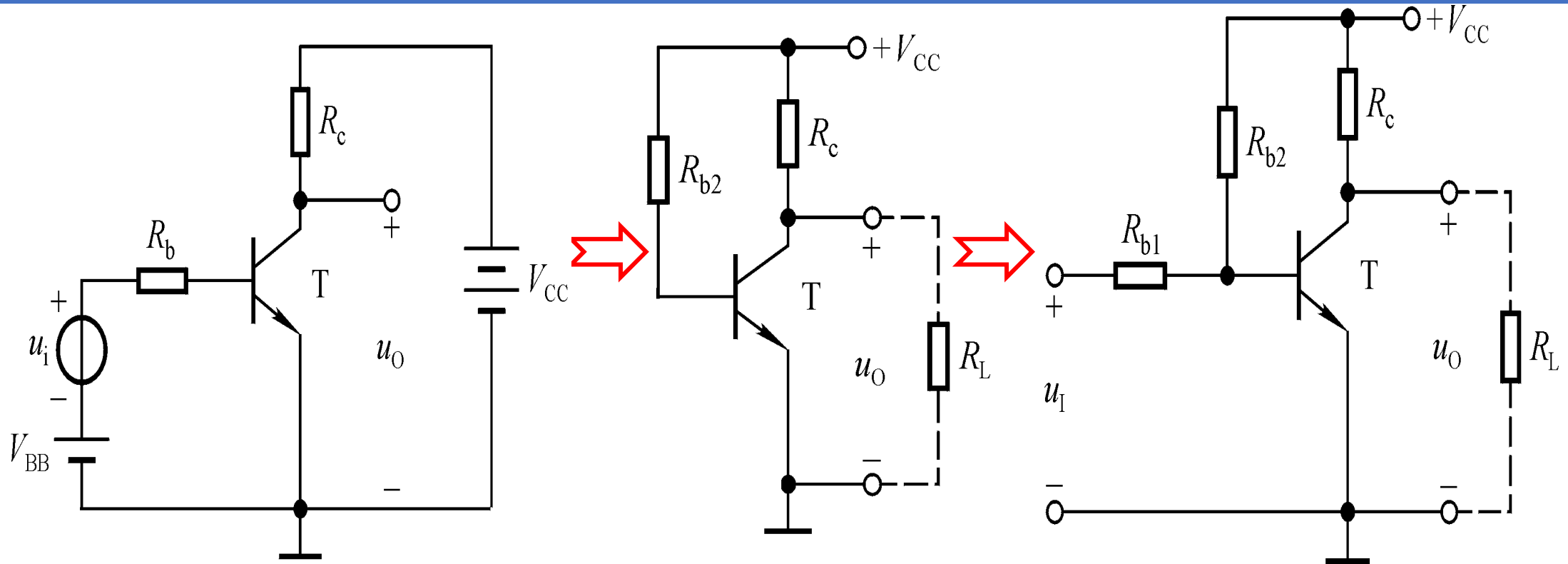


$$V_{BE} = V_{BB} - i_B R_b$$



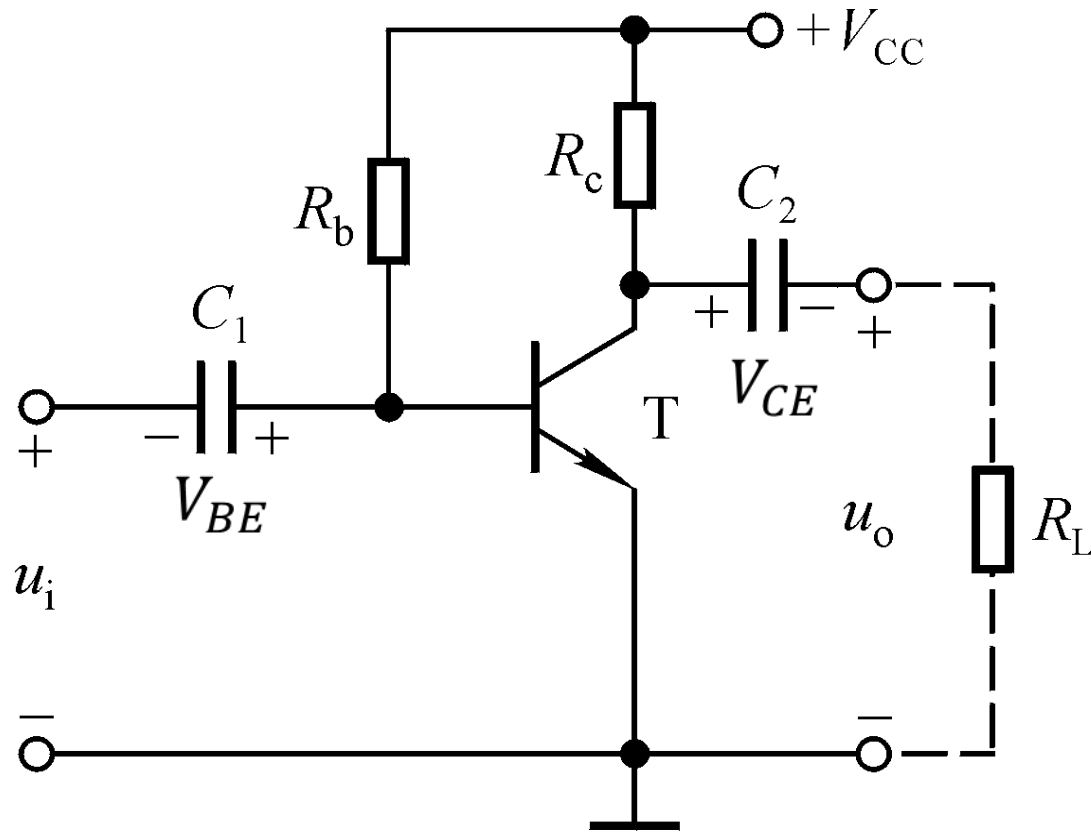
$$V_{CE} = V_{CC} - i_c R_c$$

How to provide DC & AC inputs?



How to find a voltage source which is equal to the exact biasing voltage across B-E?

How to provide DC & AC inputs?



DC:

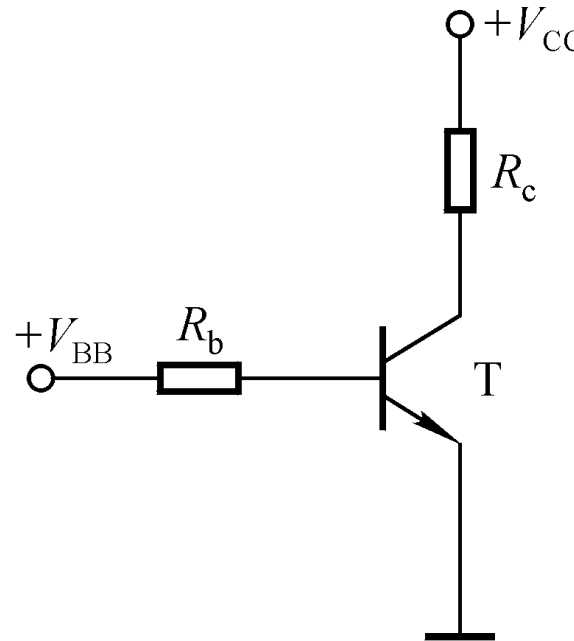
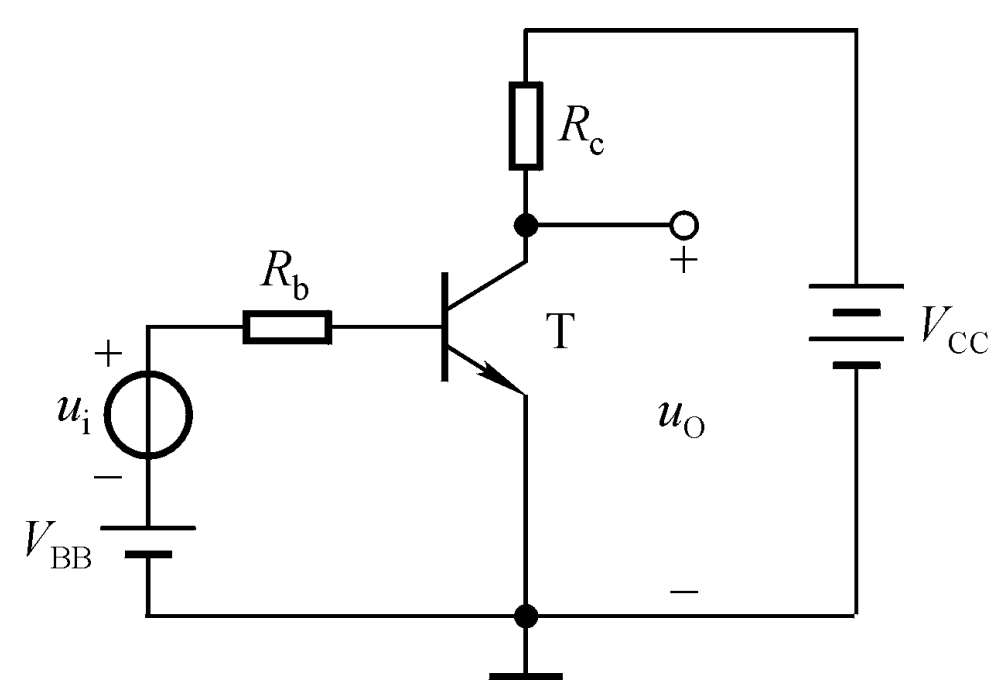
$$V_{C1} = V_{BE}, \quad V_{C2} = V_{CE}$$

AC:

$$u_{BE} = u_i + V_{BE}$$

Coupling capacitors (large enough so that they become short-circuit at signal frequencies)

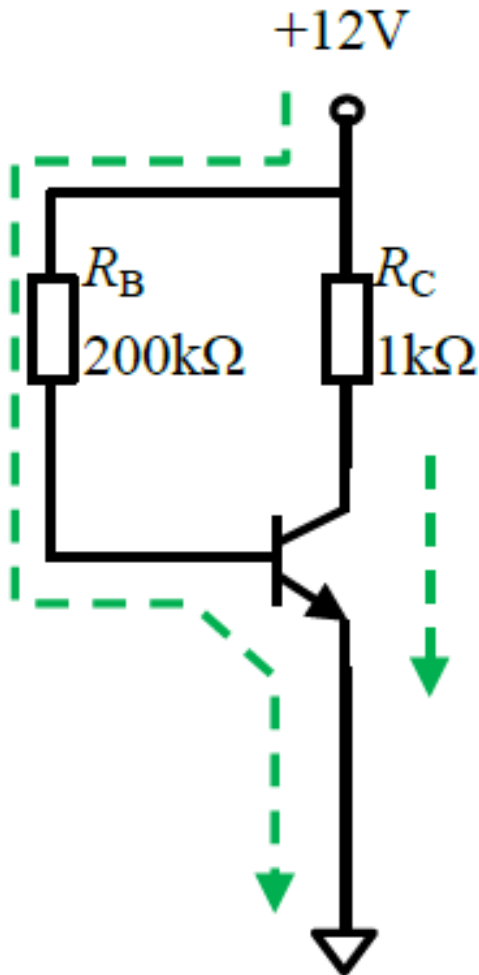
DC Analysis



$$I_{BQ} = \frac{V_{BB} - V_{BE}}{R_b}$$
$$I_{CQ} = \beta I_{BQ}$$
$$V_{CE} = V_{CC} - I_{CQ} R_c$$

$$V_{BE} = 0.6/0.7V, I_{CQ} = \beta I_{BQ}$$

DC Analysis



$$E_C = I_{BQ} \times R_B + V_{BE}$$

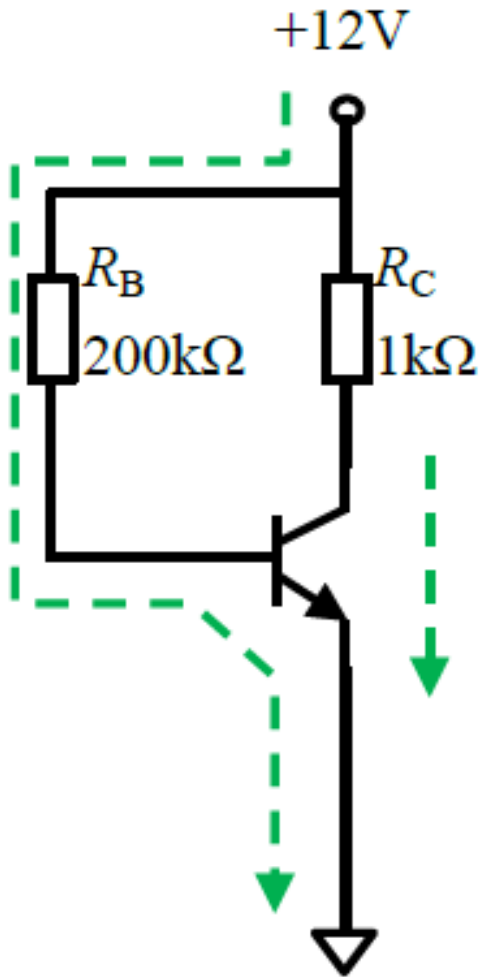
$$12 = I_{BQ} \times 200\text{k}\Omega + 0.7 \quad I_{BQ} = 56.5\mu\text{A}$$

