

L12-13: Introduction to Antennas

EE3109 Applied Electromagnetics

Lecture Notes

Dr. Alex M. H. Wong

Jan – Apr 2023

Announcement (Week 12)

- Assignment 3 is due next week
- Please do the TLQ

Announcement (Week 13)

- Assignment 3 is due today
- Make Up Lab is next Wed, 9am - 12 nn (Lau 15F)
- What to do with conceptual questions
- Please do the TLQ

Tentative Course Schedule

Week #	Week of	Course Coverage
1	Jan 9	Course Introduction, Math Review
2	Jan 16	Introduction to Maxwell Equations, Materials and Boundaries
	Jan 23	<i>Happy Chinese New Year!</i>
3	Jan 30	Wave Equations, Plane Waves, Propagation in Lossy Media
4	Feb 6	Polarizations, <i>Asst. 1a Due</i>
5	Feb 13	EM Potential, Polarizations Cont'd, Potentials <i>Asst. 1b Due</i>
6	Feb 20	<i>Test 1</i> , Reflection and Transmission
7	Feb 27	Reflection and Transmission Cont'd
8	Mar 6	Power and Velocity
9	Mar 13	Waveguides and Transmission Lines (TL), VIP in a TL, Terminated and Lossy TLs, <i>Asst. 2 Due, Lab 1 Begins</i>
10	Mar 20	Impedance in a TL, Smith Charts
11	Mar 27	Smith Charts Cont'd, Impedance Matching, Quarter-wave Transformers, <i>Lab 2 Begins</i>
12	Apr 3	<i>Test 2</i> , Introduction to Antennas
13	Apr 10	Dipole Antennas, <i>Asst. 3 Due</i>

Big Picture: Signal Refraction at Boundaries

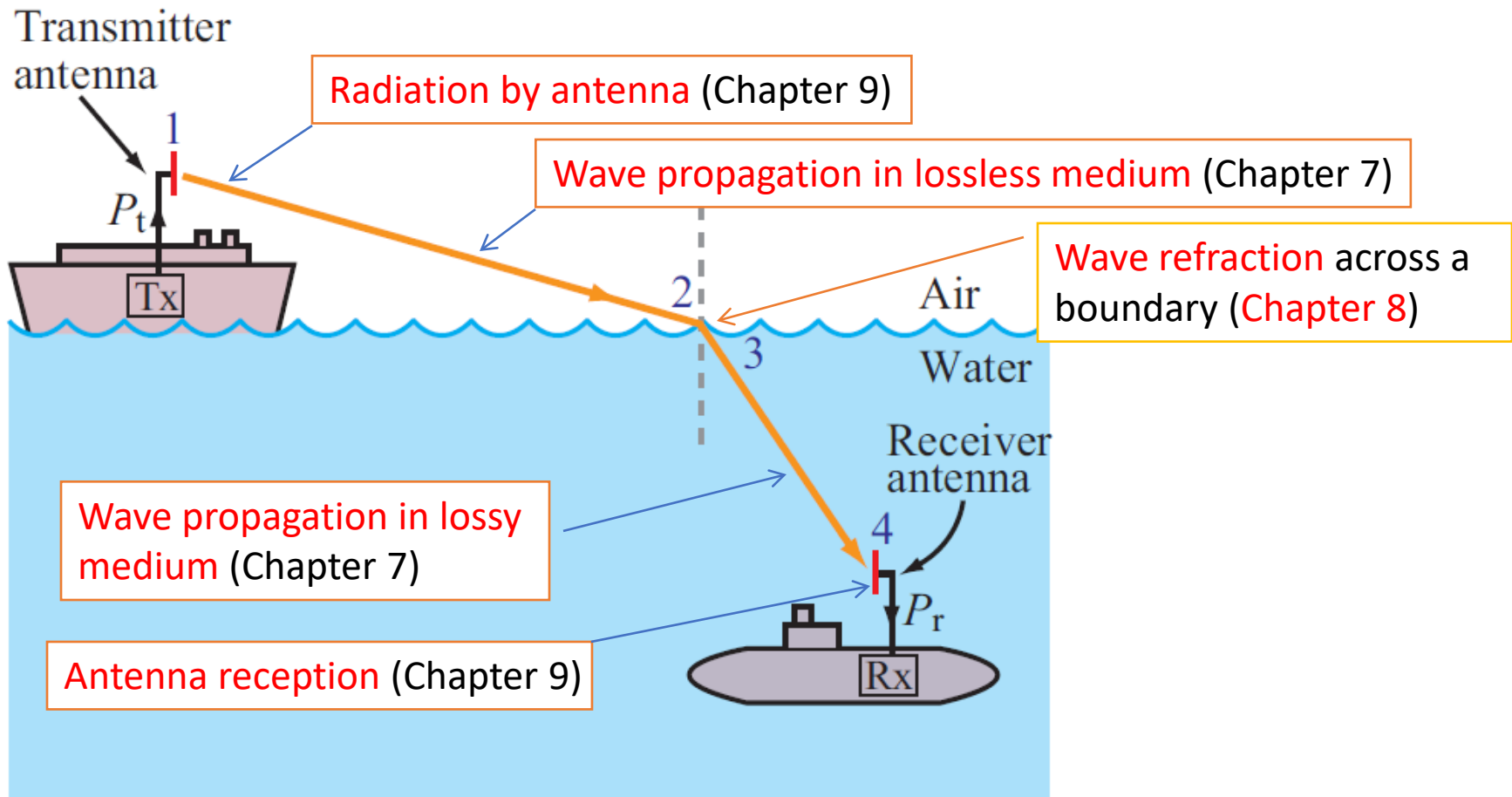


Figure 8-1: Signal path between a shipboard transmitter (Tx) and a submarine receiver (Rx).

Outline

- Course Matter
- Introduction to antennas
- The Hertzian Dipole
 - Electromagnetic (Near)-field
 - Electromagnetic Far-field
- Antenna Parameters
 - Radiation intensity $S(R, \theta, \phi) = S_{max}(R)F(\theta, \phi)$
 - Normalized radiation pattern $F(\theta, \phi)$
 - Radiated Power P_{rad}
 - Antenna Beam Parameters (β, Ω)
 - Directivity (D), Efficiency (ξ) and Gain ($G = \xi D$)
- The Half-Wavelength Dipole

Antenna Properties

1. An antenna is a **transducer** that converts a guided wave propagating on a transmission line into an electromagnetic wave propagating in an unbounded medium (usually free space), or vice versa.
2. Most antennas are **reciprocal devices**, exhibiting the same radiation pattern for transmission as for reception.
3. Being a reciprocal device, an antenna, when operating in the receiving mode, can extract from an incident wave only that component of the wave whose electric field matches the **antenna polarization** state.

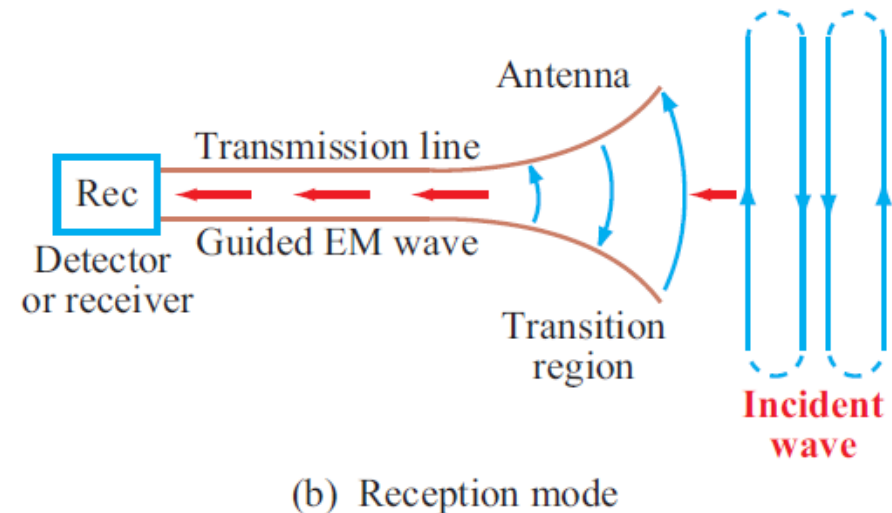
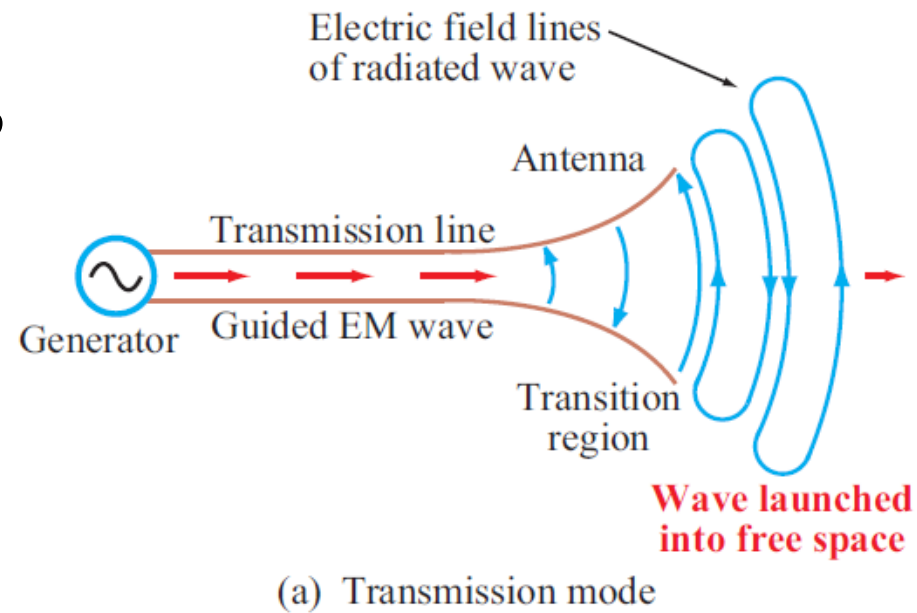
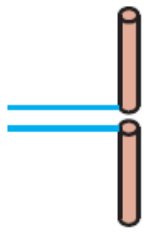
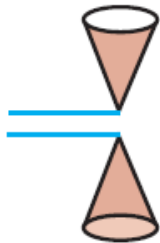


Figure 9-1: Antenna as a transducer between a guided electromagnetic wave and a free-space wave, for both transmission and reception.

