

EE3109

Answers of Assignment I

Ques. 1

(a)

(i)

$$\therefore \nabla \cdot \vec{B} = 0$$

$$\text{and } \nabla \cdot (\nabla \times \vec{A}) = 0 \text{ for any vector } \vec{A}$$

$$\therefore \text{We can define a vector } \vec{A} \text{ such that } \vec{B} = \nabla \times \vec{A}$$

(ii)

$$\therefore \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\nabla \times \left(\frac{\partial \vec{A}}{\partial t} \right) \text{ or } \nabla \times \left(\vec{E} + \frac{\partial \vec{A}}{\partial t} \right) = 0$$

$$\text{and } \nabla \times (\nabla V) = 0 \text{ for any scalar } V$$

$$\therefore \text{We can define a scalar } V \text{ such that } \vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t}$$

(iii)

From Maxwell's equations and the definitions of $\vec{A}$ and V , we can get

$$\nabla^2 \vec{A} - \mu\epsilon \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu\vec{J} + \nabla \left(\nabla \cdot \vec{A} + \mu\epsilon \frac{\partial V}{\partial t} \right)$$

The Lorentz gauge, $\nabla \cdot \vec{A} = -\mu\epsilon \frac{\partial V}{\partial t}$, can be used to simplify the equation further:

$$\nabla^2 \vec{A} - \mu\epsilon \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu\vec{J}$$

(b)

From inspection of the given phasor, $k = 15 \text{ rad/m}$

$$\text{Wave's frequency } f = \frac{\omega}{2\pi} = \frac{c_0 k}{2\pi} = 716 \text{ MHz}$$

$$\begin{aligned} \tilde{H} &= \frac{1}{\eta_0} \hat{z} \times \tilde{E} \\ &= \hat{y} \frac{30 - j20}{120\pi} e^{-j15z} = \hat{y} (0.08 - j0.05) e^{-j15z} \text{ A/m} \end{aligned}$$

$$\begin{aligned}\bar{H}(z,t) &= \text{Re} \left[\tilde{H}(z) e^{j\omega t} \right] \\ &= \hat{y} \left[0.08 \cos(4.5 \times 10^9 t - 15z) + 0.05 \sin(4.5 \times 10^9 t - 15z) \right] \text{ A/m}\end{aligned}$$

Ques. 2

(a)

$$\begin{aligned}\gamma &\approx j\omega\sqrt{\mu_0\epsilon} \left(1 - j \frac{\sigma}{2\omega\epsilon} \right) \\ &= \frac{\sigma\omega\sqrt{\mu_0\epsilon}}{2\omega\epsilon} + j\omega\sqrt{\mu_0\epsilon} = \alpha + j\beta\end{aligned}$$

Hence

$$\alpha = \frac{\sigma}{2} \sqrt{\frac{\mu_0}{\epsilon_0\epsilon_r}}$$

$$\beta = \omega\sqrt{\mu_0\epsilon_0\epsilon_r}$$

$$\begin{aligned}\eta &= \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_0}{\epsilon_0\epsilon_r}} \left(1 - j \frac{\sigma}{\omega\epsilon_0\epsilon_r} \right)^{\frac{1}{2}} \\ &= \sqrt{\frac{\mu_0}{\epsilon_0\epsilon_r}} \left(1 + j \frac{\sigma}{2\omega\epsilon_0\epsilon_r} \right)\end{aligned}$$

$$u_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu_0\epsilon_0\epsilon_r}}$$

(b)