Assume that the BJT works in forward active region

$$I_{CQ} = \beta I_{BQ} = 5.65\text{mA}$$

$$V_{CE} = E_C - I_{CQ}R_C = 12 - 5.65\text{mA} \times 1\text{k}\Omega = 6.35\text{V}$$

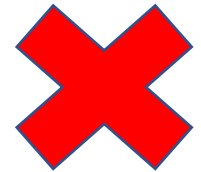
DC Analysis



What if $R_C = 5k\Omega$?

$$I_{CQ} = \beta I_{BQ} = 5.65\text{mA}$$

$$U_{CEQ} = E_C - I_{CQ}R_C$$
$$= 12 - 5.65\text{mA} \times 5k\Omega = -16.25\text{V}$$

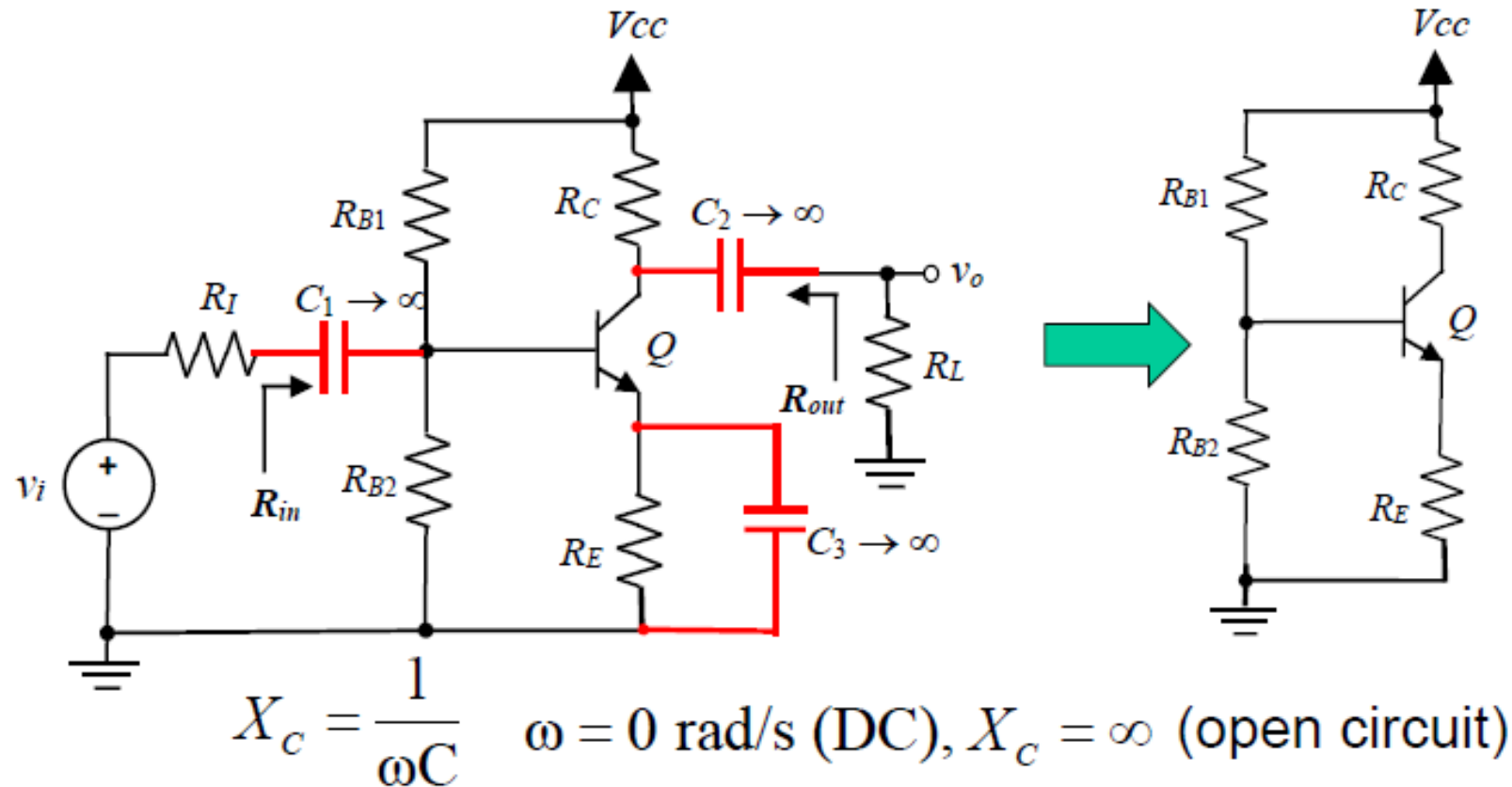


DC and AC Analysis

DC Analysis

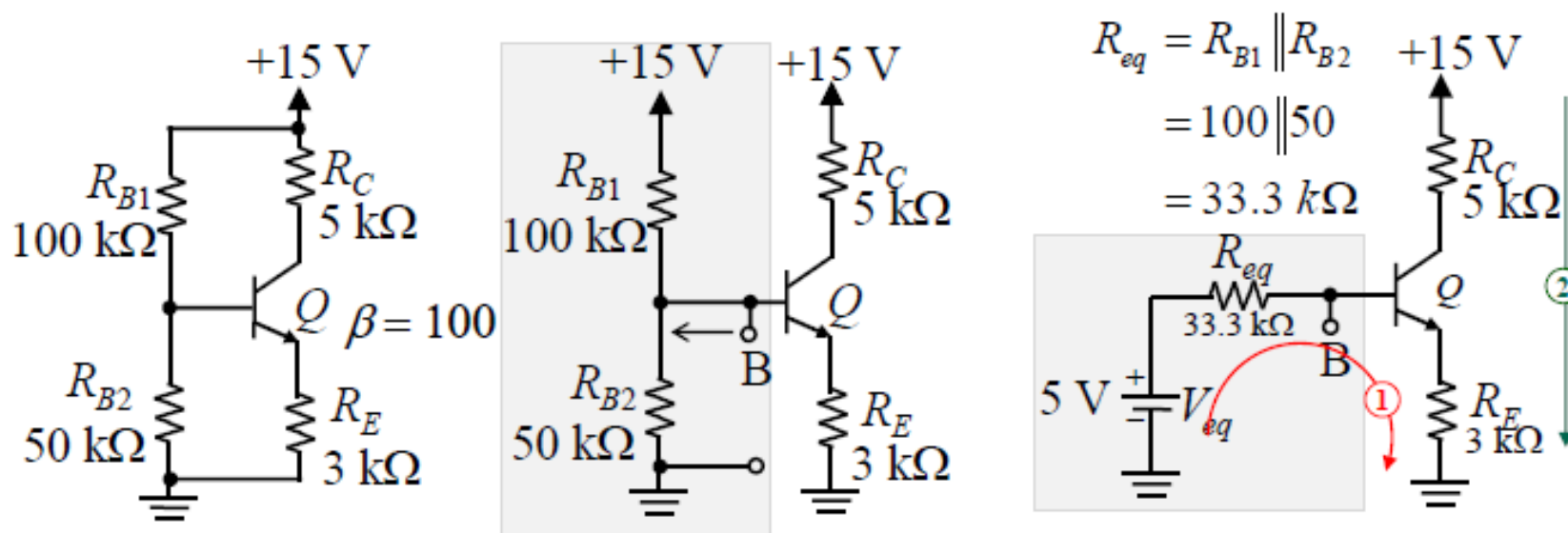
- Obtain DC equivalent circuit by replacing all capacitors by open circuits, inductors by short circuit, AC voltage sources by ground connections, and AC current sources by open circuits.
- Find Q-point from DC equivalent circuit by using appropriate circuit analysis techniques, such as Thevenin equivalent circuit, KVL, and KCL.

DC equivalent circuit for BJT CE Amplifier



All capacitors in original amplifier circuits are replaced by open circuits, disconnecting v_i , R_I , and R_L from circuit.

DC Analysis example: 4 resistor BJT biasing circuit



KVL 1: $V_{eq} = I_B R_{eq} + V_{BE} + I_E R_E$
 $5 = 33.3 I_B + 0.7 + 101 \times I_B \times 3$
 $I_B = 0.0128\text{ mA}$

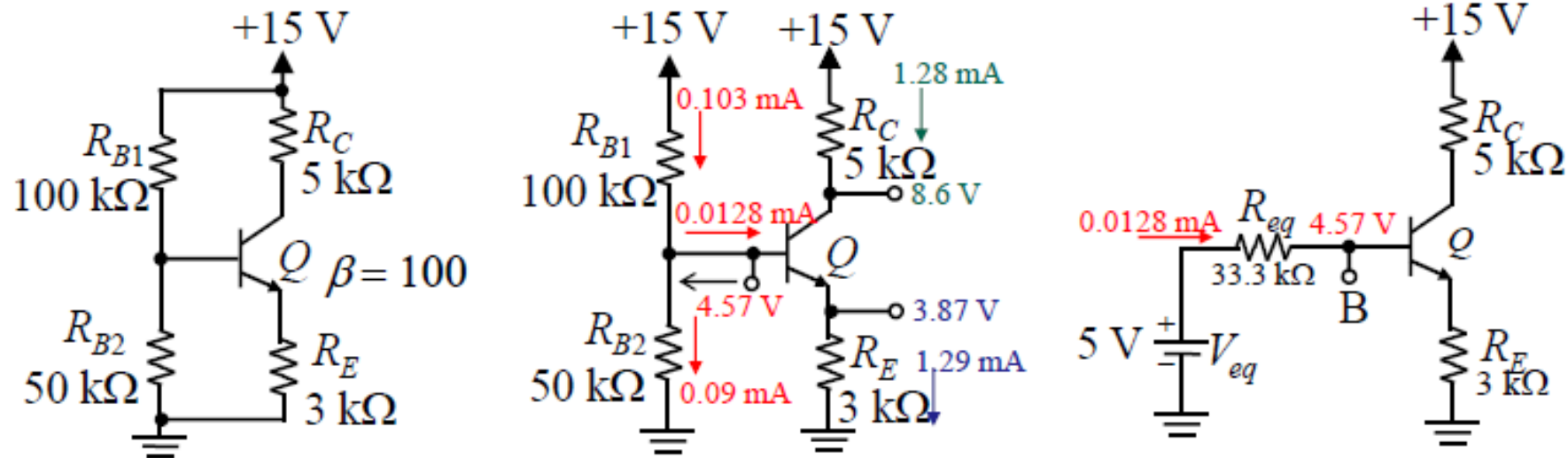
$I_C = \beta I_B = \mathbf{1.28\text{ mA}}$, $I_E = (\beta + 1) I_B = 1.29\text{ mA}$