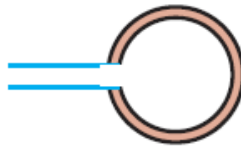
Examples of Antennas



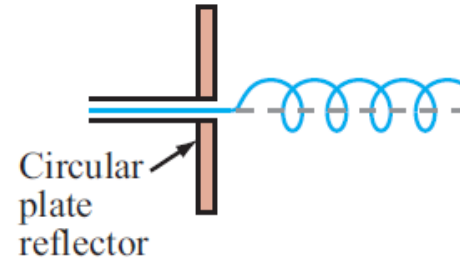
(a) Thin dipole



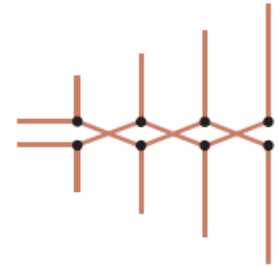
(b) Biconical dipole



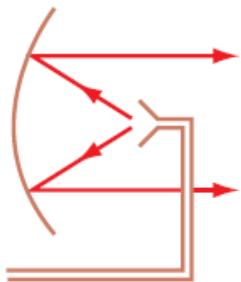
(c) Loop



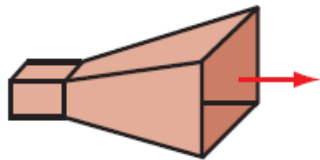
(d) Helix



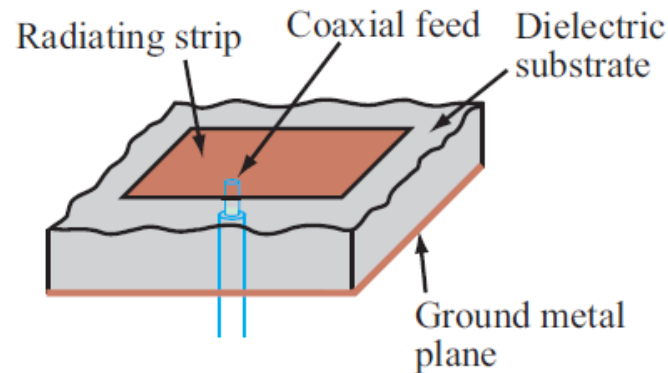
(e) Log-periodic



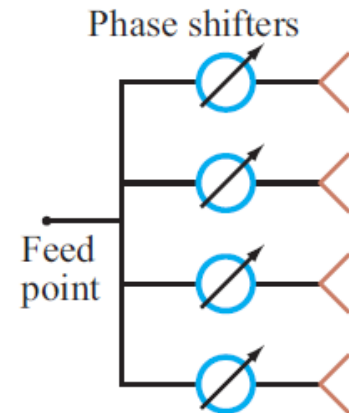
(f) Parabolic dish reflector



(g) Horn



(h) Microstrip

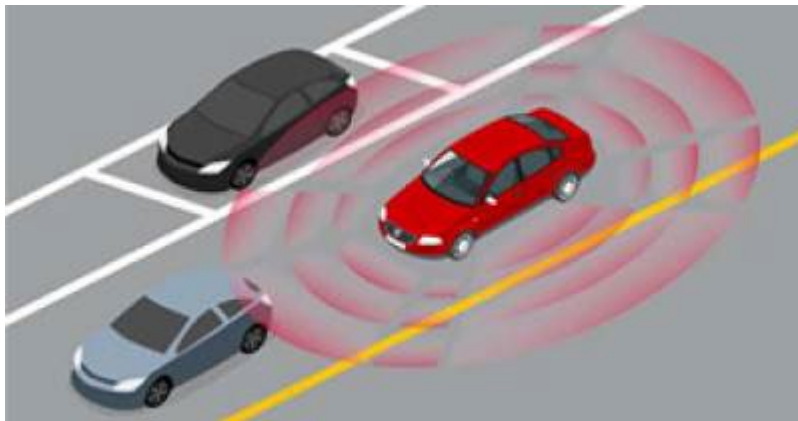


(i) Antenna array

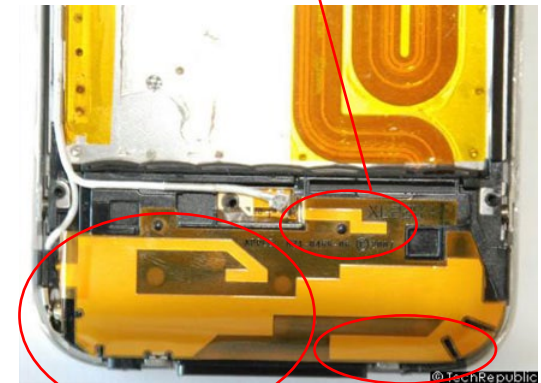
Examples of Antennas



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<http://bpaddock.com/lib/exe/fetch.php/emf/mmwavecarradiation.jpg?w=400&tok=01ba58>



Bluetooth, WiFi
antenna

GSM 850, EGSM
900 antenna

DCS 1800, PCS
1900 antenna

<http://m.zdnet.com.au/photos-cracking-open-the-iphone-339279684.htm>



https://live.staticflickr.com/8500/8334641612_8c6b2bddd_b.jpg

Far-Field Approximation

1. In close proximity to a radiating source, the wave is **spherical** in shape, but at a far distance, it becomes approximately a **plane wave** as seen by a receiving antenna.

2. The far-field approximation **simplifies the math**.

3. The distance beyond which the far-field approximation is valid is called the **far-field range** (will be defined later).

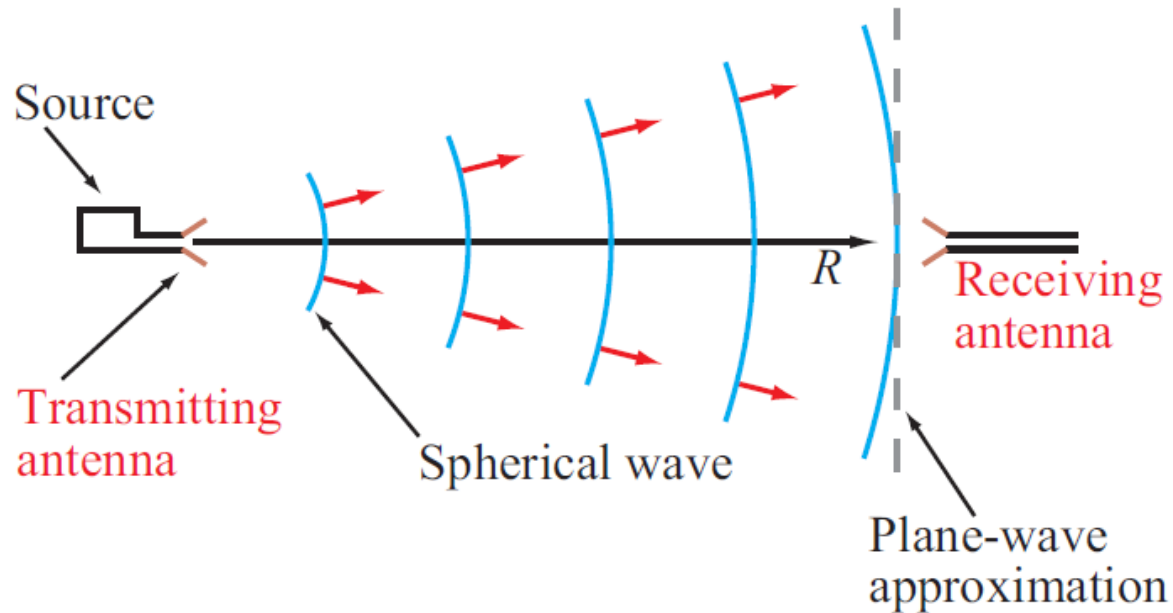
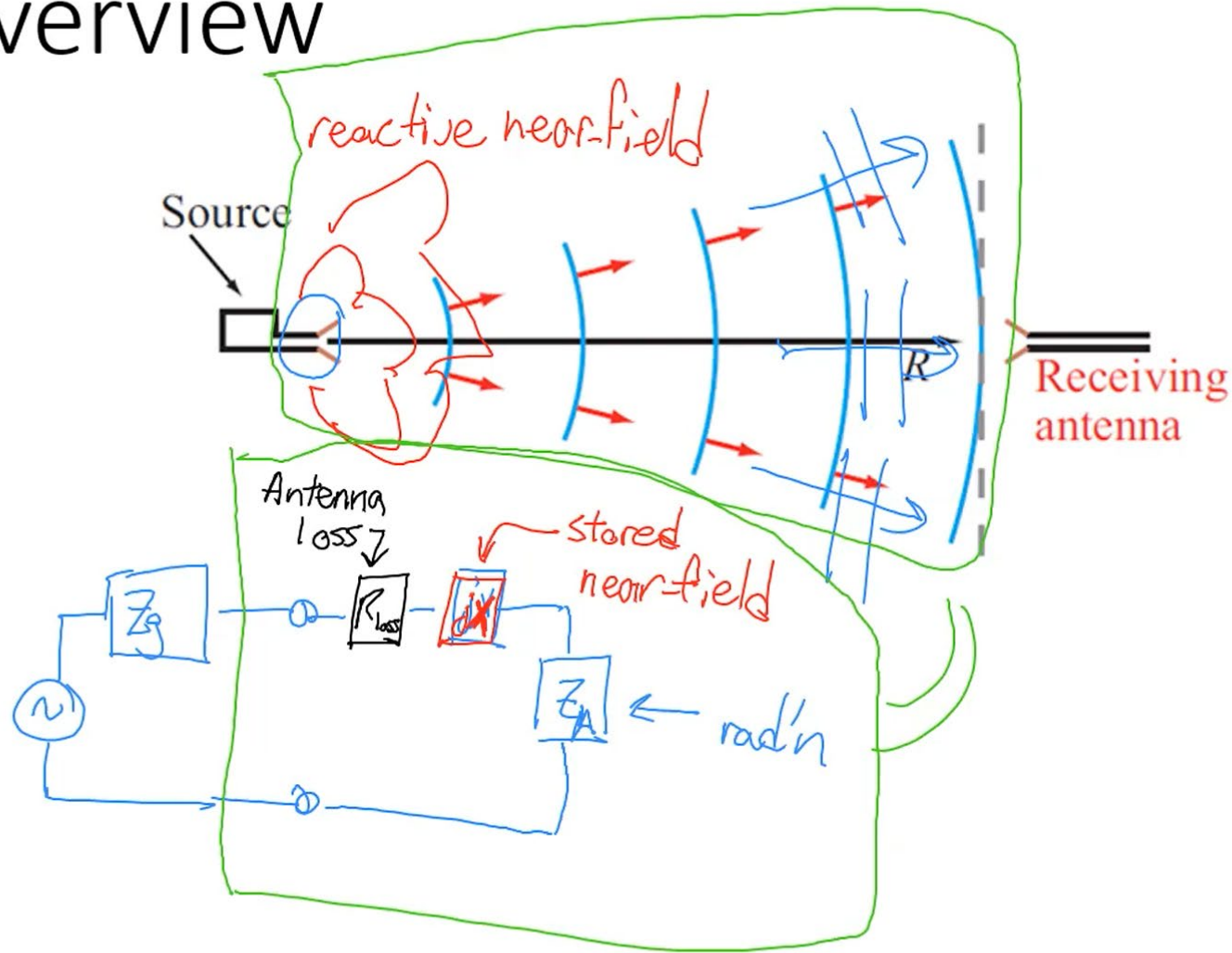


Figure 9-3: Far-field plane-wave approximation.

