

Block A: Unit 2

Circuit Analysis Methods

G. Rizzoni Ch 3.2-3.4, 3.6

Block A: Unit 2

This unit is organized into the following 4 sections:

- Nodal voltage analysis
- Mesh current analysis
- Superposition

The previous unit of this block was a light introduction to basic circuit theory. This included a description of the 3 basic laws in circuit theory. Simply knowing the 3 laws will not get you far. It is more important to be able to apply the laws to analyze and design circuits. In this unit, we will cover 3 commonly used approaches to analyze basic circuits.

I. Nodal Voltage Analysis (NVA)

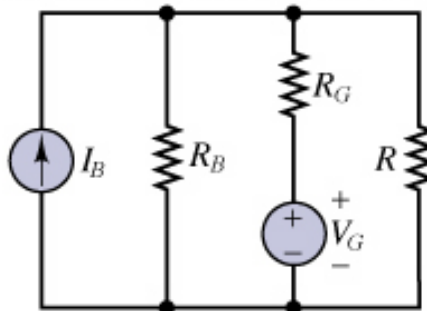
KCL & Ohm's law in action

The method of nodal voltage analysis is an application of KCL and Ohm's together. The unknown variables that you will solve for are node voltages. We do this by applying KCL at a node, but expressing the unknown currents as unknown node voltages. Let us see how this works in the example on the next page.

Example on how to do Nodal Voltage Analysis

For the following circuit (Problem 3.18 from Rizzoni), we will use Nodal Voltage Analysis to find the voltage across the current source, V_S .

Given: $I_B = 12\text{A}$, $V_G = 12\text{V}$, $R_G = 0.3\Omega$, $R_B = 1\Omega$, $R = 0.23\Omega$



By examining the above circuit, which nodes do you not know the voltage? The only unknown is just V_S .

Apply KCL at V_S :

$$I_B = \frac{V_S}{R_B} + \frac{V_S - V_G}{R_G} + \frac{V_S}{R}$$

$$12 = \frac{V_S}{1} + \frac{V_S}{0.23} + \frac{V_S - 12}{0.3}$$

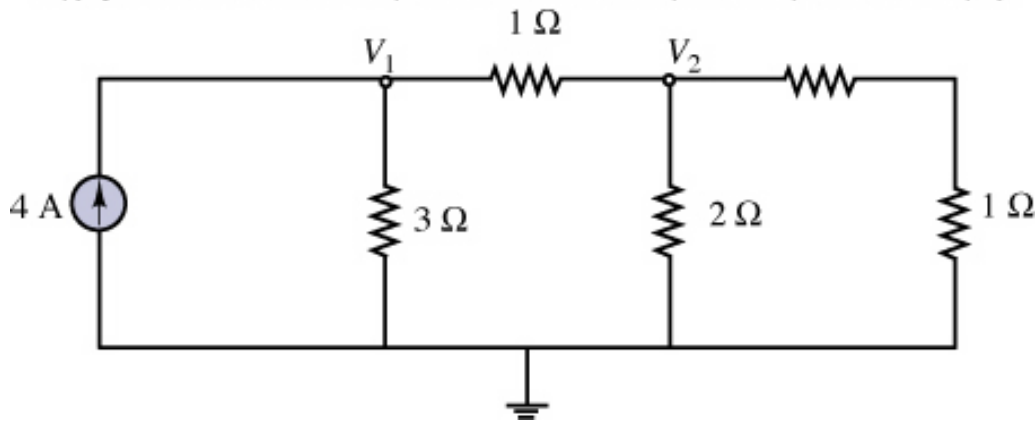
$$V_S = 5.99\text{V}$$

Note that the negative terminal of V_G has been used as a reference (0V) in our working. The value of V_S is then referenced to this node. This is useful since we are only interested in the voltage difference, and not so much of the value of the voltage.

Worked Example on NVA 1

Use nodal voltage analysis to find V_1 and V_2

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Apply KCL at V_1 :

$$4 = \frac{V_1}{3} + \frac{V_1 - V_2}{1}$$
$$12 = 4V_1 - V_2 \quad (1)$$

Apply KCL at V_2 :

$$\frac{V_1 - V_2}{1} = \frac{V_2}{2} + \frac{V_2}{2}$$
$$V_1 = 2V_2 \quad (2)$$

Solve equations (1) and (2):

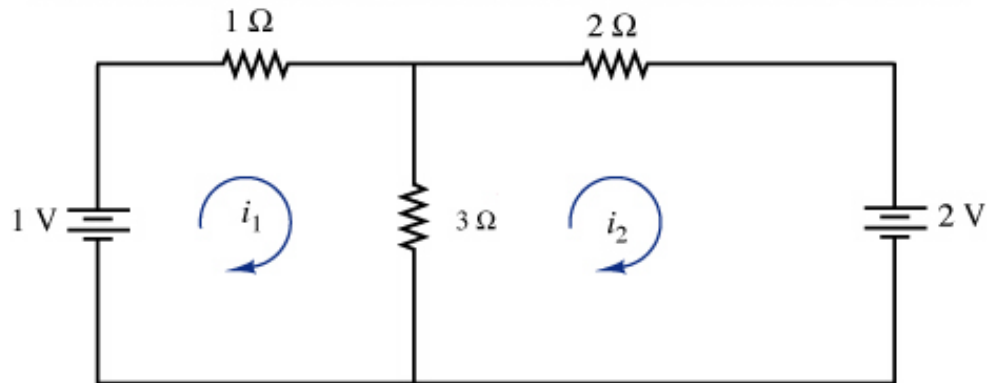
$$V_1 = 4.8 \text{ V}, V_2 = 2.4 \text{ V}$$

CHECK: Substitute back into (1) and (2) to verify your answer.