KVL 2: $V_{CE} = 15 - I_C R_C - I_E R_E = 15 - 1.28 \times 5 - 1.29 \times 3 = \mathbf{4.73\text{ V}}$

$$V_{eq} = \left(\frac{R_{B2}}{R_{B1} + R_{B2}} \right) V_{CC}$$

$$= \frac{50}{100 + 50} \times 15 = 5\text{ V}$$

DC Analysis example: 4 resistor BJT biasing circuit



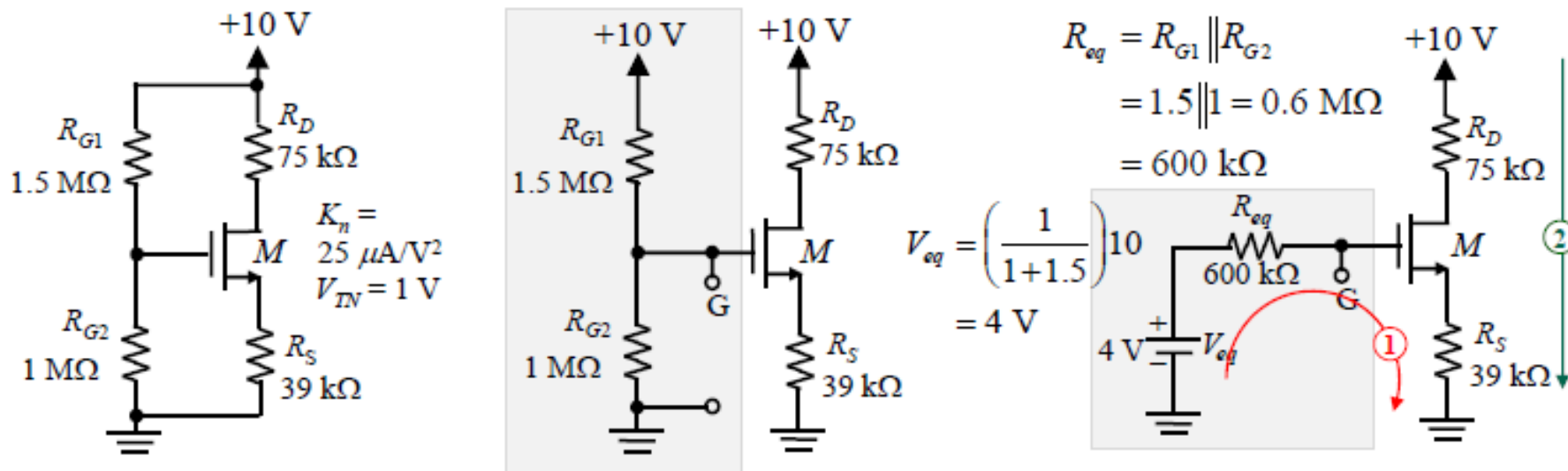
$$V_B = V_{BE} + I_E R_E = 0.7 + 3.87 = 4.57\text{ V}$$

$$V_C = V_{CC} - I_C R_C = 15 - 1.28 \times 5 = 8.6\text{ V}$$

$$V_{BC} = V_B - V_C = 4.57 - 8.6 = -4.03\text{ V}$$

BCJ is reverse biased, Q is indeed in active mode as had been assumed.

DC Analysis example: 4 resistor MOSFET biasing circuit



KVL 1: Since $I_G = 0$, $V_{eq} = V_{GS} + I_D R_S$

$$4 = V_{GS} + 0.5 \times 25 \mu (V_{GS} - 1)^2 \times 39 \text{ k}$$

$$V_{GS}^2 + 0.05 V_{GS} - 7.21 = 0$$

$$V_{GS} = -2.71 \text{ or } 2.66 \text{ V}$$

Since $V_{GS} = -2.71 < V_{TN} = 1$, $V_{GS} = 2.66 \text{ V}$.

$$I_D = (4 - 2.66)/39\text{k} = \mathbf{34.4 \mu\text{A}}$$

KVL 2: $V_{DS} = 10 - I_D R_D - I_D R_S$

$$V_{DS} = 10 - 0.0344 \times (75 + 39) = \mathbf{6.08 \text{ V}}$$

Since $V_{DS} > V_{GS} - V_{TN} = 1.66$,

***M* is in saturation region.**

DC and AC Analysis

AC Analysis

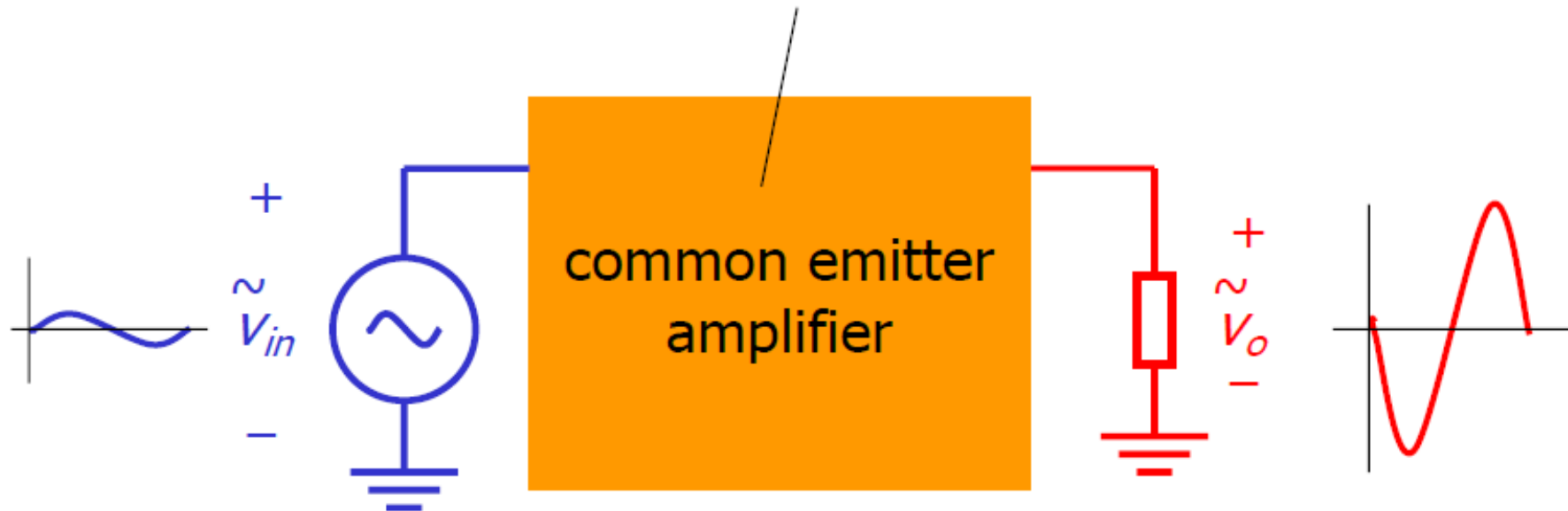
- Obtain AC equivalent circuit by replacing all capacitors by short circuits, inductors by open circuits, DC voltage sources by ground connections, and DC current sources by open circuits.
- Replace transistor by its small signal model.
- Use small signal AC equivalent to analyse AC characteristics of amplifier.

Combine end results of DC and AC analyses to yield total voltages and currents in the network.

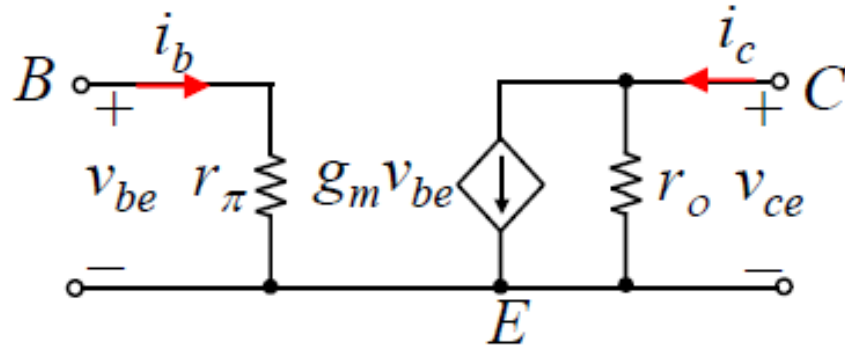
AC analysis: Small signal model

- We are interested in AC signal.

Can we create a simple circuit just to look at ac signals?



AC analysis: Small signal BJT model



- This hybrid-pi small-signal model is the intrinsic low-frequency representation of the BJT.
- Small-signal parameters are controlled by the Q-point and are independent of geometry of BJT.

Transconductance:

$$g_m = \frac{I_C}{V_T} \approx 40 I_C$$

$$\text{where } V_T = \frac{kT}{q} \approx 25 \text{ mV@}25^\circ\text{C}$$

Input resistance:

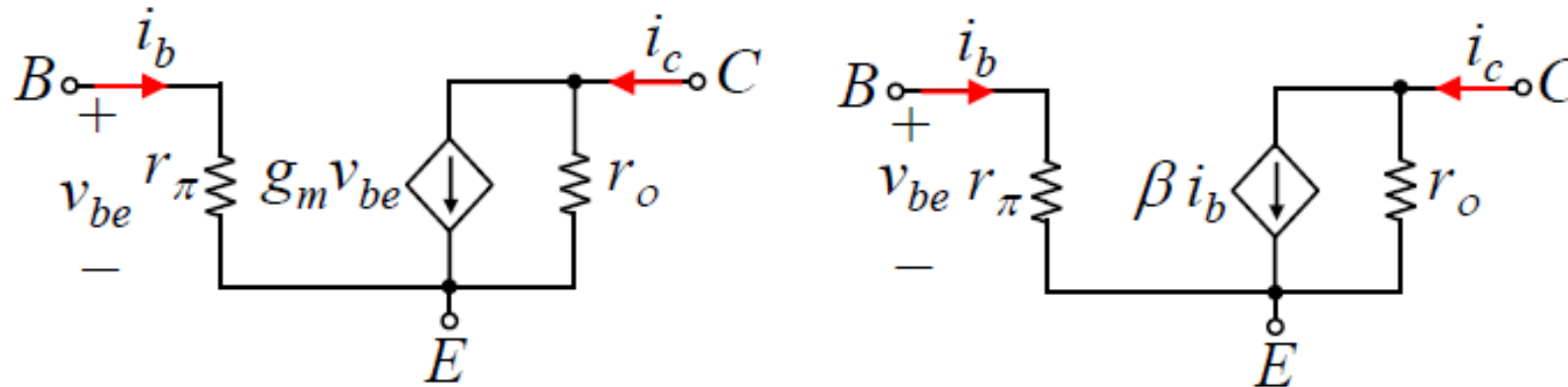
$$r_\pi = \frac{\beta}{g_m}$$

Output resistance:

$$r_o = \frac{V_A + V_{CE}}{I_C} \approx \frac{V_A}{I_C} \text{ if } V_A \gg V_{CE}$$

AC analysis: Small signal BJT model

Voltage-controlled current source $g_m v_{be}$ can be transformed into current-controlled current source.



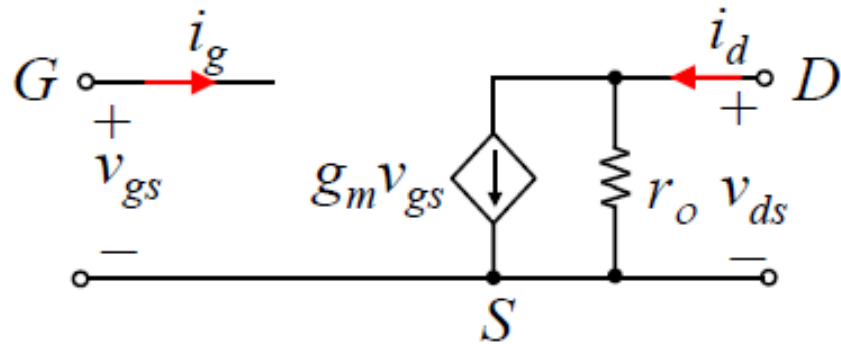
$$v_{be} = i_b r_\pi$$

$$g_m v_{be} = g_m i_b r_\pi = \beta i_b$$

$$i_c = \beta i_b + \frac{v_{ce}}{r_o} \approx \beta i_b$$

Basic relationship $i_c = \beta i_b$ is useful for both dc and ac analysis when BJT is in forward-active region.