(i)

$$\alpha = \frac{\sigma}{2} \sqrt{\frac{\mu_0}{\epsilon_0\epsilon_r}} = 0.0377 \text{ (Np/m)}$$

$$u_p = \frac{1}{\sqrt{\mu_0\epsilon_0\epsilon_r}} = \frac{c_0}{\sqrt{\epsilon_r}} = 1.5 \times 10^8 \text{ (m/s)}$$

$$\beta = \omega\sqrt{\mu_0\epsilon_0\epsilon_r} = \frac{\omega}{u_p} = 12.572 \text{ (rad/m)}$$

$$\eta = \sqrt{\frac{\mu_0}{\epsilon_0\epsilon_r}} \left(1 + j \frac{\sigma}{2\omega\epsilon_0\epsilon_r} \right) = 188.4 + j0.565 \text{ or } 188.4 \angle 0.17^\circ \text{ } (\Omega)$$

(ii)

$$\begin{aligned}\bar{E}(z) &= \bar{x}60e^{-0.0377z}e^{-j12.572z} \\ \bar{H} &= \frac{1}{\eta} \hat{z} \times \bar{E}(z) = \hat{y} \frac{60}{188.4} e^{-j0.003} e^{-0.0377z} e^{-j12.572z} \\ &= \hat{y}0.32e^{-0.038z} e^{-j(12.57z+0.003)} \text{ (A/m)}\end{aligned}$$

(iii)

$$\bar{P}_{av} = \frac{1}{2} \text{Re}\{\bar{E} \times \bar{H}^*\} = \hat{z}9.6e^{-0.076z} \text{ (W/m}^2\text{)}$$

Ques. 3

(a)

(i)

The wave has linear polarization because the two E components has the same phase difference $e^{j\pi/4}$.

(ii)

$$\begin{aligned}\tilde{H} &= \frac{1}{\eta_0} \hat{x} \times \tilde{E} = \frac{3}{120\pi} \hat{z} e^{j(\pi/4-25x)} - \frac{2}{120\pi} \hat{y} e^{j(\pi/4-25x)} \\ &= 8 \times 10^{-3} \hat{z} e^{j(\pi/4-25x)} - 5.3 \times 10^{-3} \hat{y} e^{j(\pi/4-25x)} \text{ A/m}\end{aligned}$$

(b)

(i)

$$\beta = 50$$

$$u_p = \frac{\omega}{\beta} = 0.6c \Rightarrow f = \omega / 2\pi = 0.6c\beta / 2\pi = 1.43\text{GHz}$$

(ii)

$$u_p = \frac{c}{\sqrt{\epsilon_r}} = 0.6c \Rightarrow \epsilon_r = 2.778$$

(iii) LCP

(iv)

$$\begin{aligned}\bar{E}(t, z) &= \hat{x}20 \cos(9 \times 10^9 t - 50z) - \hat{y}10 \sin(9 \times 10^9 t - 50z) \\ \bar{H} &= \frac{1}{\eta} \hat{z} \times \bar{E} = \frac{\sqrt{2.778}}{120\pi} \left[\hat{y}20 \cos(9 \times 10^9 t - 50z) + \hat{x}10 \sin(9 \times 10^9 t - 50z) \right] \\ &= \hat{y}0.088 \cos(9 \times 10^9 t - 50z) + \hat{x}0.044 \sin(9 \times 10^9 t - 50z) \text{ (A/m)}\end{aligned}$$

Ques. 4

(a)

For a time-harmonic source moving towards a stationary observer, the apparent frequency of the received wave is:

$$f' \approx f \left(1 + \frac{u}{c} \cos \theta \right)$$

In Doppler radar involving a moving target, the signal undergoes the Doppler shift when impinging upon the target. This target becomes the “source” of the reflected waves. Therefore the Doppler Effect is encountered a second time and the overall effect is double. Hence, the frequency shift is:

$$\begin{aligned} \Delta f &= \frac{2uf}{c} \cos \theta \\ &= \frac{2uf}{c} \cos 30^\circ = \frac{\sqrt{3}uf}{c} \end{aligned}$$

(b)

$$\Delta f = 3\text{kHz}, f = 12\text{GHz}$$

$$u = \frac{c\Delta f}{\sqrt{3}f} \cong 156\text{km/hr}$$