Overview



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The Hertzian Dipole

A *Hertzian dipole* is a thin, linear conductor whose length l is very short compared with the wavelength λ ; l should not exceed $\lambda/50$.

This restriction allows us to treat the current along the length of the conductor as constant, even though it has to decay to zero at the ends of the wire.

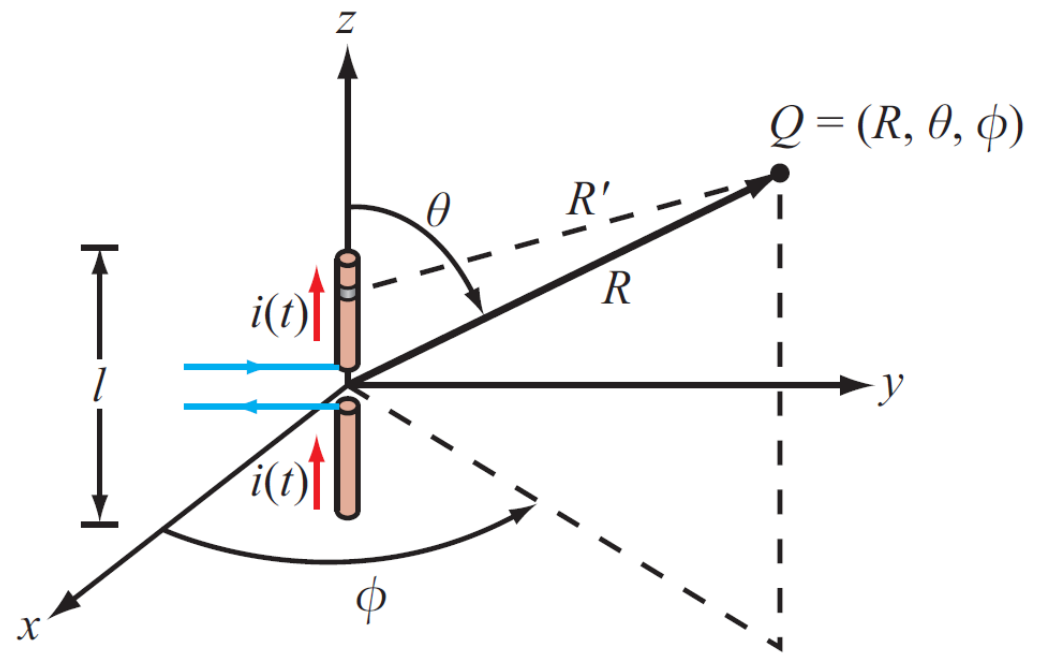


Figure 9-4: Short dipole placed at the origin of a spherical coordinate system.

Fields Radiated by Hertzian Dipole

Current along dipole:

$$i(t) = I_0 \cos \omega t = \Re[I_0 e^{j\omega t}]$$

Magnetic Vector Potential:

$$\tilde{\mathbf{A}}(\mathbf{R}) = \frac{\mu_0}{4\pi} \int_{V'} \frac{\tilde{\mathbf{J}} e^{-jkR'}}{R'} dV'$$

With: $\tilde{\mathbf{J}} = \hat{\mathbf{z}}(I_0/s)$

$$dV' = s dz$$

$$R' \simeq R$$

$$\tilde{\mathbf{A}} = \frac{\mu_0}{4\pi} \frac{e^{-jkR}}{R} \int_{-l/2}^{l/2} \hat{\mathbf{z}} I_0 dz$$

$$= \hat{\mathbf{z}} \frac{\mu_0}{4\pi} I_0 l \left(\frac{e^{-jkR}}{R} \right),$$

Given $\mathbf{A}$, we can determine $\mathbf{E}$ and $\mathbf{H}$

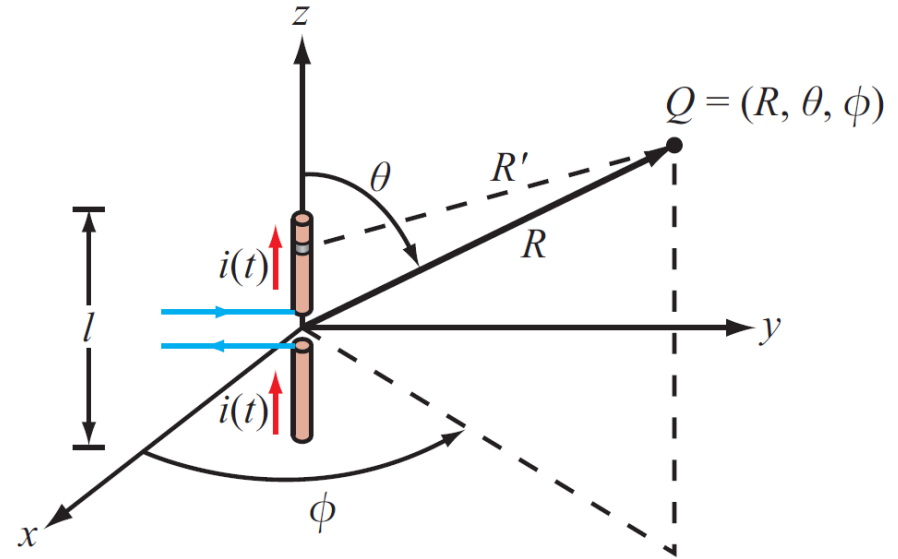


Figure 9-4: Short dipole placed at the origin of a spherical coordinate system.

Fields Radiated by Hertzian Dipole (cont.)

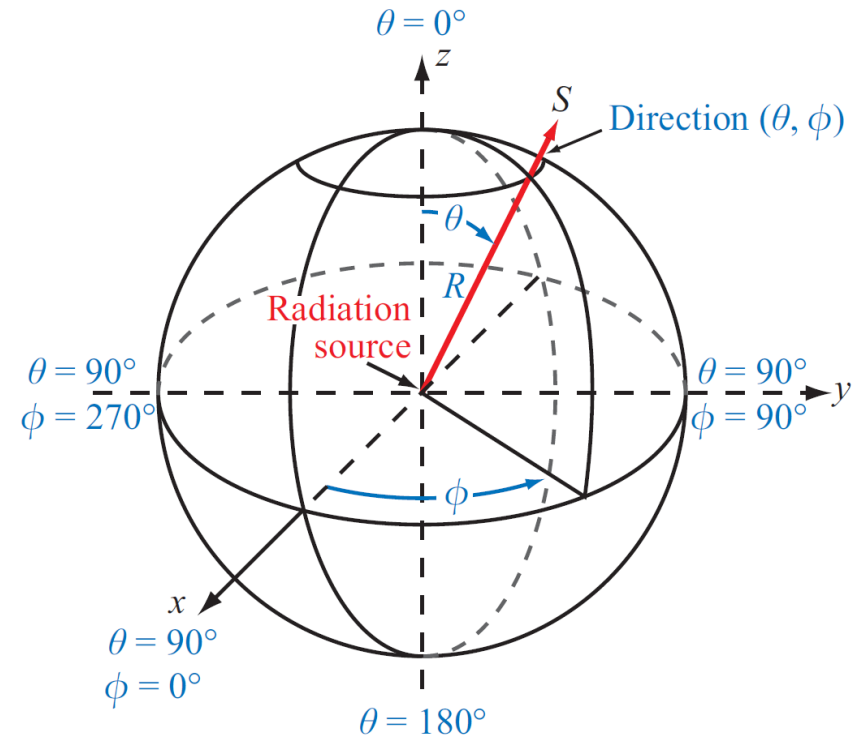
$$\begin{aligned}\tilde{\mathbf{A}} &= \frac{\mu_0}{4\pi} \frac{e^{-jkR}}{R} \int_{-l/2}^{l/2} \hat{\mathbf{z}} I_0 dz \\ &= \hat{\mathbf{z}} \frac{\mu_0}{4\pi} I_0 l \left(\frac{e^{-jkR}}{R} \right),\end{aligned}$$

Upon converting z to spherical coordinates:

$$\hat{\mathbf{z}} = \hat{\mathbf{R}} \cos \theta - \hat{\boldsymbol{\theta}} \sin \theta$$

we have:

$$\begin{aligned}\tilde{\mathbf{A}} &= (\hat{\mathbf{R}} \cos \theta - \hat{\boldsymbol{\theta}} \sin \theta) \frac{\mu_0 I_0 l}{4\pi} \left(\frac{e^{-jkR}}{R} \right) \\ &= \hat{\mathbf{R}} \tilde{A}_R + \hat{\boldsymbol{\theta}} \tilde{A}_\theta + \hat{\boldsymbol{\phi}} \tilde{A}_\phi,\end{aligned}$$



with

$$\begin{aligned}\tilde{A}_R &= \frac{\mu_0 I_0 l}{4\pi} \cos \theta \left(\frac{e^{-jkR}}{R} \right), \\ \tilde{A}_\theta &= -\frac{\mu_0 I_0 l}{4\pi} \sin \theta \left(\frac{e^{-jkR}}{R} \right), \\ \tilde{A}_\phi &= 0.\end{aligned}$$

Fields Radiated by Hertzian Dipole (cont.)

with

$$\tilde{A}_R = \frac{\mu_0 I_0 l}{4\pi} \cos \theta \left(\frac{e^{-jkR}}{R} \right),$$

$$\tilde{A}_\theta = -\frac{\mu_0 I_0 l}{4\pi} \sin \theta \left(\frac{e^{-jkR}}{R} \right),$$