AC analysis: Small signal MOS model



- Since gate is insulated from channel by gate-oxide, input resistance = ∞ .
- Small-signal parameters are controlled by the Q-point.
- MOSFET transconductance is geometry dependent.

$$i_D = \frac{K_n}{2} (v_{GS} - V_{TN})^2 (1 + \lambda v_{DS})$$

$$g_m = \left. \frac{\partial i_D}{\partial v_{GS}} \right|_{Q-pt} = K_n (V_{GS} - V_{TN}) (1 + \lambda V_{DS})$$

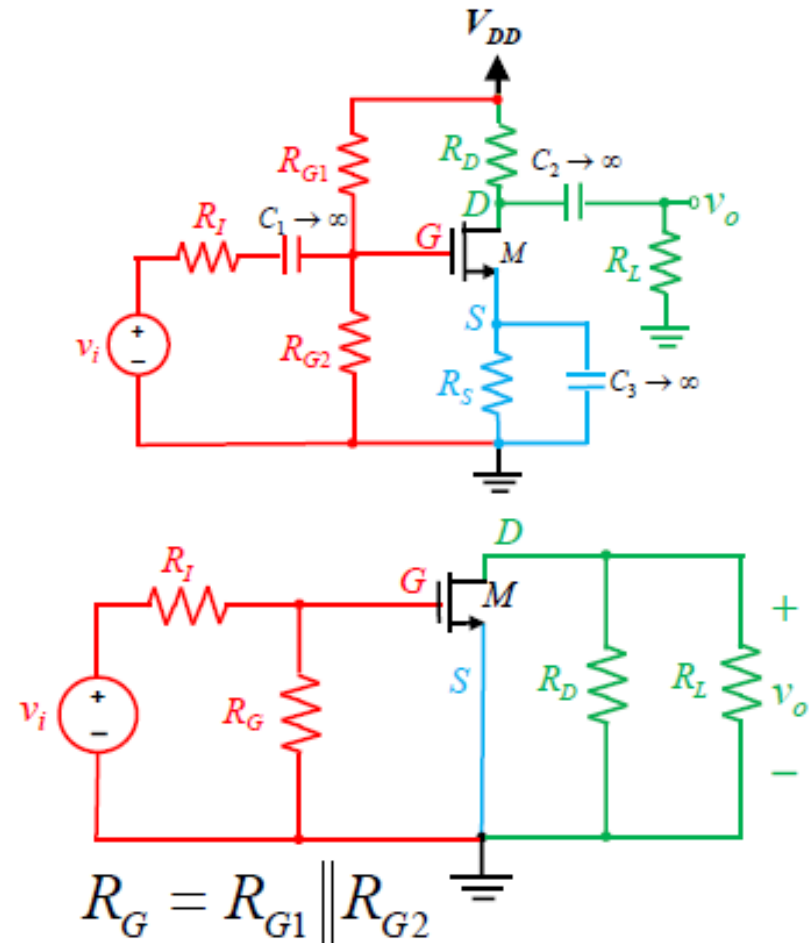
where $K_n = \mu_n C_{ox} \left(\frac{W}{L} \right)$

$$g_m = \frac{I_D}{\frac{V_{GS} - V_{TN}}{2}} \approx \sqrt{2K_n I_D}$$

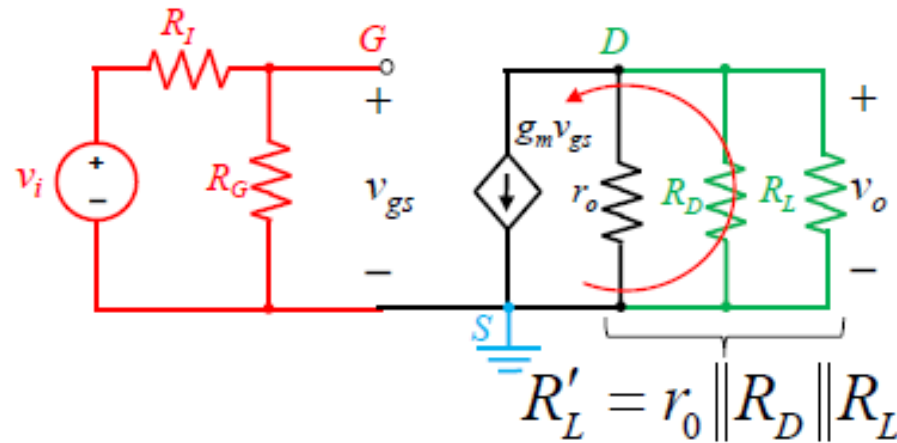
$$r_o = \frac{\frac{1}{\lambda} + V_{DS}}{I_D} \approx \frac{1}{\lambda I_D} \text{ if } \frac{1}{\lambda} \gg V_{DS}$$

Small-signal analysis of C-S amplifier fully bypassed R_s

- AC equivalent circuit is constructed by assuming that all capacitances have zero impedance at signal frequency and dc voltage sources represent ac grounds.
- The small signal parameters, g_m and r_o of the MOSFET is calculated at the Q-point, I_D and V_{DS} .



C-S amplifier fully bypassed Rs: Voltage Gain



Terminal voltage gain between gate and drain is:

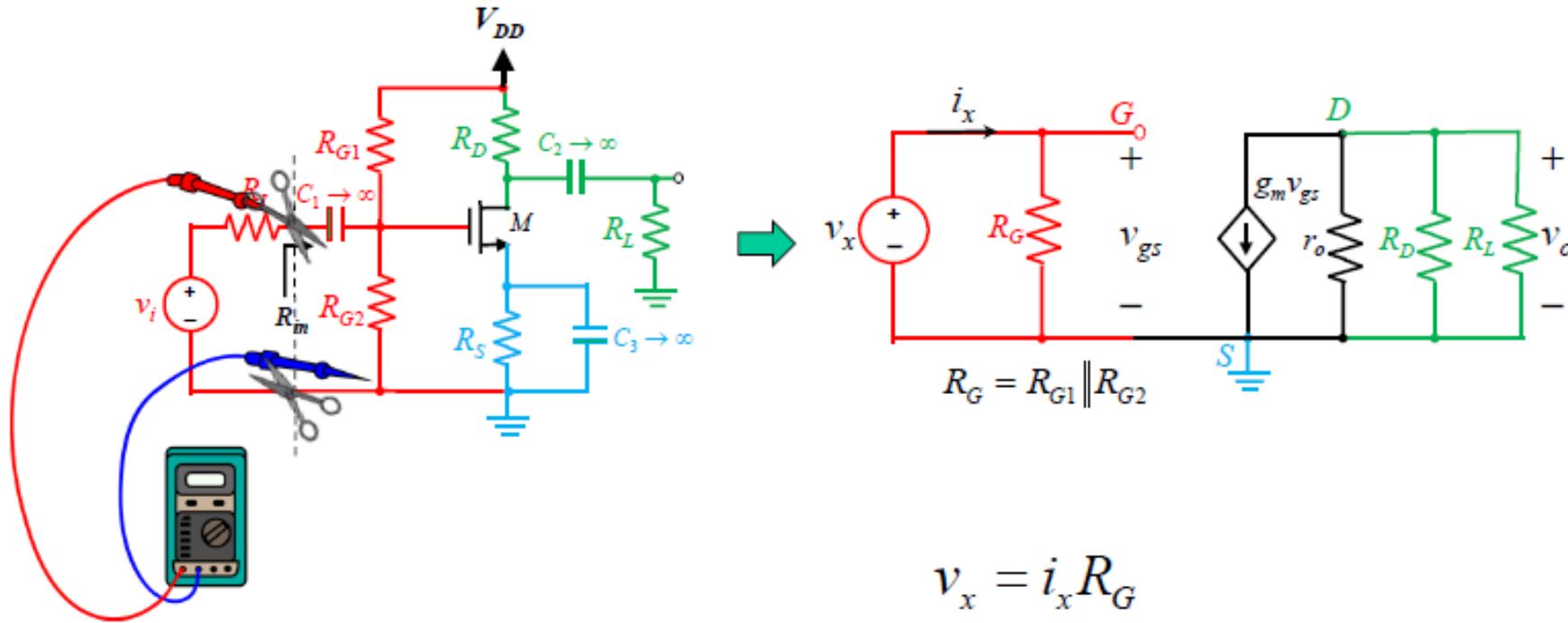
$$A_{vt} = \frac{v_d}{v_g} = \frac{-g_m v_{gs} R'_L}{v_{gs}} = -g_m R'_L$$

Overall voltage gain from source v_i to output voltage v_o across R_L is:

$$A_v = \frac{v_o}{v_i} = \frac{v_o}{v_g} \times \frac{v_g}{v_i} = A_{vt} \times \frac{v_g}{v_i}$$

$$= -g_m R'_L \left(\frac{R_G}{R_I + R_G} \right)$$

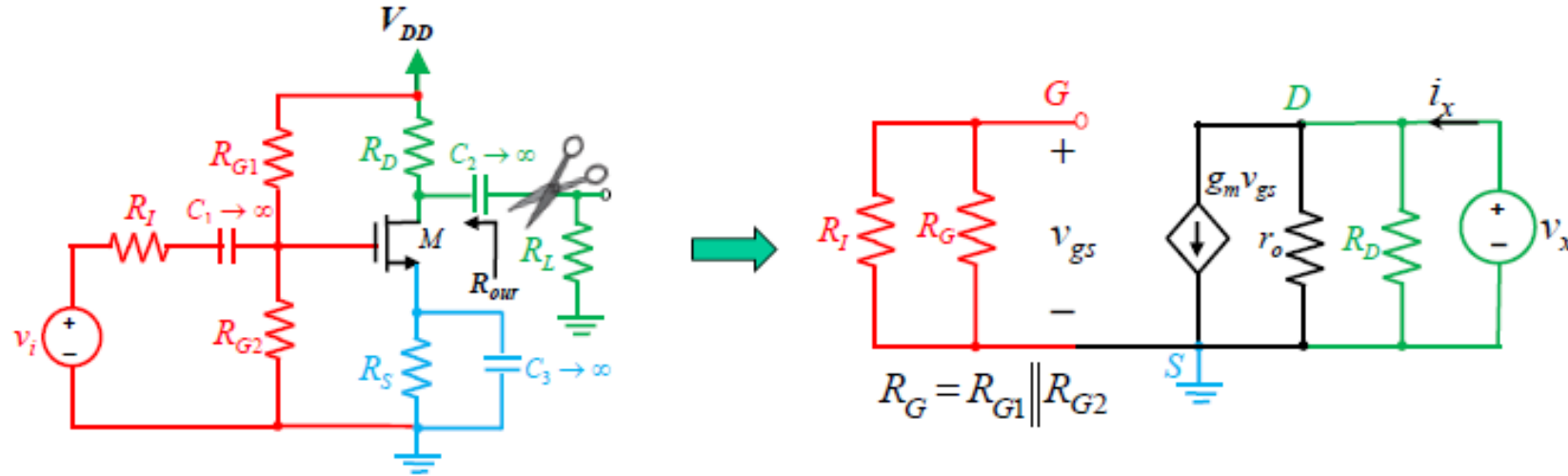
C-S amplifier fully bypassed R_s : Input Resistance