Alternatively, combine all the resistors to find V_1 first. Then combine the 3 resistors on the right followed by voltage divider rule to find V_2 . We did not have to use nodal voltage analysis for this question.

Worked Example on NVA 2

Use nodal voltage analysis to find the current through the 3Ω resistor



Objective: Find current through 3Ω resistor

Strategy: Find voltage across 3Ω resistor (labeled as V_A) and then use this to find the current through it (Ohm's law).

Apply KCL at V_A :

$$\frac{1 - V_A}{1} + \frac{2 - V_A}{2} = \frac{V_A}{3}$$
$$6 - 6V_A + 6 - 3V_A = 2V_A$$
$$V_A = \frac{12}{11} \text{ V}$$

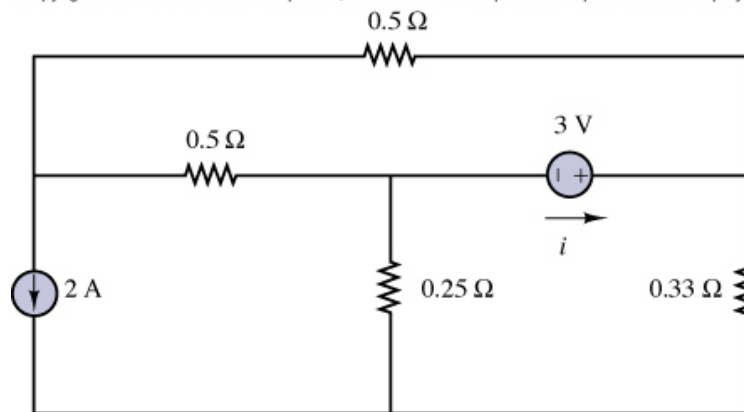
Current through the 3Ω resistor:

$$I_A = (12/11)/3 = 4/11 \text{ A}$$

Worked Example on NVA 3

Use nodal voltage analysis to find the current through the 3V source.

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Apply KCL at V_1 :

$$2 = \frac{V_2 - V_1}{0.5} + \frac{V_3 - V_1}{0.5}$$
$$1 = V_2 + V_3 - 2V_1 \quad (1)$$

Apply KCL at V_2 :

$$\frac{V_1 - V_2}{0.5} = \frac{V_2}{0.25} +$$
$$2V_1 - 6V_2 = i \quad (2)$$

Apply KCL at V_3 :

$$\frac{V_3}{0.33} + \frac{V_3 - V_1}{0.5} = i$$
$$\frac{166}{33}V_3 - 2V_1 = i \quad (3)$$

In the process of deriving equation (2), we introduced a new variable (i), thus increasing the number of required equations from 3 to 4. We need one more equation.

For the last equation, we use the relationship between V_2 and V_3 through the 3V voltage source:

$$V_3 - V_2 = 3V \quad (4)$$

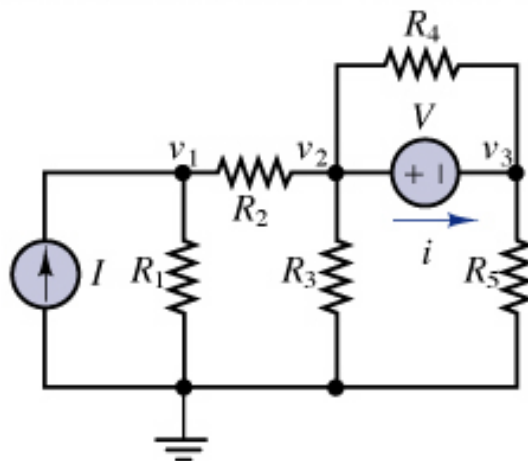
Finally, we solve equations (1) – (4): $i = 8.31A$

In summary, we used 4 equations to solve this problem. Keep in mind the question on whether this was the best approach to use.

Worked Example on NVA 4

Determine all unknown nodal voltages in the following circuit using nodal voltage analysis.