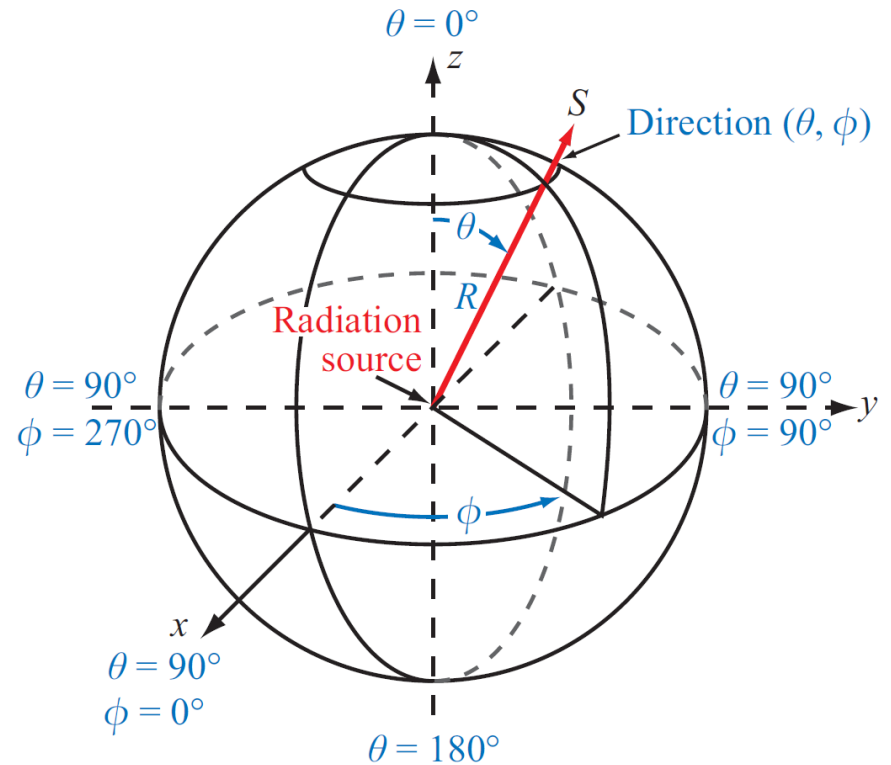
$$\tilde{A}_\phi = 0.$$

Application of:

$$\tilde{\mathbf{H}} = \frac{1}{\mu_0} \nabla \times \tilde{\mathbf{A}},$$

$$\tilde{\mathbf{E}} = \frac{1}{j\omega\epsilon_0} \nabla \times \tilde{\mathbf{H}},$$

leads to:



$$\tilde{H}_\phi = \frac{I_0 l k^2}{4\pi} e^{-jkR} \left[\frac{j}{kR} + \frac{1}{(kR)^2} \right] \sin \theta,$$

$$\tilde{E}_R = \frac{2I_0 l k^2}{4\pi} \eta_0 e^{-jkR} \left[\frac{1}{(kR)^2} - \frac{j}{(kR)^3} \right] \cos \theta,$$

$$\tilde{E}_\theta = \frac{I_0 l k^2}{4\pi} \eta_0 e^{-jkR} \left[\frac{j}{kR} + \frac{1}{(kR)^2} - \frac{j}{(kR)^3} \right] \sin \theta$$

Radiated Electric Field

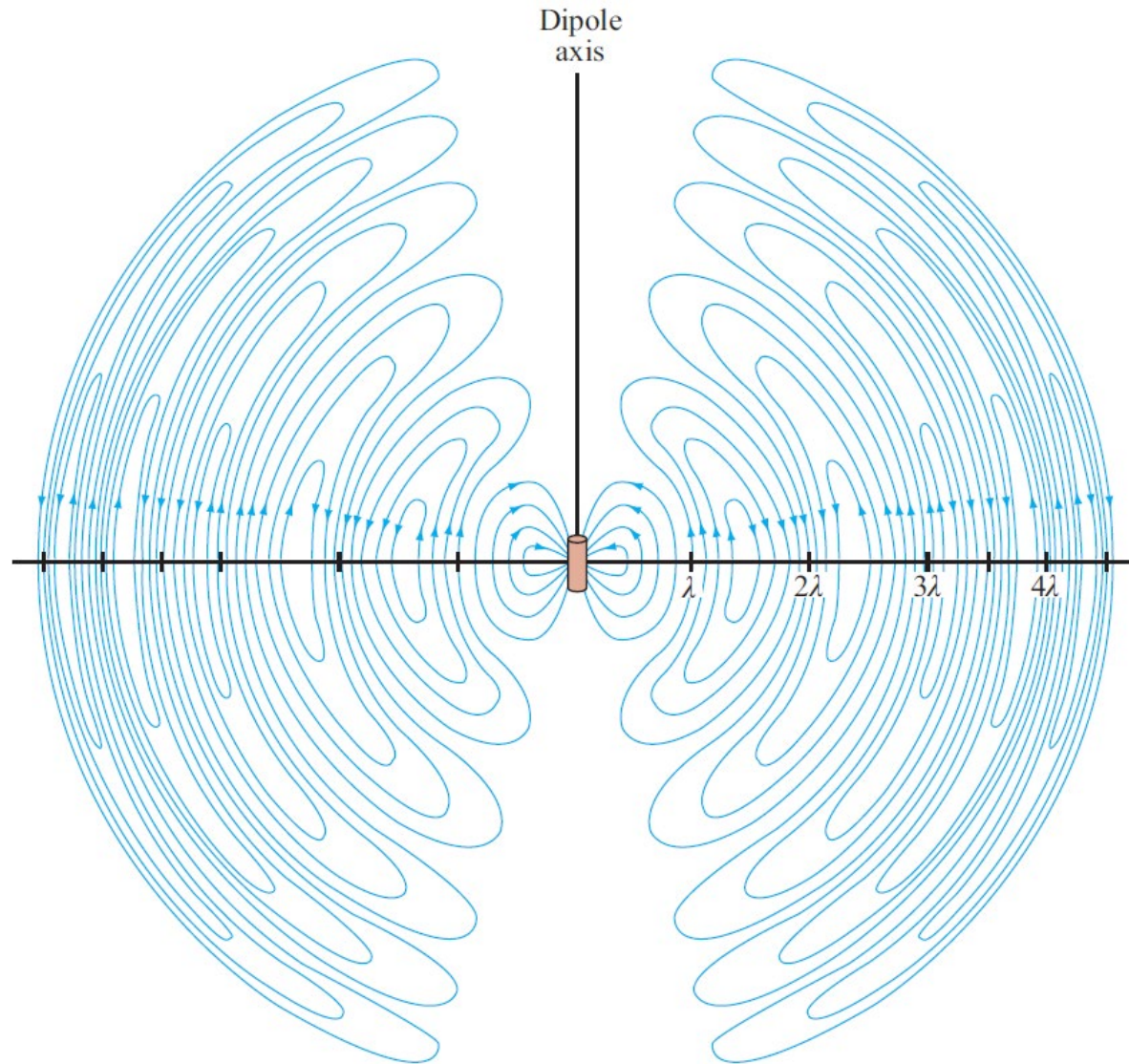


Figure 9-6: Electric field lines surrounding an oscillating dipole at a given instant.

Hertzian Dipole—Far-Field Approximation

At any range R :

$$\tilde{H}_\phi = \frac{I_0 l k^2}{4\pi} e^{-jkR} \left[\frac{j}{kR} + \frac{1}{(kR)^2} \right] \sin \theta,$$

$$\tilde{E}_R = \frac{2I_0 l k^2}{4\pi} \eta_0 e^{-jkR} \left[\frac{1}{(kR)^2} - \frac{j}{(kR)^3} \right] \cos \theta,$$

$$\tilde{E}_\theta = \frac{I_0 l k^2}{4\pi} \eta_0 e^{-jkR} \left[\frac{j}{kR} + \frac{1}{(kR)^2} - \frac{j}{(kR)^3} \right] \sin \theta$$

At distances R such that $R \gg \lambda$ or, equivalently, $kR = 2\pi R/\lambda \gg 1$

$$\tilde{E}_\theta = \frac{j I_0 l k \eta_0}{4\pi} \left(\frac{e^{-jkR}}{R} \right) \sin \theta \quad (\text{V/m}),$$

$$\tilde{H}_\phi = \frac{\tilde{E}_\theta}{\eta_0} \quad (\text{A/m}),$$

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Normalized Radiation Intensity

Electric and Magnetic Fields

$$\tilde{E}_\theta = \frac{j I_0 l k \eta_0}{4\pi} \left(\frac{e^{-jkR}}{R} \right) \sin \theta \quad (\text{V/m}),$$

$$\tilde{H}_\phi = \frac{\tilde{E}_\theta}{\eta_0} \quad (\text{A/m}),$$

Average Power Density

$$\mathbf{S}_{\text{av}} = \frac{1}{2} \Re \left(\tilde{\mathbf{E}} \times \tilde{\mathbf{H}}^* \right) \quad (\text{W/m}^2).$$

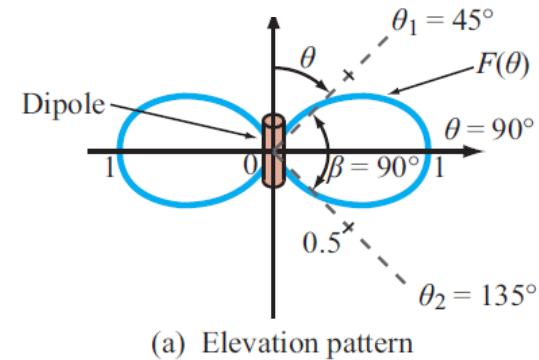
$$\mathbf{S}_{\text{av}} = \hat{\mathbf{R}} S(R, \theta),$$

$$S(R, \theta) = \left(\frac{\eta_0 k^2 I_0^2 l^2}{32\pi^2 R^2} \right) \sin^2 \theta$$

$$= S_0 \sin^2 \theta \quad (\text{W/m}^2).$$

Normalized Radiation Intensity

$$F(\theta, \phi) = \frac{S(R, \theta, \phi)}{S_{\text{max}}} \quad (\text{dimensionless})$$



$$S_{\text{max}} = S_0 = \frac{\eta_0 k^2 I_0^2 l^2}{32\pi^2 R^2}$$

$$= \frac{15\pi I_0^2}{R^2} \left(\frac{l}{\lambda} \right)^2$$

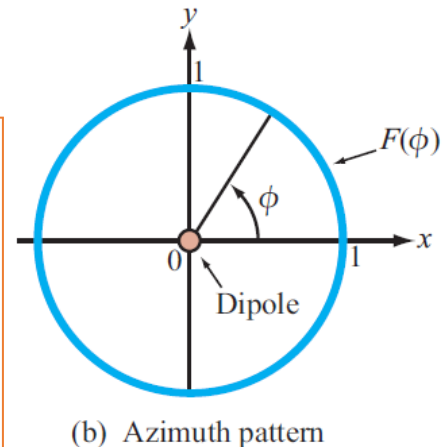


Figure 9-7: Radiation patterns of a short dipole.

Radiation Pattern of Hertzian Dipole

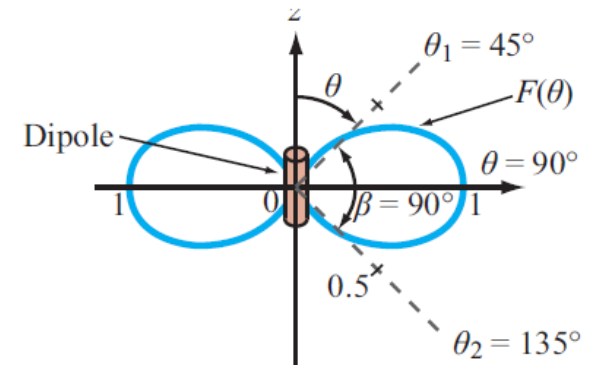
$$S(R, \theta) = \left(\frac{\eta_0 k^2 I_0^2 l^2}{32\pi^2 R^2} \right) \sin^2 \theta$$

$$= S_0 \sin^2 \theta \quad (\text{W/m}^2)$$

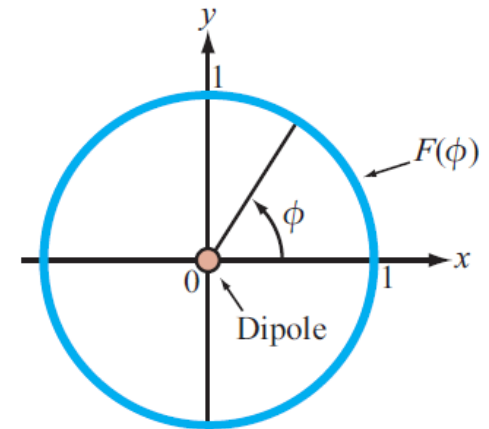
$$F(\theta, \phi) = \frac{S(R, \theta, \phi)}{S_{\max}}$$

$$F(\theta, \phi) = F(\theta) = \sin^2 \theta$$

*No energy is radiated by the short dipole along the direction of the dipole axis and maximum radiation ($F = 1$) occurs in the **broadside direction** ($\theta = 90^\circ$). Since $F(\theta)$ is independent of ϕ , the pattern is doughnut-shaped in θ - ϕ space.*



(a) Elevation pattern



(b) Azimuth pattern

Figure 9-7: Radiation patterns of a short dipole.