$$v_x = i_x R_G$$

$$R_{in} = \frac{v_x}{i_x} = R_G$$

C-S amplifier fully bypassed R_s : Output Resistance



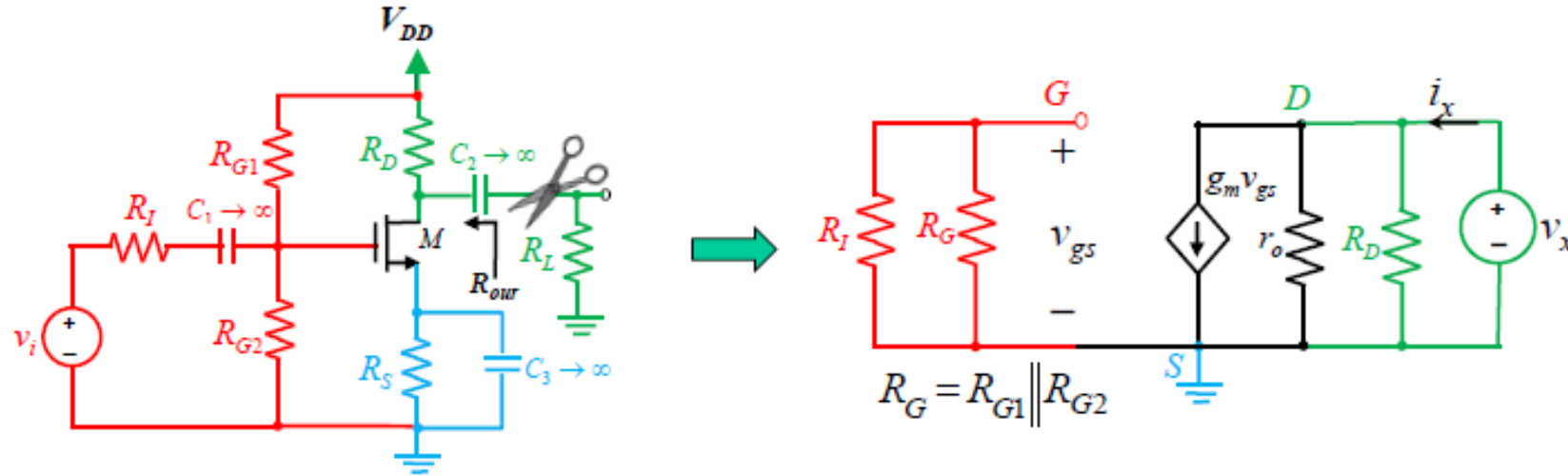
Since $v_{gs} = 0$, $g_m v_{gs} = 0$.

$$v_x = i_x (R_D \parallel r_o)$$

$$R_{out} = \frac{v_x}{i_x} = R_D \parallel r_o$$

$$R_{out} \approx R_D \text{ if } r_o \gg R_D$$

C-S amplifier fully bypassed R_s : Output Resistance



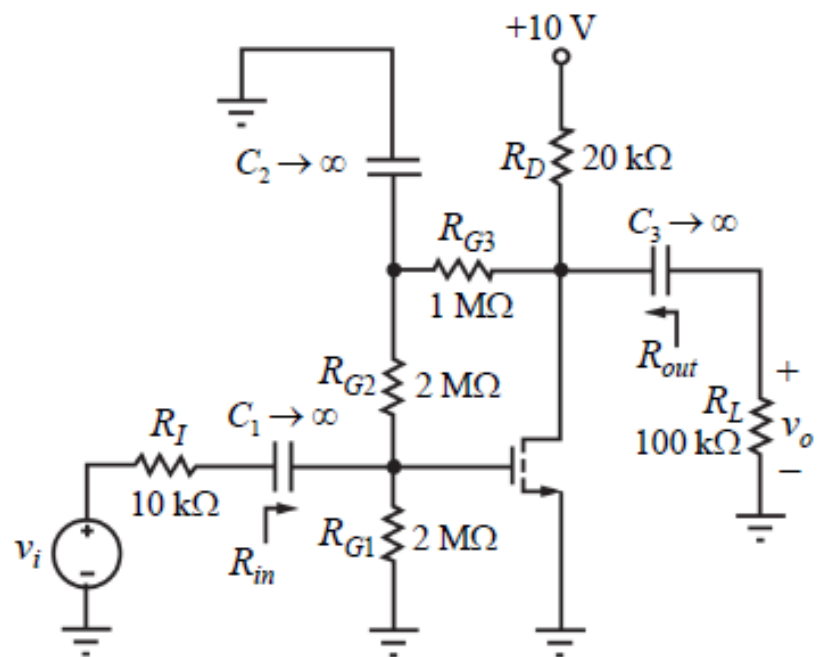
Since $v_{gs} = 0$, $g_m v_{gs} = 0$.

$$v_x = i_x (R_D \parallel r_o)$$

$$R_{out} = \frac{v_x}{i_x} = R_D \parallel r_o$$

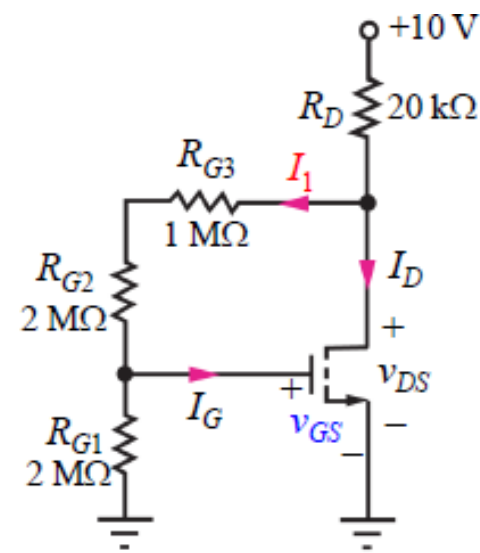
$$R_{out} \approx R_D \text{ if } r_o \gg R_D$$

C-S amplifier fully bypassed R_s : Example



Problem: Find voltage gain, input and output resistances.

Given: $K_n = 500 \mu\text{A}/\text{V}^2$, $V_{TN} = 1\text{V}$,
 $\lambda = 0.0167 \text{ V}^{-1}$



DC Analysis:

$$I_1 = \frac{V_{DS}}{5 \times 10^6}$$

$$V_{GS} = I_1 \times 2 \times 10^6 = 0.4V_{DS}$$

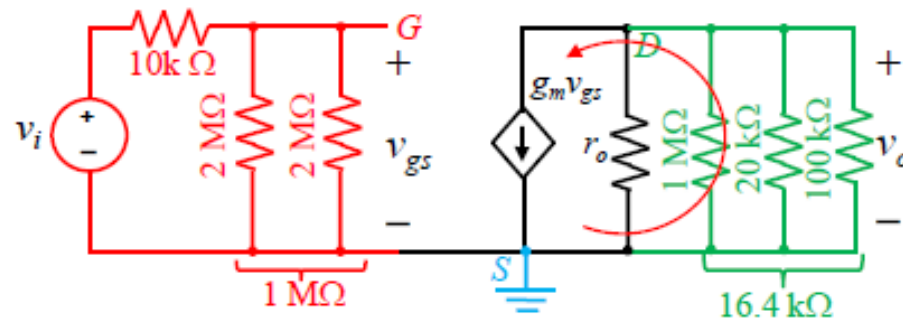
$$I_D = \frac{500 \mu}{2} (V_{GS} - 1)^2 \quad (1)$$

$$V_{DS} = 10 - 20 \times 10^3 (I_D + I_1) \quad (2)$$

$$\therefore V_{DS} = 5\text{V}, V_{GS} = 2\text{V}, I_D = 250 \mu\text{A}$$

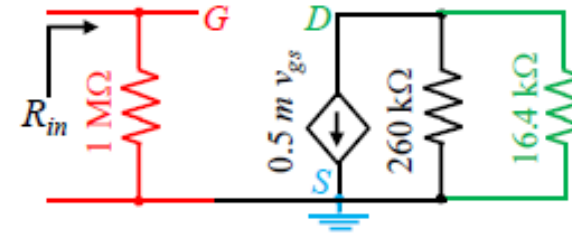
C-S amplifier fully bypassed Rs: Example

Small Signal Equivalent Circuit:

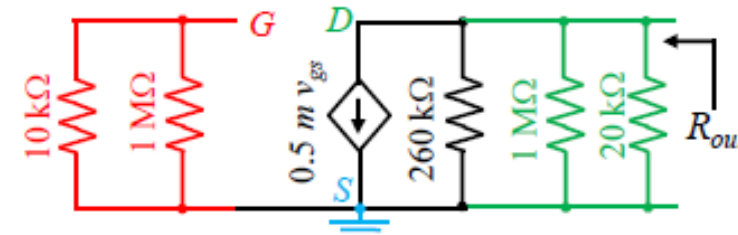


$$g_m \approx \sqrt{2 \times 500 \mu \times 250 \mu} = 0.5 \text{ mS}$$

$$r_o = \frac{1}{\frac{0.0167}{250 \mu} + 5} = 260 \text{ k}\Omega$$



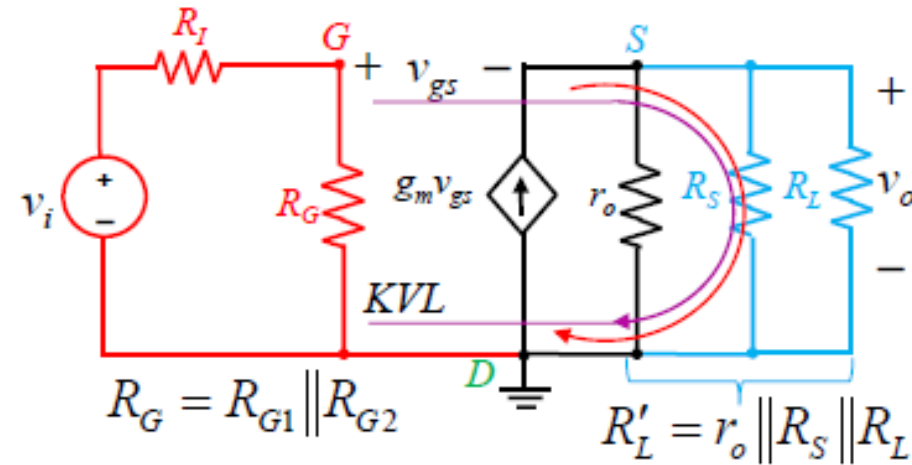
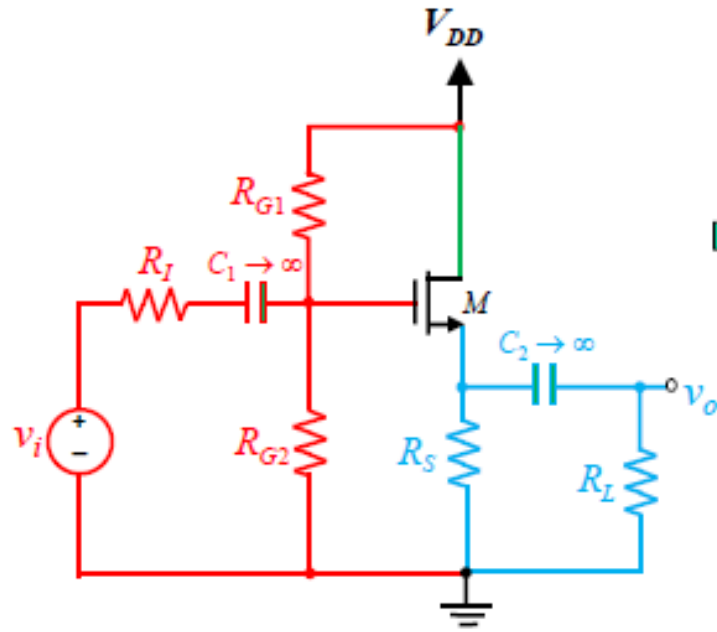
$$R_{in} = 2M \parallel 2M = 1M\Omega$$



$$R_{out} = 260k \parallel 1M \parallel 20k = 18.2k\Omega$$

$$A_v = A_{vt} \left(\frac{R_{in}}{R_I + R_{in}} \right) = \frac{-(0.5m)v_{gs} (260k \parallel 1M \parallel 20k \parallel 100k)}{v_{gs}} \left(\frac{1M}{10k + 1M} \right) = -7.93$$

