

EE1001 Fundamentals of Digital Techniques

Assignment 2 (Solution)

1.

a) $p \wedge q$

b) $\sim p \wedge \sim q$

2.

a)

p	q	$\sim p$	$\sim p \wedge q$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

b)

p	q	r	$\sim q$	$\sim q \vee r$	$p \wedge (\sim q \vee r)$
T	T	T	F	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	T	F
F	T	F	F	F	F
F	F	T	T	T	F
F	F	F	T	T	F

c)

3.

p	q	r	$q \vee r$	$p \wedge q$	$p \wedge r$	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	T	F	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	T

same truth values

The truth table shows that $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$ always have the same truth values. Therefore they are logically equivalent. This proves the distributive law for $\wedge$ over $\vee$.

4.

- a) The train is not late and my watch is not fast.
 b) $x \leq -2$ or $x \geq 7$.

5.

p	q	$\sim p$	$\sim q$	$p \wedge \sim q$	$\sim p \vee q$	$(p \wedge \sim q) \wedge (\sim p \vee q)$
T	T	F	F	F	T	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	F	T	F

↑

Its truth values are all F's, so $(p \wedge \sim q) \wedge (\sim p \vee q)$ is a contradiction.

6.

a)

$$\begin{aligned}
 \sim (p \vee \sim q) \vee (\sim p \wedge \sim q) &\equiv (\sim p \wedge \sim (\sim q)) \vee (\sim p \wedge \sim q) && \text{De Morgan's law} \\
 &\equiv (\sim p \wedge q) \vee (\sim p \wedge \sim q) && \text{double negative law} \\
 &\equiv \sim p \wedge (q \vee \sim q) && \text{distributive law} \\
 &\equiv \sim p \wedge \mathbf{t} && \text{negation law for } \vee \\
 &\equiv \sim p && \text{identity law for } \wedge
 \end{aligned}$$

b)

$$\begin{aligned}
 (p \wedge (\sim (\sim p \vee q))) \vee (p \wedge q) &\equiv (p \wedge (\sim (\sim p) \wedge \sim q)) \vee (p \wedge q) && \text{De Morgan's law} \\
 &\equiv (p \wedge (p \wedge \sim q)) \vee (p \wedge q) && \text{double negative law} \\
 &\equiv ((p \wedge p) \wedge \sim q) \vee (p \wedge q) && \text{associative law for } \wedge \\
 &\equiv (p \wedge \sim q) \vee (p \wedge q) && \text{idempotent law for } \wedge \\
 &\equiv p \wedge (\sim q \vee q) && \text{distributive law} \\
 &\equiv p \wedge (q \vee \sim q) && \text{commutative law for } \vee \\
 &\equiv p \wedge \mathbf{t} && \text{negation law for } \vee \\
 &\equiv p && \text{identity law for } \wedge
 \end{aligned}$$

7.

$$\begin{aligned} \text{a) } p \rightarrow (q \vee r) &\equiv \sim p \vee (q \vee r) \\ &\equiv (\sim p \vee q) \vee r \\ &\equiv \sim(p \wedge \sim q) \vee r \\ &\equiv (p \wedge \sim q) \rightarrow r \end{aligned}$$

- b) "If n is prime and n is not odd, then n is 2."
(Alternatively, "If n is prime and n is even, then n is 2.")

8. "If a computer program runs correctly, then it contains no syntax errors."
No. Even if a computer program is free of syntax errors, it may contain logical errors that could make it run incorrectly.

9. Statements (ii), (iii), and (v) must be true.

10.

$$\begin{aligned} \text{a) } p \vee (p \wedge q) \\ &\equiv p \quad (\text{This is just the Absorption Law.}) \end{aligned}$$

$$\begin{aligned} \text{b) } ((p \rightarrow q) \wedge p) \rightarrow q \\ &\equiv ((\sim p \vee q) \wedge p) \rightarrow q \\ &\equiv ((\sim p \wedge p) \vee (q \wedge p)) \rightarrow q \\ &\equiv (\mathbf{f} \vee (q \wedge p)) \rightarrow q \\ &\equiv (q \wedge p) \rightarrow q \\ &\equiv \sim(q \wedge p) \vee q \\ &\equiv (\sim q \vee \sim p) \vee q \\ &\equiv \mathbf{t} \vee \sim p \\ &\equiv \mathbf{t} \end{aligned}$$