Antenna Radiation Characteristics

1. By virtue of **reciprocity**, a receiving antenna has the same directional antenna pattern as the pattern that it exhibits when operated in the transmission mode.

2. Total Radiated Power

Differential area

$$dA = R^2 \sin \theta \, d\theta \, d\phi$$

Solid Angle

$$d\Omega = \frac{dA}{R^2} = \sin \theta \, d\theta \, d\phi \quad (\text{sr}).$$

Power radiated through dA

$$dP_{\text{rad}} = R^2 S(R, \theta, \phi) \, d\Omega$$

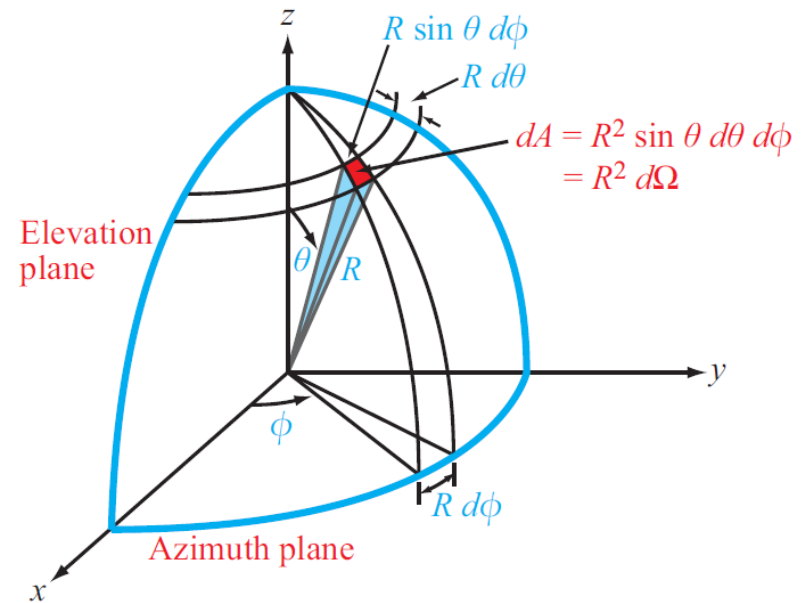


Figure 9-8: Definition of solid angle $d\Omega = \sin \theta \, d\theta \, d\phi$.

$$\begin{aligned} P_{\text{rad}} &= R^2 \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} S(R, \theta, \phi) \sin \theta \, d\theta \, d\phi \\ &= R^2 S_{\text{max}} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} F(\theta, \phi) \sin \theta \, d\theta \, d\phi \\ &= R^2 S_{\text{max}} \iint_{4\pi} F(\theta, \phi) \, d\Omega \quad (\text{W}), \end{aligned}$$

Total Radiated Power

Example of 3-D Pattern

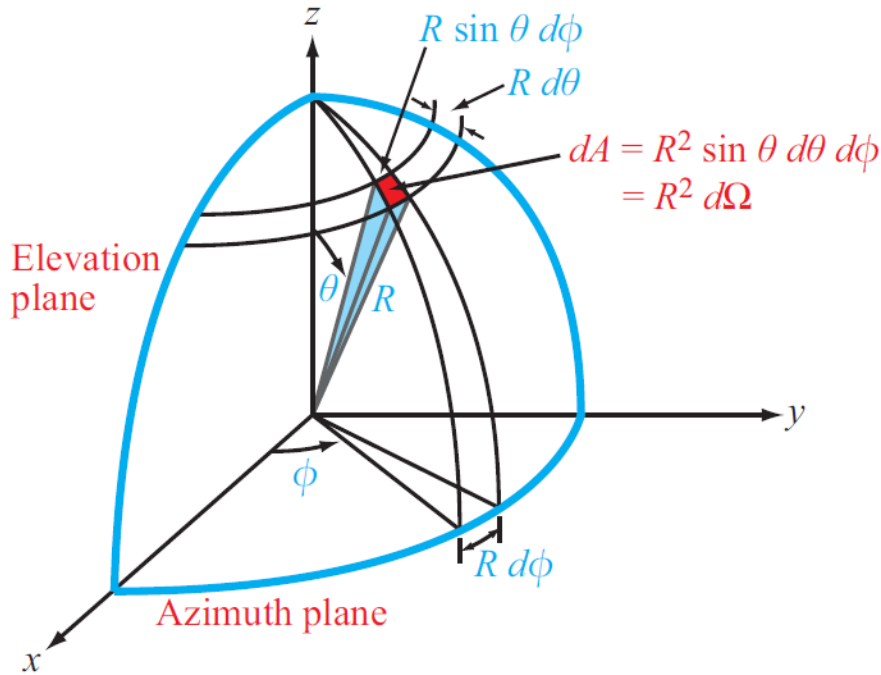


Figure 9-8: Definition of solid angle $d\Omega = \sin \theta d\theta d\phi$.

Principal planes:

1. **Elevation plane** (x-z and y-z planes)
2. **Azimuth plane** (x-y plane)

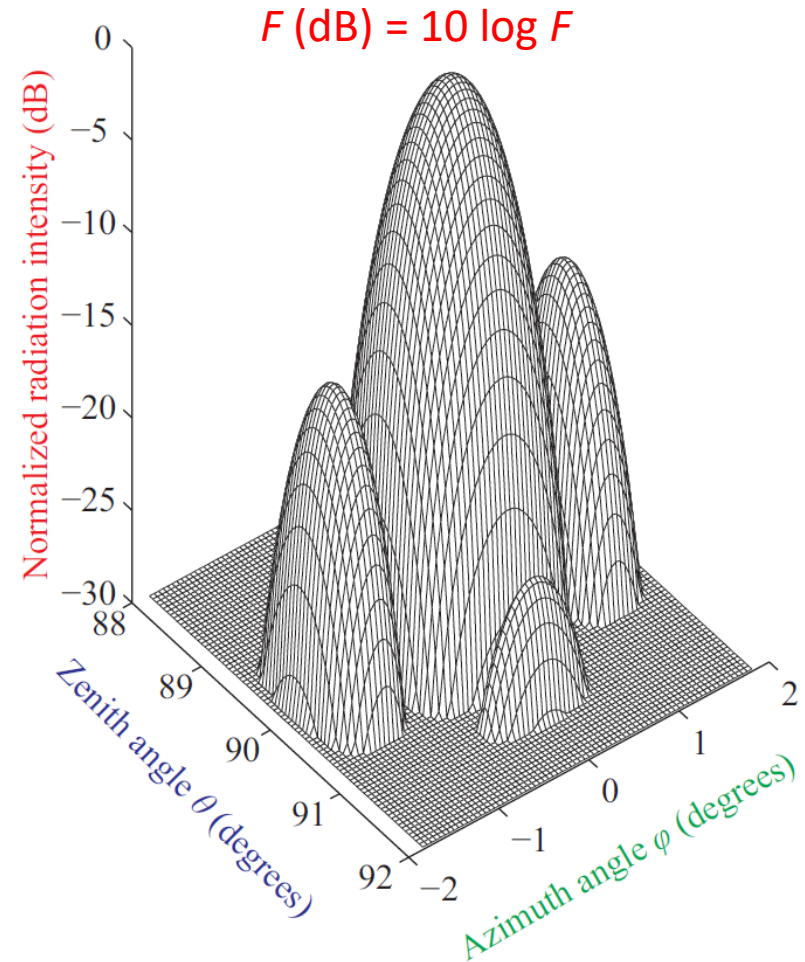
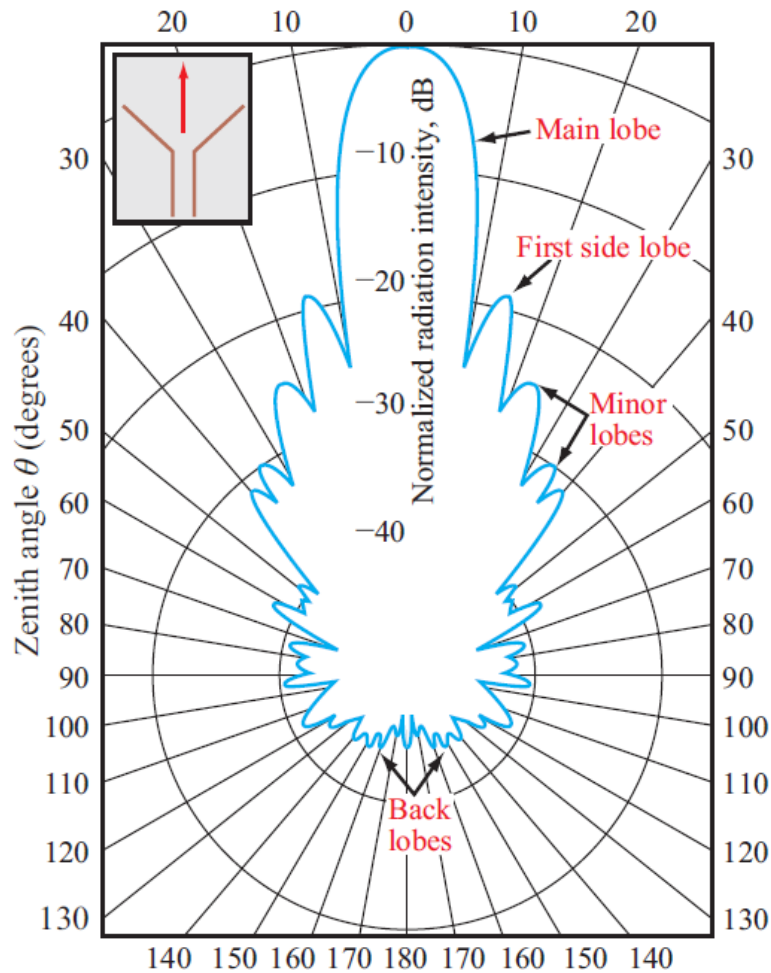
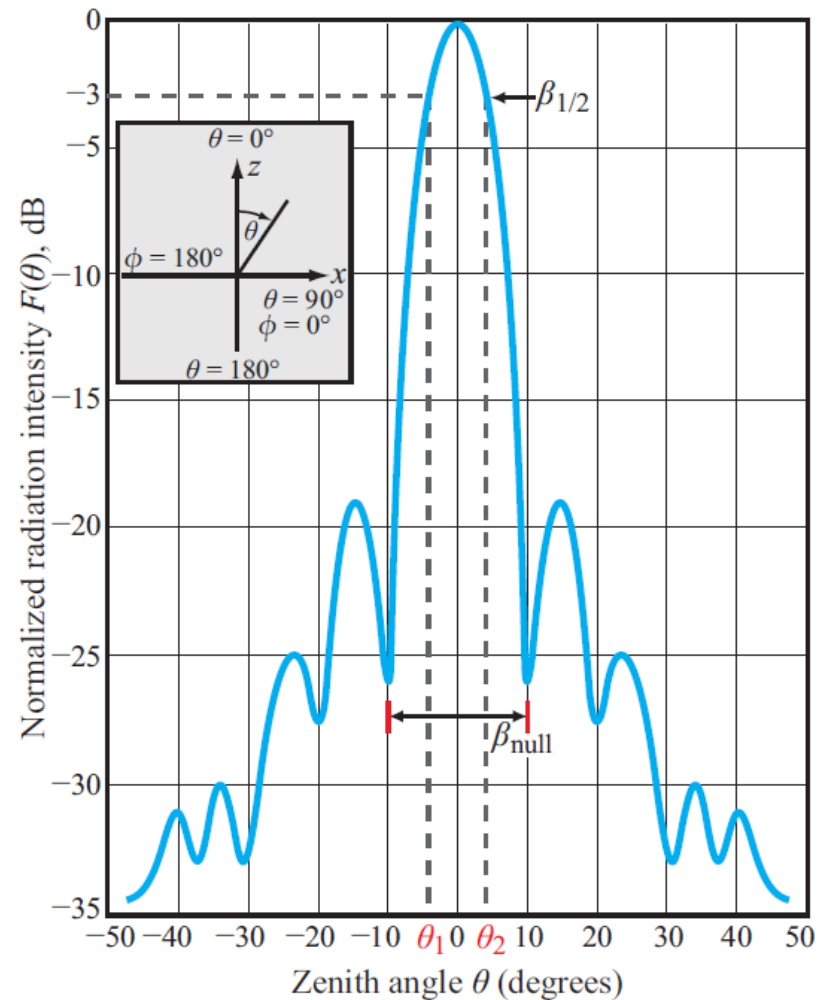