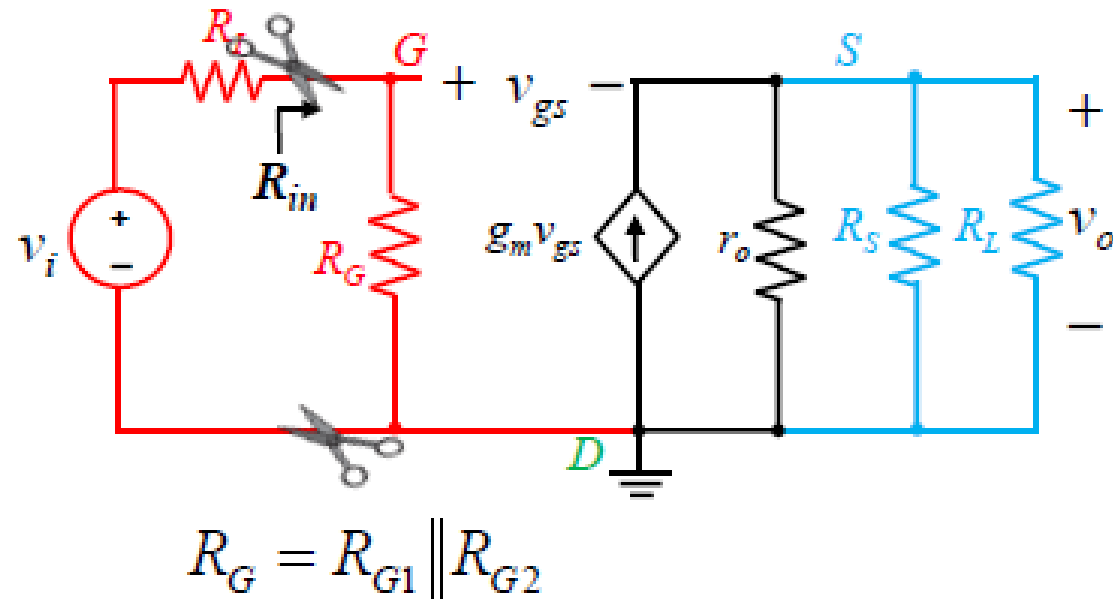
C-D Amplifier (Voltage follower): Terminal Voltage Gain



$$A_{vt} = \frac{v_s}{v_g} = \frac{g_m v_{gs} R'_L}{v_{gs} + g_m v_{gs} R'_L} = \frac{g_m R'_L}{1 + g_m R'_L}$$

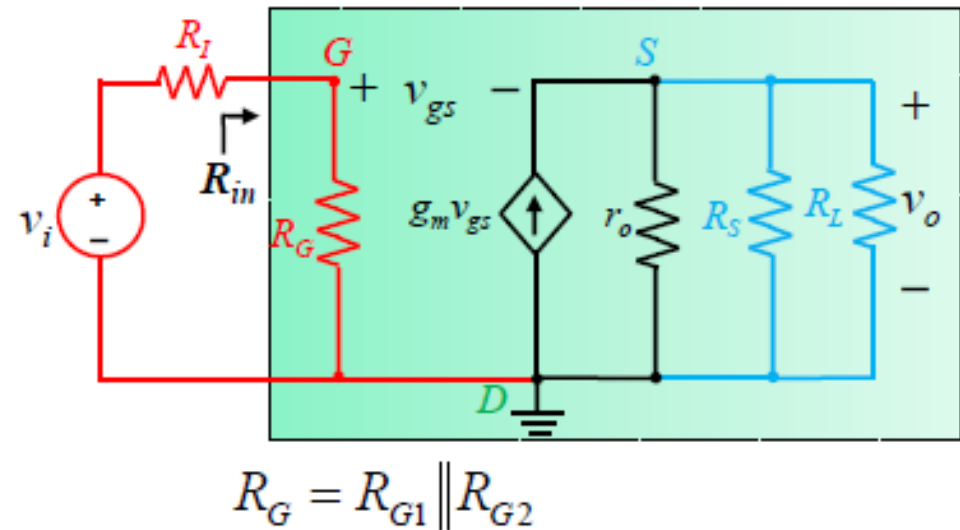
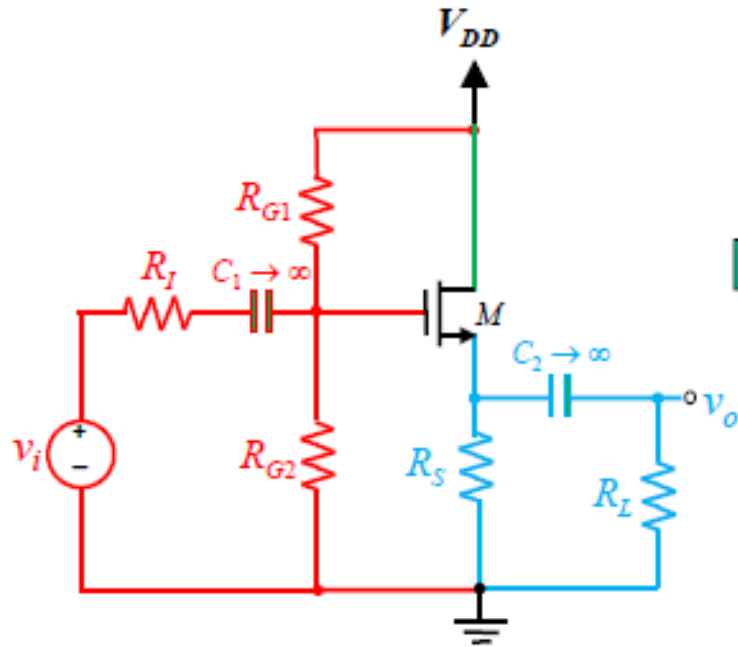
$$\text{If } g_m R'_L \gg 1, A_{vt} \approx 1 \Rightarrow v_o \approx v_g.$$

C-D Amplifier (Voltage follower): Input Resistance



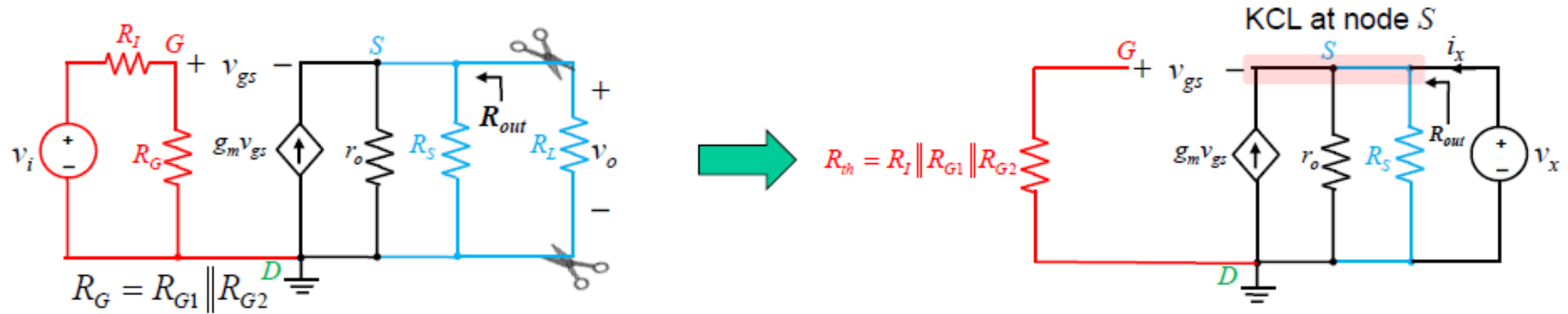
$$R_{in} = R_G$$

C-D Amplifier (Voltage follower): Overall Voltage Gain



$$A_v = \frac{v_o}{v_i} = \frac{v_o}{v_g} \times \frac{v_g}{v_i} = A_{vt} \times \frac{R_G}{R_I + R_G}$$

C-D Amplifier (Voltage follower): Output Resistance



$$i_x = \frac{v_x}{R_S} + \frac{v_x}{r_o} - g_m v_{gs}$$

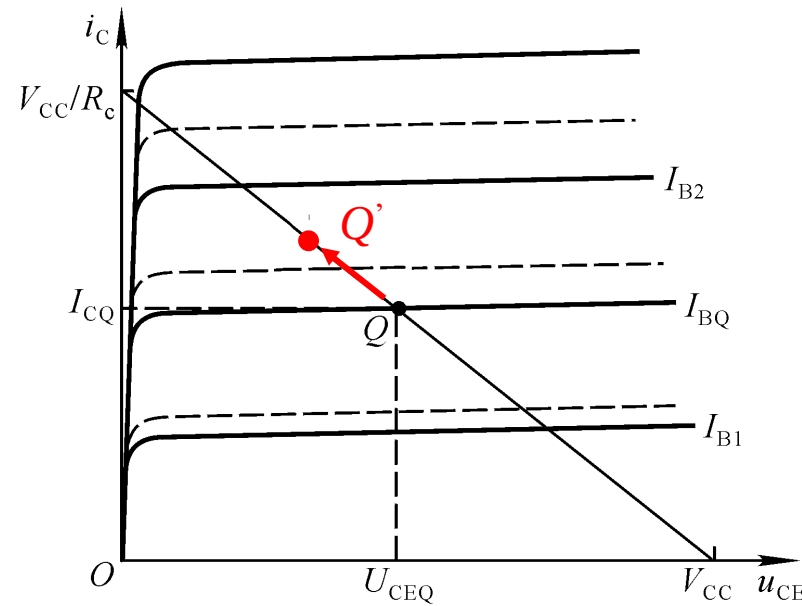
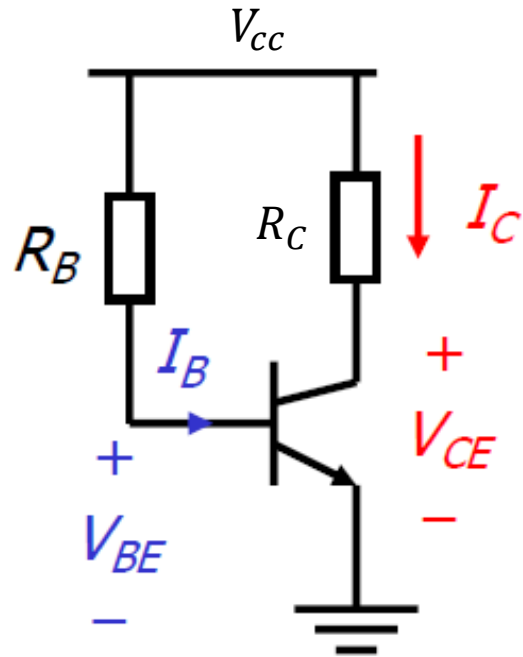
$$v_{gs} = -v_x$$

$$i_x = \frac{v_x}{R_S} + \frac{v_x}{r_o} + g_m v_x = \left(\frac{1}{R_S} + \frac{1}{r_o} + g_m \right) v_x$$

$$R_{out} = \frac{v_x}{i_x} = \left(\frac{1}{R_S} + \frac{1}{r_o} + g_m \right)^{-1}$$

$$= R_S \parallel r_o \parallel \frac{1}{g_m}$$

Role of feedback: Stabilizing Q point

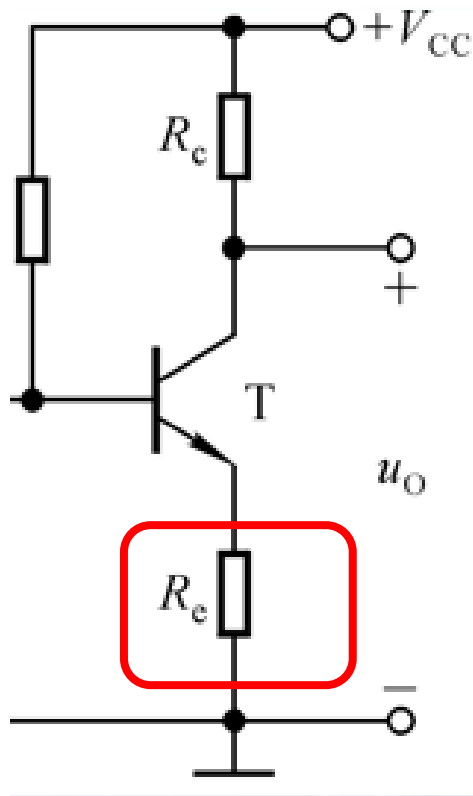


What if temperature increases?

$$T \text{ (} ^\circ\text{C} \text{)} \rightarrow \beta \uparrow \rightarrow I_C \uparrow$$

$$Q \Rightarrow Q'$$

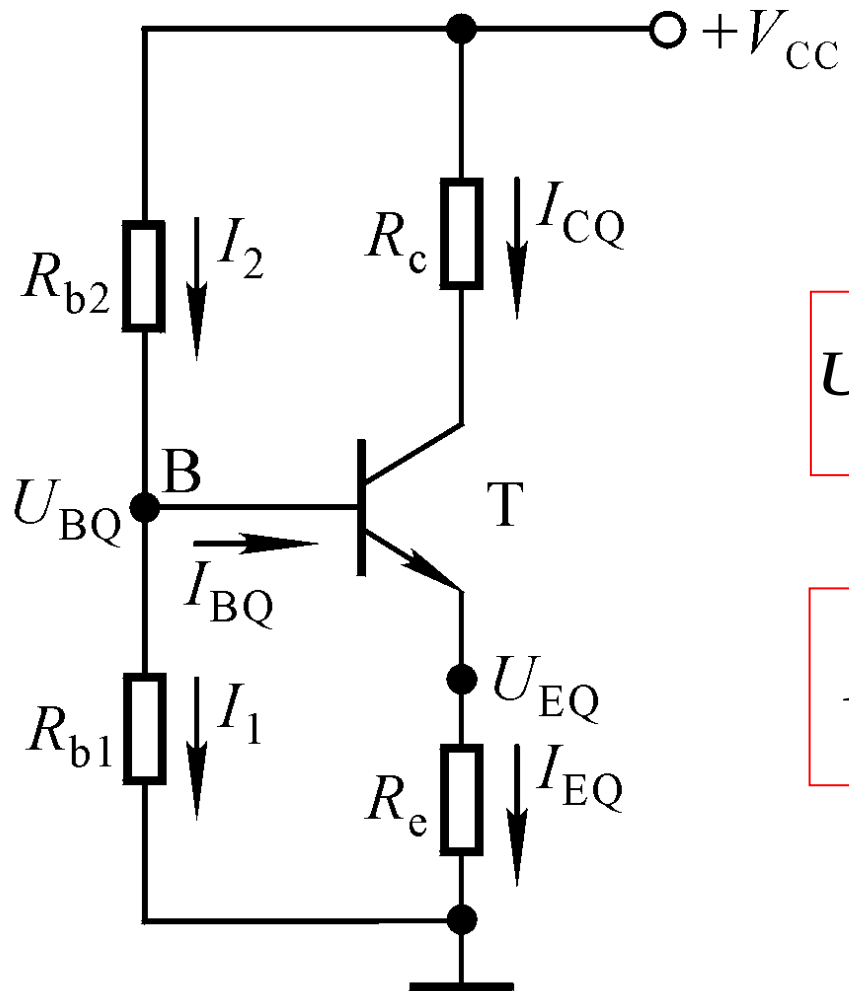
Emitter degeneration: Feedback



$$T \uparrow \rightarrow \beta \uparrow \rightarrow I_C \uparrow \rightarrow I_E \uparrow \\ \rightarrow U_{EQ} \uparrow \rightarrow I_B \downarrow$$

Feedback

Emitter degeneration: 4 resistor circuit



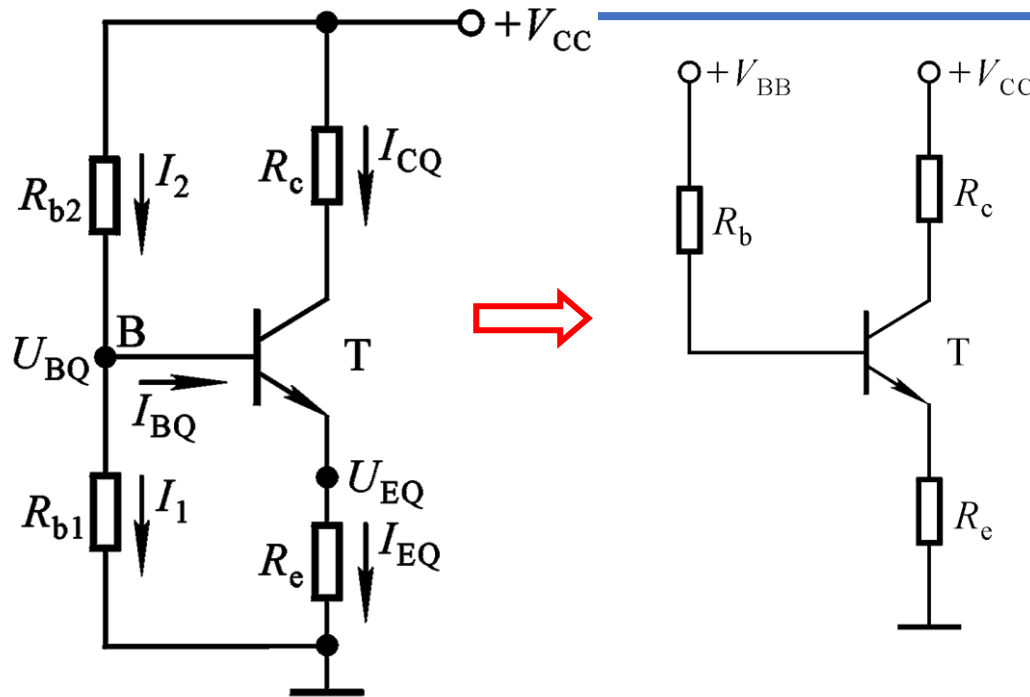
$$I_1 \gg I_B, \quad I_1 \approx I_2$$

$$U_{BQ} \approx \frac{R_{b1}}{R_{b1} + R_{b2}} \cdot V_{CC}$$

U_{BQ} is a constant

$$I_{EQ} = \frac{U_{BQ} - U_{BEQ}}{R_e}$$

Emitter degeneration: 4 resistor circuit



$$V_{BB} = \frac{R_{b1}}{R_{b1} + R_{b2}} \cdot V_{CC}$$

$$R_b = R_{b1} // R_{b2}$$

$$V_{BB} = I_{BQ}R_b + V_{BE} + I_{EQ}R_e$$

$$U_{BQ} \approx \frac{R_{b1}}{R_{b1} + R_{b2}} \cdot V_{CC}$$

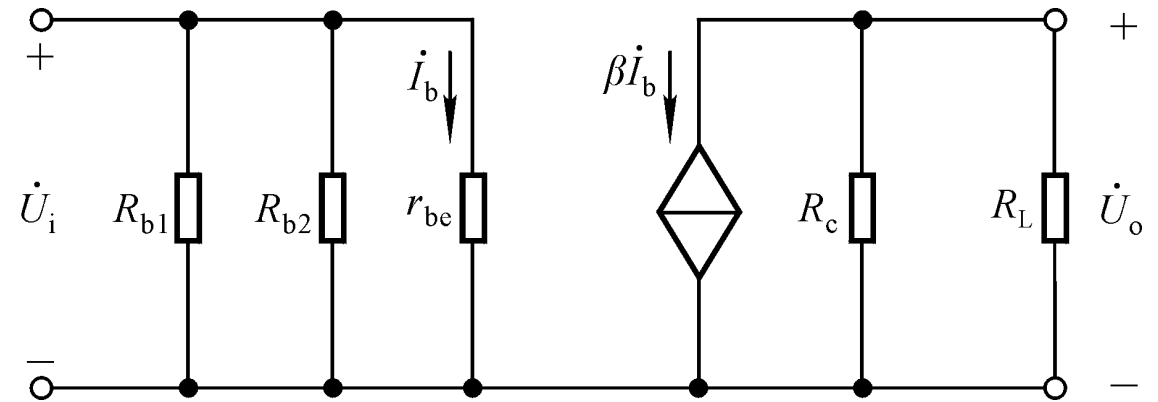
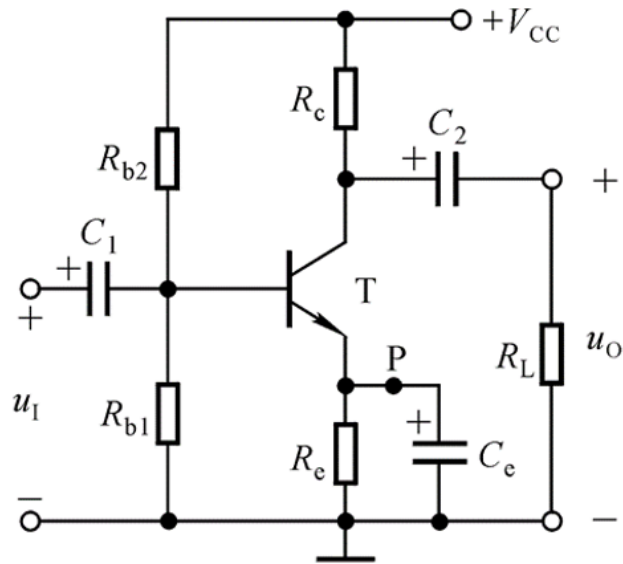
$$I_{EQ} = \frac{U_{BQ} - U_{BEQ}}{R_e}$$