Figure 9-9: Three-dimensional pattern of a narrow-beam antenna.

Polar and Rectangular Plots



(a) Polar diagram



(b) Rectangular plot

Beam Antennas: Antennas with a Single Main Lobe

How “wide” is the antenna beam?

1. Pattern solid angle

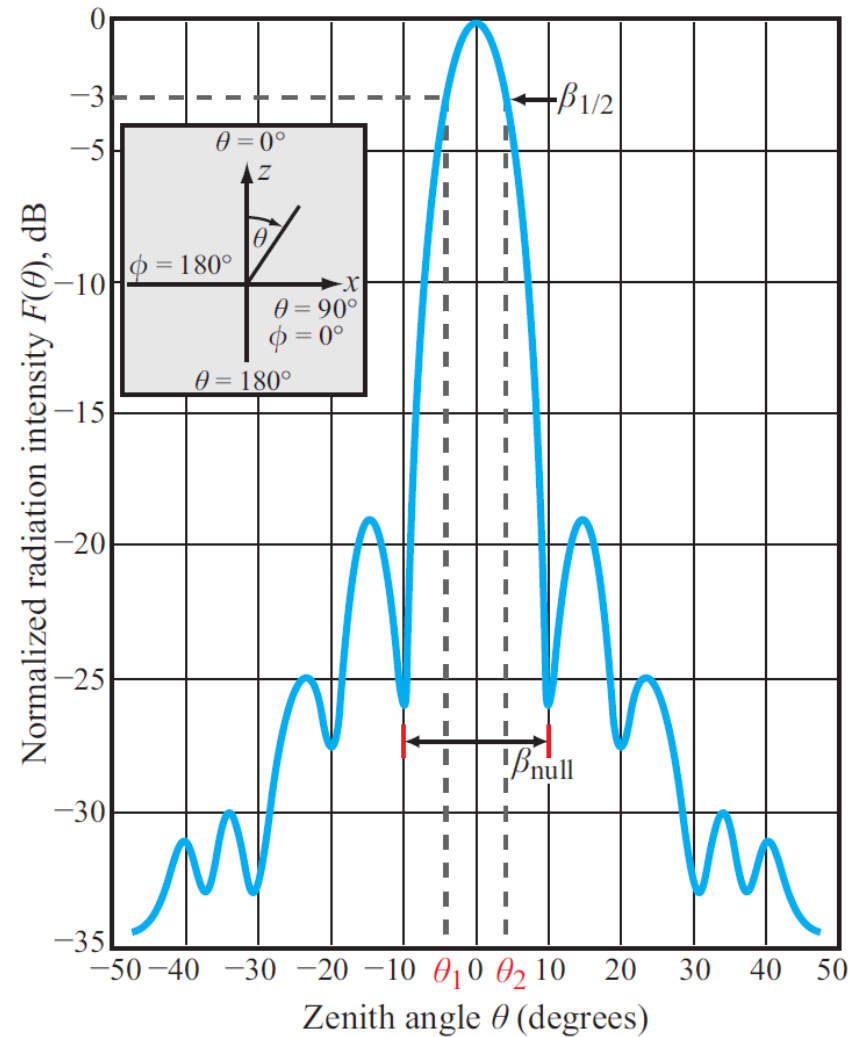
$$\Omega_p = \iint_{4\pi} F(\theta, \phi) d\Omega \quad (\text{sr}).$$

2. Half-power beamwidth

$$\beta = \theta_2 - \theta_1,$$

where θ_1 and θ_2 are the *half-power angles* at which $F(\theta, 0) = 0.5$ (with θ_2 denoting the larger value and θ_1 denoting the smaller one, as shown in the figure). If the pattern

Since 0.5 corresponds to -3 dB, the half power beamwidth is also called the 3-dB beamwidth.



Antenna Directivity D

$$\Omega_p = \iint_{4\pi} F(\theta, \phi) d\Omega \quad (\text{sr}). \quad \text{Antenna pattern solid angle}$$

For an isotropic antenna with $F(\theta, \phi) = 1$ in all directions, $\Omega_p = 4\pi$ (sr).

$$\begin{aligned} D &= \frac{F_{\max}}{F_{\text{av}}} && \text{Directivity} \\ &= \frac{1}{\frac{1}{4\pi} \iint_{4\pi} F(\theta, \phi) d\Omega} \\ &= \frac{4\pi}{\Omega_p} \quad (\text{dimensionless}) \end{aligned}$$

the narrower Ω_p of an antenna pattern is, the greater is the directivity. For an isotropic antenna, $\Omega_p = 4\pi$; hence, its directivity $D_{\text{iso}} = 1$.

Approximating D with the 3dB Beamwidth

Equivalent Solid Angle

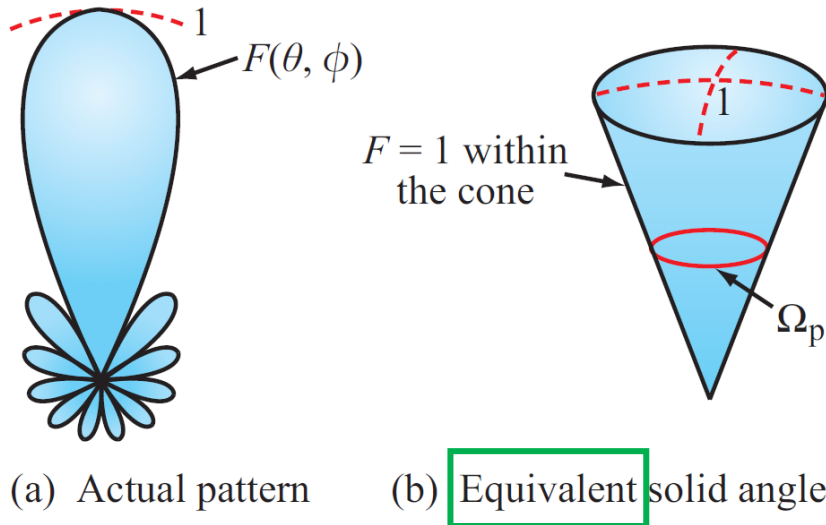


Figure 9-11: The pattern solid angle Ω_p defines an equivalent cone over which all the radiation of the actual antenna is concentrated with uniform intensity equal to the maximum of the actual pattern.

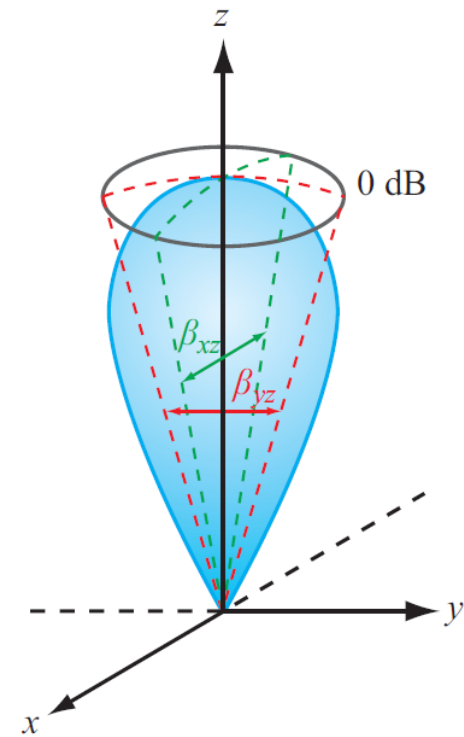


Figure 9-12: The solid angle of a unidirectional radiation pattern is approximately equal to the product of the half-power beamwidths in the two principal planes; that is, $\Omega_p \simeq \beta_{xz} \beta_{yz}$.

For an antenna with a single main lobe pointing in the z -direction as shown in Fig. 9-12, Ω_p may be approximated as the product of the half-power beamwidths β_{xz} and β_{yz} (in radians):

$$\Omega_p \simeq \beta_{xz} \beta_{yz}, \quad (9.25)$$

and therefore

$$D = \frac{4\pi}{\Omega_p} \simeq \frac{4\pi}{\beta_{xz} \beta_{yz}}. \quad (9.26)$$

Example 9-1: Antenna Radiation Properties

Determine (a) the direction of maximum radiation, (b) pattern solid angle, (c) directivity, and (d) half-power beamwidth in the y - z plane for an antenna that radiates only into the upper hemisphere with normalized radiation intensity given by $F(\theta, \phi) = \cos^2 \theta$.