$$I_{BQ} = \frac{I_{EQ}}{1 + \beta}$$

$$V_{CE} = V_{CC} - I_{CQ}R_c - I_{EQ}R_e$$

$$\approx V_{CC} - I_{EQ}(R_c + R_e)$$

4 resistor circuit: AC analysis with Re bypass



C_e : bypass capacitor

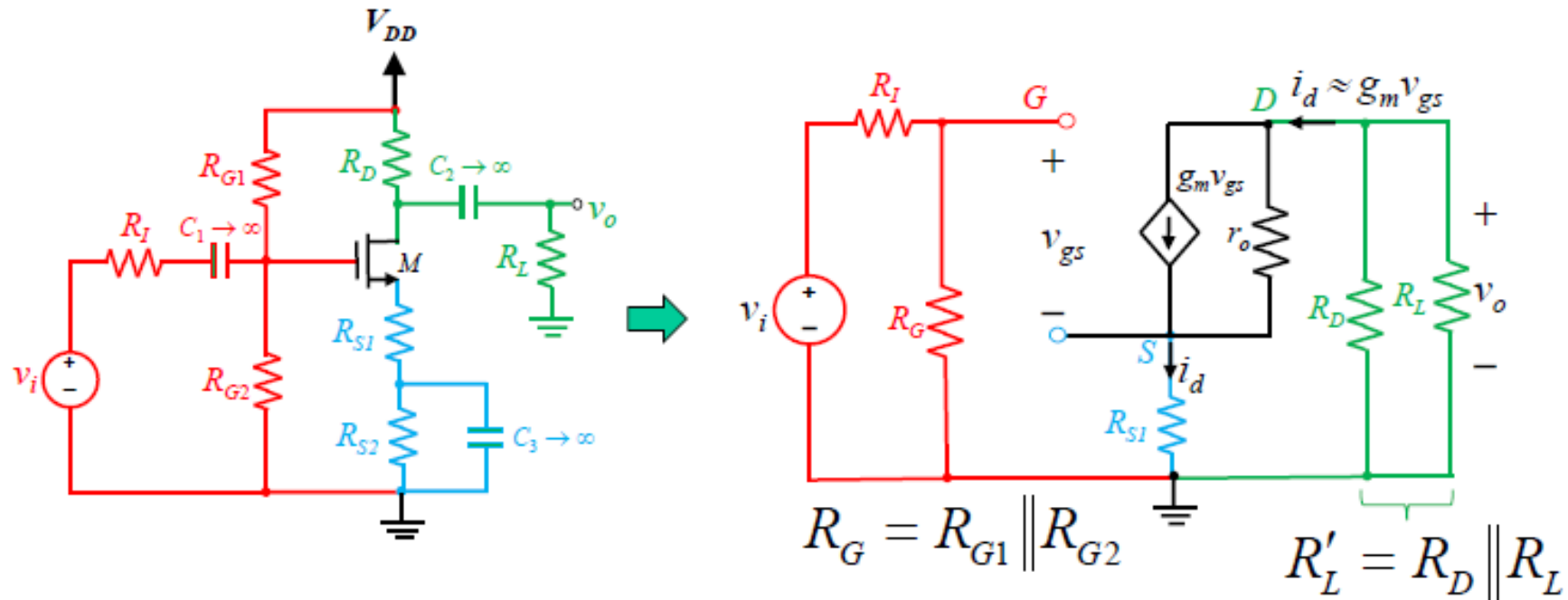
$$R_i = R_{b1} // R_{b2} // r_{be}$$

$$R_o = R_c$$

$$\dot{A}_u = \frac{\dot{U}_o}{\dot{U}_i} = -\frac{\beta R'_L}{r_{be}}$$

Same as C-E amplifier

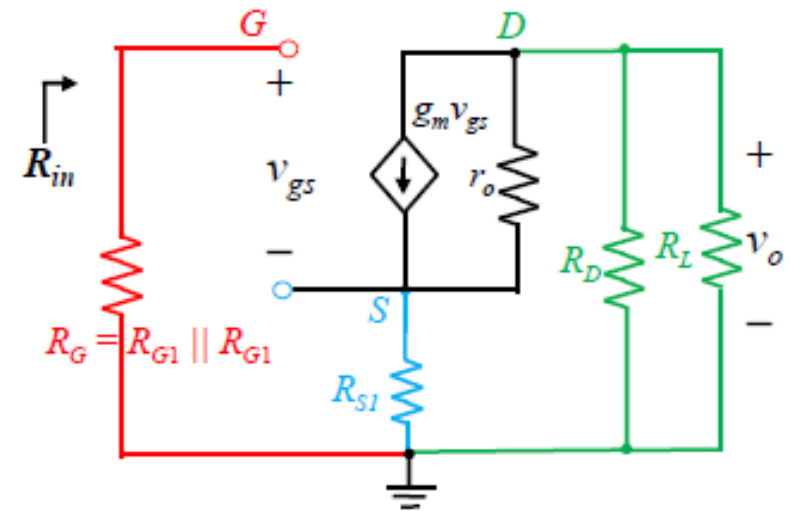
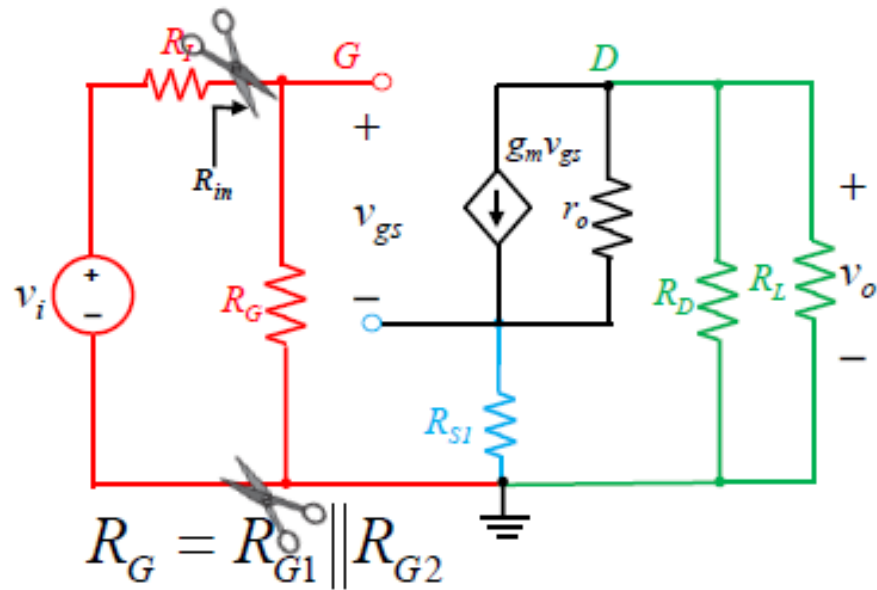
4 resistor circuit without bypass: Terminal voltage gain



$$\because i_{r_o} \ll g_m v_{gs}, i_d \approx g_m v_{gs}$$

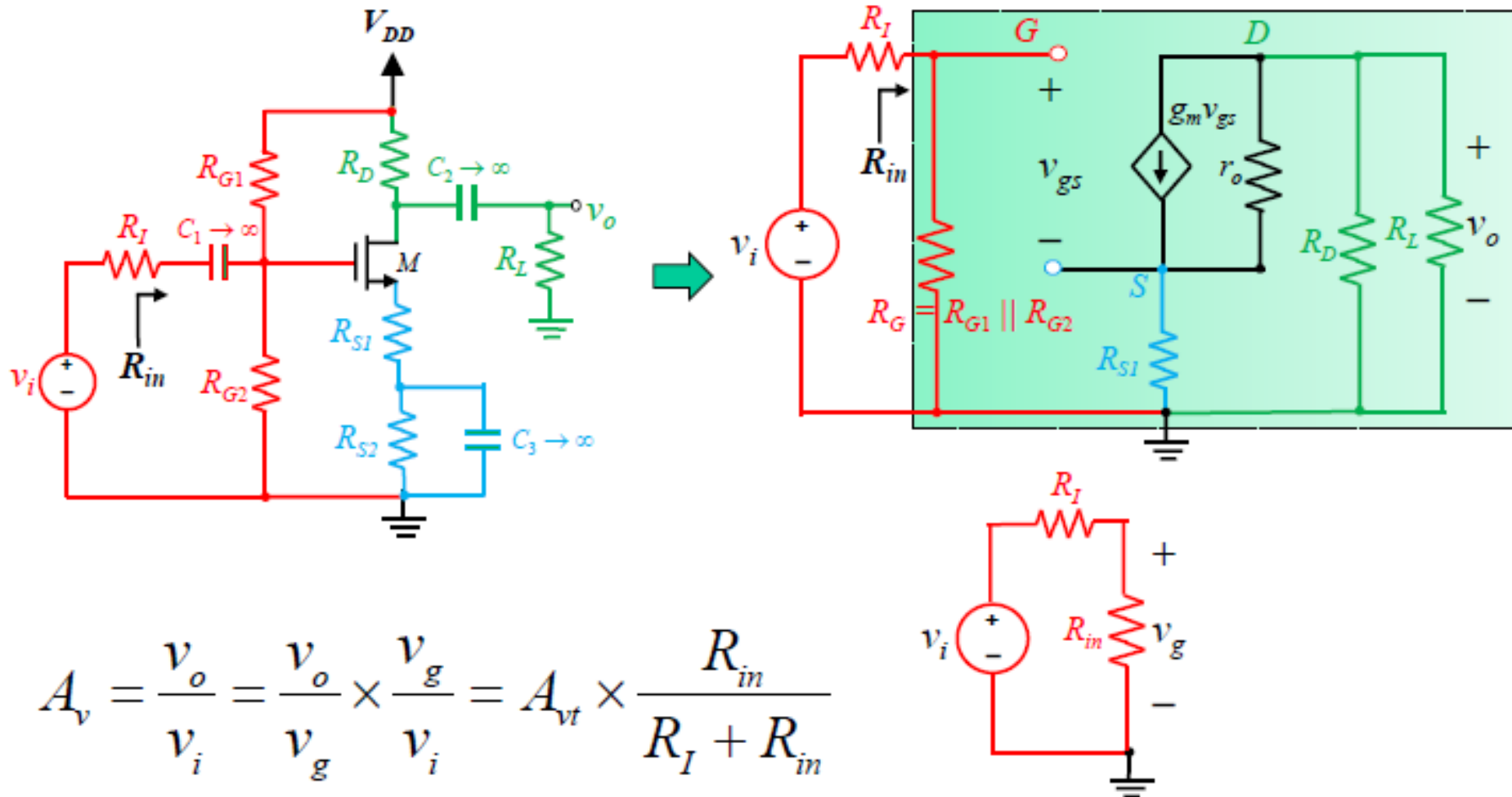
$$A_{vt} = \frac{v_d}{v_g} = \frac{-i_d R'_L}{v_{gs} + i_d R_{S1}} \approx \frac{-g_m v_{gs} R'_L}{v_{gs} + g_m v_{gs} R_{S1}} = \frac{-g_m R'_L}{1 + g_m R_{S1}}$$

4 resistor circuit without bypass: Input resistance

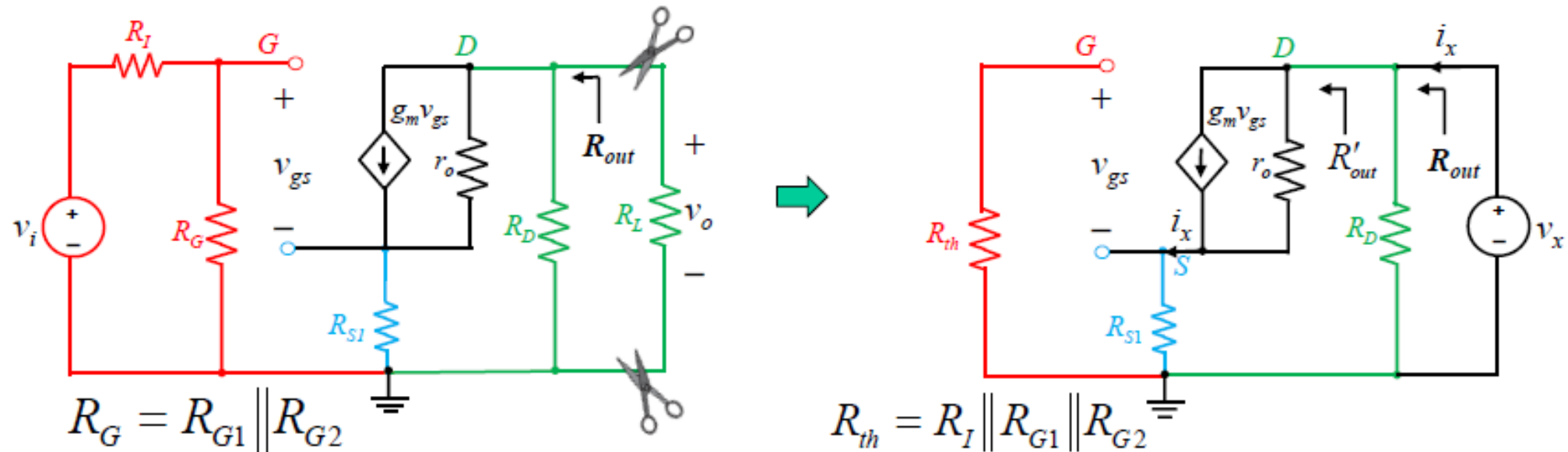


$$R_{in} = R_G$$

4 resistor circuit without bypass: Overall voltage gain



4 resistor circuit without bypass: Output resistance



$$v_x = \underbrace{\left(i_x - g_m v_{gs}\right)}_{\text{current through } r_o} r_o + v_s$$

$$v_s = i_x R_{S1}$$

$$\because v_g = 0, v_{gs} = -v_s = -i_x R_{S1}$$

$$v_x = (i_x + g_m i_x R_{S1}) r_o + i_x R_{S1}$$

$$R'_{out} = \frac{v_x}{i_x} \approx (1 + g_m R_{S1}) r_o$$

$$R_{out} = R'_{out} \parallel R_D$$