Solution: The statement that the antenna radiates through only the upper hemisphere is equivalent to

$$F(\theta, \phi) = F(\theta) = \begin{cases} \cos^2 \theta & \text{for } 0 \leq \theta \leq \pi/2 \\ & \text{and } 0 \leq \phi \leq 2\pi, \\ 0 & \text{elsewhere.} \end{cases}$$

(a) The function $F(\theta) = \cos^2 \theta$ is independent of ϕ and is maximum when $\theta = 0^\circ$. A polar plot of $F(\theta)$ is shown in Fig. 9-13.

(b) From Eq. (9.21), the pattern solid angle Ω_p is given by

$$\begin{aligned} \Omega_p &= \iint_{4\pi} F(\theta, \phi) d\Omega \\ &= \int_{\phi=0}^{2\pi} \left[\int_{\theta=0}^{\pi/2} \cos^2 \theta \sin \theta d\theta \right] d\phi \\ &= \int_{\phi=0}^{2\pi} \left[-\frac{\cos^3 \theta}{3} \right]_0^{\pi/2} d\phi \\ &= \int_0^{2\pi} \frac{1}{3} d\phi = \frac{2\pi}{3} \quad (\text{sr}). \end{aligned}$$

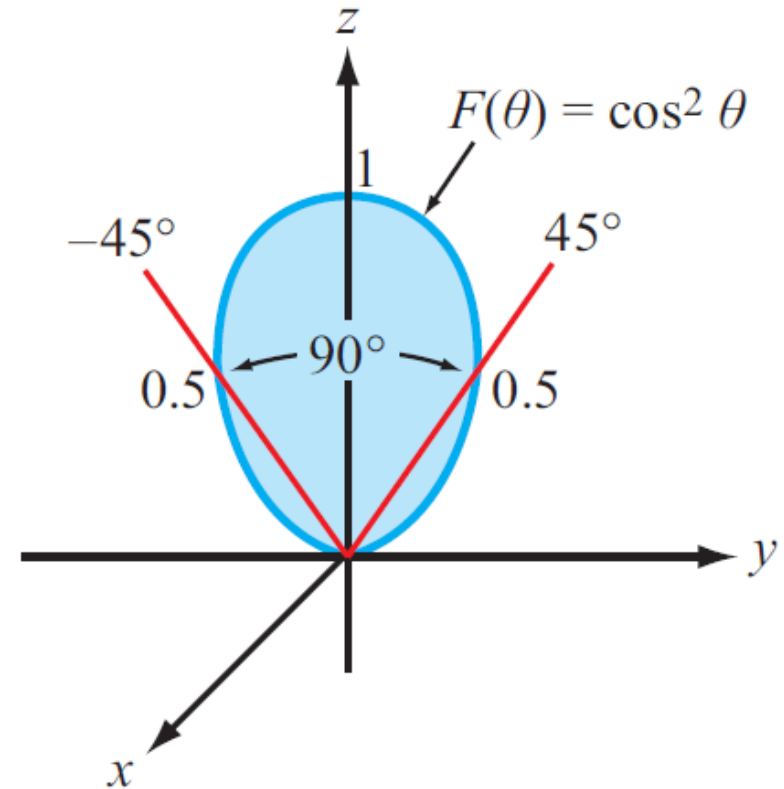


Figure 9-13: Polar plot of $F(\theta) = \cos^2 \theta$.

Cont.

Example 9-1: Antenna Radiation Properties (cont.)

Determine (a) the direction of maximum radiation, (b) pattern solid angle, (c) directivity, and (d) half-power beamwidth in the y - z plane for an antenna that radiates only into the upper hemisphere with normalized radiation intensity given by $F(\theta, \phi) = \cos^2 \theta$.

(c) Application of Eq. (9.23) gives

$$D = \frac{4\pi}{\Omega_p} = 4\pi \left(\frac{3}{2\pi} \right) = 6,$$

which corresponds to $D \text{ (dB)} = 10 \log 6 = 7.78 \text{ dB}$.

(d) The half-power beamwidth β is obtained by setting $F(\theta) = 0.5$. That is,

$$F(\theta) = \cos^2 \theta = 0.5,$$

which gives the half-power angles $\theta_1 = -45^\circ$ and $\theta_2 = 45^\circ$. Hence,

$$\beta = \theta_2 - \theta_1 = 90^\circ.$$

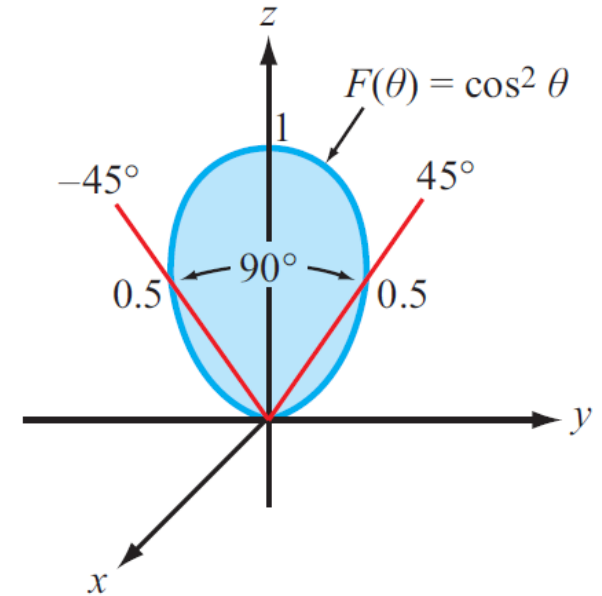


Figure 9-13: Polar plot of $F(\theta) = \cos^2 \theta$.

Example 9-2: Directivity of a Hertzian Dipole

Calculate the directivity of a Hertzian dipole.

Solution: Application of Eq. (9.23) with $F(\theta) = \sin^2 \theta$ [from Eq. (9.15)] gives

$$\begin{aligned} D &= \frac{4\pi}{\iint_{4\pi} F(\theta, \phi) \sin \theta \, d\theta \, d\phi} \\ &= \frac{4\pi}{\int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \sin^3 \theta \, d\theta \, d\phi} = \frac{4\pi}{8\pi/3} = 1.5, \end{aligned}$$

or, equivalently, 1.76 dB.

Radiation Efficiency and Gain

Radiation efficiency

Of the total power P_t (transmitter power) supplied to the antenna, a part, P_{rad} , is radiated out into space, and the remainder, P_{loss} , is dissipated as heat in the antenna structure. The *radiation efficiency* ξ is defined as the ratio of P_{rad} to P_t :

$$\xi = \frac{P_{\text{rad}}}{P_t} \quad (\text{dimensionless}). \quad (9.27)$$

Antenna gain G

$$G = \xi D \quad (\text{dimensionless}). \quad (9.29)$$

The gain accounts for ohmic losses in the antenna material, whereas the directivity does not. For a lossless antenna, $\xi = 1$, and $G = D$.

Antenna Radiation and Loss Resistances

Since $P_t = P_{\text{rad}} + P_{\text{loss}}$, it follows that R_{in} can be defined as the sum of a *radiation resistance* R_{rad} and a *loss resistance* R_{loss} ,

$$R_{\text{in}} = R_{\text{rad}} + R_{\text{loss}}, \quad (9.32)$$

with

$$P_{\text{rad}} = \frac{1}{2} I_0^2 R_{\text{rad}}, \quad (9.33a)$$

$$P_{\text{loss}} = \frac{1}{2} I_0^2 R_{\text{loss}}, \quad (9.33b)$$

$$\xi = \frac{P_{\text{rad}}}{P_t} = \frac{P_{\text{rad}}}{P_{\text{rad}} + P_{\text{loss}}} = \frac{R_{\text{rad}}}{R_{\text{rad}} + R_{\text{loss}}}.$$

Example 9-3: Radiation Resistance and Efficiency of a Hertzian Dipole

A 4-cm-long center-fed dipole is used as an antenna at 75 MHz. The antenna wire is made of copper and has a radius $a = 0.4$ mm. From Eqs. (7.92a) and (7.94), the loss resistance of a circular wire of length l is given by

$$R_{\text{loss}} = \frac{l}{2\pi a} \sqrt{\frac{\pi f \mu_c}{\sigma_c}}, \quad (9.35)$$

where μ_c and σ_c are the magnetic permeability and conductivity of the wire, respectively. Calculate the radiation resistance and the radiation efficiency of the dipole antenna.

Solution: At 75 MHz,

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{7.5 \times 10^7} = 4 \text{ m}.$$

The length to wavelength ratio is $l/\lambda = 4 \text{ cm}/4 \text{ m} = 10^{-2}$. Hence, this is a short dipole. From Eq. (9.24),

$$P_{\text{rad}} = \frac{4\pi R^2}{D} S_{\text{max}}. \quad (9.36)$$

For the Hertzian dipole, S_{max} is given by Eq. (9.14), and from Example 9-2 we established that $D = 1.5$. Hence,

(Equation for S_0 on slide 14)

$$P_{\text{rad}} = \frac{4\pi R^2}{1.5} \times \frac{15\pi I_0^2}{R^2} \left(\frac{l}{\lambda}\right)^2 = 40\pi^2 I_0^2 \left(\frac{l}{\lambda}\right)^2. \quad (9.37)$$

Cont.

Example 9-3 (cont.)

Recap:

For any antenna: $P_{\text{rad}} = \frac{1}{2} I_0^2 R_{\text{rad}}$

For the Hertzian dipole: $P_{\text{rad}} = \frac{4\pi R^2}{1.5} \times \frac{15\pi I_0^2}{R^2} \left(\frac{l}{\lambda}\right)^2 = 40\pi^2 I_0^2 \left(\frac{l}{\lambda}\right)^2$

Combining both gives:

$$R_{\text{rad}} = 80\pi^2 (l/\lambda)^2 \quad (\Omega). \quad (9.38)$$

For $l/\lambda = 10^{-2}$, $R_{\text{rad}} = 0.08 \Omega$.

Next, we determine the loss resistance R_{loss} . For copper, Appendix B gives $\mu_c \simeq \mu_0 = 4\pi \times 10^{-7} \text{ H/m}$ and $\sigma_c = 5.8 \times 10^7 \text{ S/m}$. Hence,

$$\begin{aligned} R_{\text{loss}} &= \frac{l}{2\pi a} \sqrt{\frac{\pi f \mu_c}{\sigma_c}} \\ &= \frac{4 \times 10^{-2}}{2\pi \times 4 \times 10^{-4}} \left(\frac{\pi \times 75 \times 10^6 \times 4\pi \times 10^{-7}}{5.8 \times 10^7} \right)^{1/2} \\ &= 0.036 \Omega. \end{aligned}$$

Therefore, the radiation efficiency is

$$\xi = \frac{R_{\text{rad}}}{R_{\text{rad}} + R_{\text{loss}}} = \frac{0.08}{0.08 + 0.036} = 0.69.$$

Thus, the dipole is 69% efficient.

Outline

- Course Matter
- Introduction to antennas
- The Hertzian Dipole
 - Electromagnetic (Near)-field
 - Electromagnetic Far-field
- Antenna Parameters
 - Radiation intensity $S(R, \theta, \phi) = S_{max}(R)F(\theta, \phi)$
 - Normalized radiation pattern $F(\theta, \phi)$
 - Radiated Power P_{rad}
 - Antenna Beam Parameters (β, Ω)
 - Directivity (D), Efficiency (ξ) and Gain ($G = \xi D$)
- The Half-Wavelength Dipole

Half-Wave Dipole

1. Current in half-wave dipole

$$i(t) = I_0 \cos \omega t \cos kZ = \Re \left[I_0 \cos kZ e^{j\omega t} \right]$$

whose phasor is

$$\tilde{I}(z) = I_0 \cos kz, \quad -\lambda/4 \leq z \leq \lambda/4$$

2. For Hertzian dipole of length l , E field is:

$$\tilde{E}_\theta = \frac{j I_0 l k \eta_0}{4\pi} \left(\frac{e^{-jkR}}{R} \right) \sin \theta \quad (\text{V/m})$$

3. Each length element dz of half-wave dipole is like a Hertzian dipole, radiating a field

$$d\tilde{E}_\theta(z) = \frac{jk\eta_0}{4\pi} \tilde{I}(z) dz \left(\frac{e^{-jks}}{s} \right) \sin \theta_s$$

4. For the entire dipole, the total radiated field is

$$\tilde{E}_\theta = \int_{z=-\lambda/4}^{\lambda/4} d\tilde{E}_\theta$$

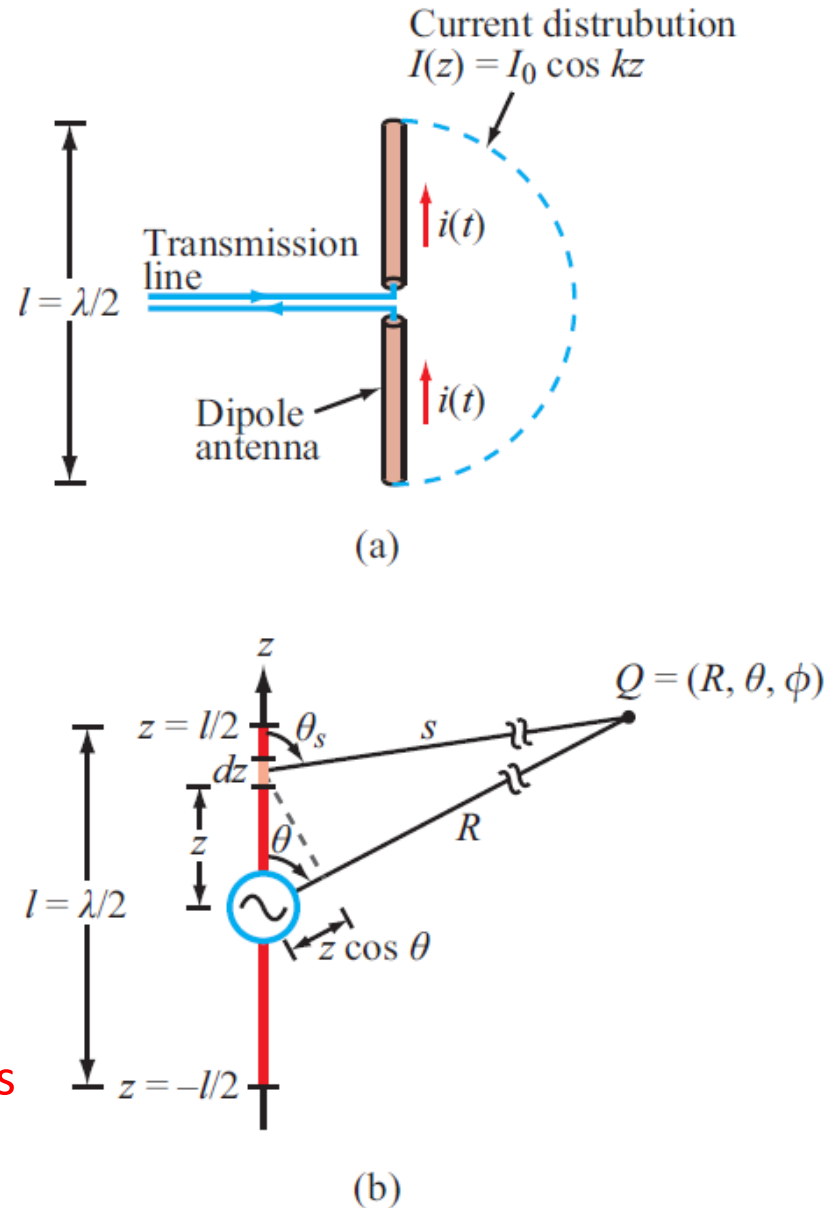


Figure 9-14: Center-fed half-wave dipole.

Half-Wave Dipole (cont.)

Integration leads to:

$$\tilde{E}_\theta = j 60 I_0 \left\{ \frac{\cos[(\pi/2) \cos \theta]}{\sin \theta} \right\} \left(\frac{e^{-jkR}}{R} \right),$$

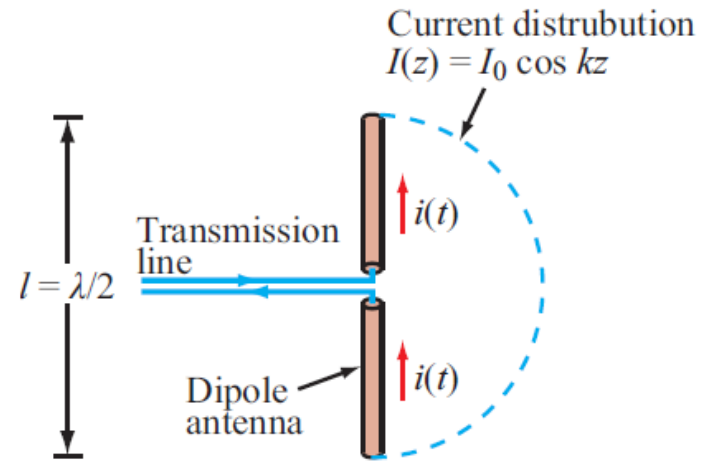
$$\tilde{H}_\phi = \frac{\tilde{E}_\theta}{\eta_0}.$$

$$S(R, \theta) = \frac{|\tilde{E}_\theta|^2}{2\eta_0}$$

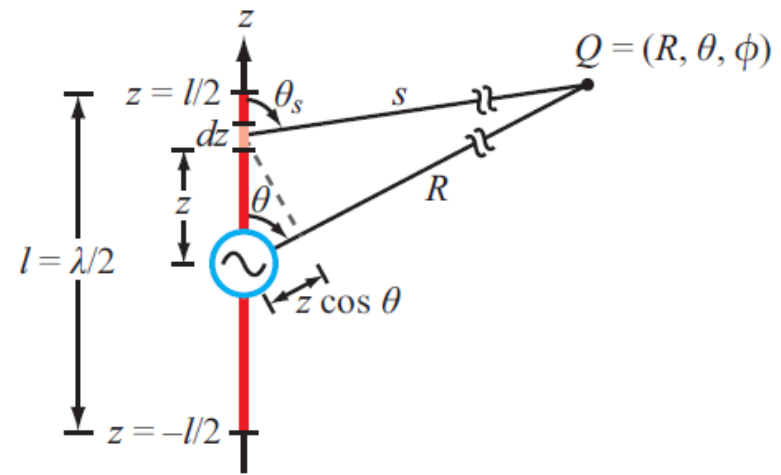
$$= \frac{15 I_0^2}{\pi R^2} \left\{ \frac{\cos^2[(\pi/2) \cos \theta]}{\sin^2 \theta} \right\}$$

$$= S_0 \left\{ \frac{\cos^2[(\pi/2) \cos \theta]}{\sin^2 \theta} \right\}$$

$$F(\theta) = \frac{S(R, \theta)}{S_0} = \left\{ \frac{\cos[(\pi/2) \cos \theta]}{\sin \theta} \right\}^2$$



(a)

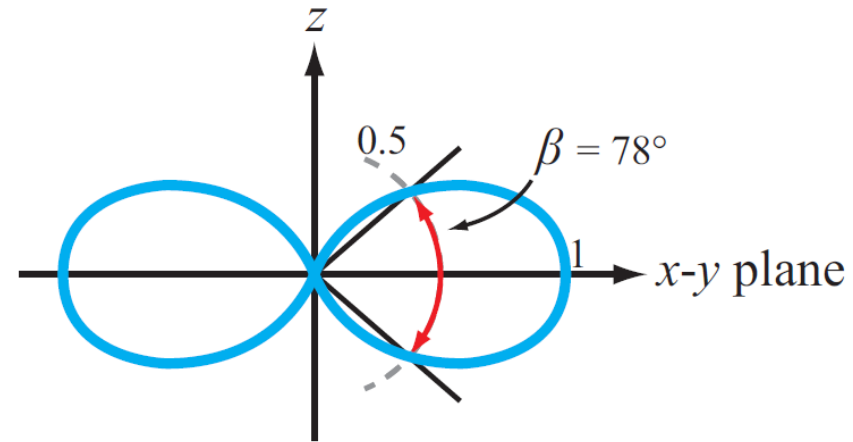


(b)

Radiation Pattern of Half-Wave Dipole

$$F(\theta) = \frac{S(R, \theta)}{S_0} = \left\{ \frac{\cos[(\pi/2) \cos \theta]}{\sin \theta} \right\}^2$$

Radiation pattern resembles that of the Hertzian dipole. Its beamwidth is slightly narrower, 78 degrees compared with 90 degrees for the Hertzian dipole.



(a) $l = \lambda/2$

Its directivity is slightly larger (1.64 compared with 1.5 for the short dipole), but its radiation resistance is 73Ω (as will be shown later in Section 9-3.2), which is orders of magnitude larger than that of a short dipole.

Half-Wavelength Dipole Fact Table

Half-Wavelength Dipole	
S_0	$\frac{15I_0^2}{\pi R^2}$
F	$\left[\frac{\cos\left(\frac{\pi}{2} \cos \theta\right)}{\sin \theta} \right]^2$
Ω_p	7.66
D	1.64 (2.15 dB)
3dB BW	78°
R_{rad}	73.1 Ω

Summary

- In this class, we briefly introduced the concept of antennas
- We looked at the fields emanating from a Hertzian dipole and a half-wavelength dipole
- Along the way, we introduced the following topics
 - Near-field vs. far-field
 - Antenna Pattern
 - Beamwidth
 - Directivity, Efficiency and Gain
 - Radiated vs. loss power
 - Radiation vs. Loss resistance

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thank you!