Let $I = 0.2A$, $R_1 = 200\Omega$, $R_2 = 75\Omega$, $R_3 = 25\Omega$, $R_4 = 50\Omega$, $R_5 = 100\Omega$, $V = 10V$



Apply KCL at V_1 :

$$I = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2}$$
$$0.2 = \frac{V_1}{200} + \frac{V_1 - V_2}{75}$$
$$\frac{1}{5} = \frac{11}{600}V_1 - \frac{1}{75}V_2 \quad (1)$$

Apply KCL at V_2 (as well as V_3)

$$\frac{V_1 - V_2}{R_2} = \frac{V_2}{R_3} +$$
$$\frac{V_1 - V_2}{75} = \frac{V_2}{25} + \frac{V_3}{100}$$
$$V_1 - 4V_2 = \frac{3}{4}V_3 \quad (2)$$

By recognizing that the current flowing out of V_2 via R_4 and V is the same as the current flowing into V_3 , we save the step of writing one more equation for i .

Relationship of V_2 and V_3 through source V :

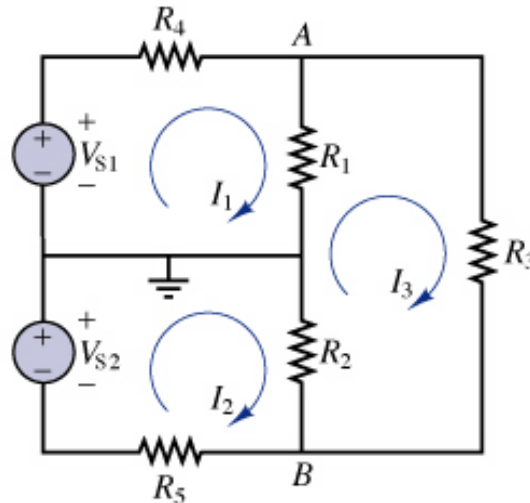
$$V_2 - V_3 = V \quad (3)$$

Check if we have all the equations we need (3 variables solved with 3 equations). Solve equations (1), (2), (3):

$$V_1 = 14.24\text{V}, V_2 = 4.58\text{V}, V_3 = -5.42\text{V}$$

Worked Example on NVA 5 *Lab 2

Determine the voltage difference across terminals A-B. All the resistors have the same value of $5\text{k}\Omega$ and both supply voltages are 10V .



Note that we CANNOT merge any resistors in this case. We will use NVA to solve this problem.

Apply KCL at A:

Apply KCL at B:

Solving the two equations: $V_A = -V_B = 2.5\text{V}$ (V_B simply mirrors V_A) $\Rightarrow V_{AB} = 5\text{V}$

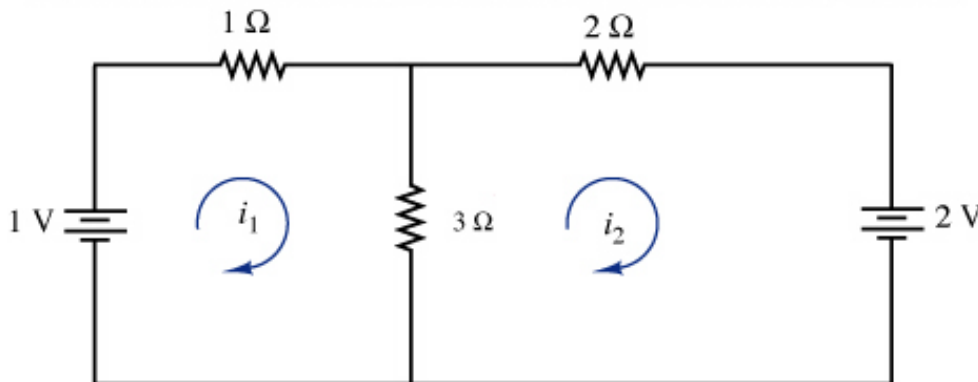
II. Mesh Current Analysis (MCA)

KVL & Ohm's law in action

The method of mesh current analysis is an application of KVL and Ohm's together. The unknown variables that you will solve for are mesh currents. We do this by applying KVL around a given mesh. From KVL, the sum of voltage drops and rise must equal zero. We here express the voltage differences using the current going through each branch in the mesh. Let us see how this works in the following example.

Example on how to do Mesh Current Analysis

Instead of using nodal voltage analysis, we will now use mesh current analysis to find the current through the 3Ω resistor



In the above figure, the unknown mesh currents that are to be determined have already been marked out. There are 2 meshes and hence 2 mesh currents. To perform mesh current analysis, we apply KVL to each of these 2 meshes as follows:

Apply KVL to mesh 1:

We follow the defined direction of the mesh current as we add up the voltages

$$\begin{aligned} 1 &= i_1(1 + 3) - 3i_2 \\ 4i_1 - 3i_2 &= 1 \end{aligned} \quad (1)$$

Notice that since voltage of the source increases in the direction of the mesh current, we use a positive sign. A useful tip is to keep voltages of sources on one side of the equation, and keep voltages of resistors on the other side (What goes up must equal what goes down).

Since i_1 is defined opposite to i_2 through the 3Ω resistor, we use a negative sign.

Apply KVL to mesh 2:

Once again we follow the defined direction of the mesh current as we add up the voltages

$$\begin{aligned} -2 &= i_2(2 + 3) - 3i_1 \\ 5i_2 - 3i_1 &= -2 \end{aligned} \quad (2)$$

This time, we see the voltage of the source decreases in the defined direction, thus we use a negative sign.

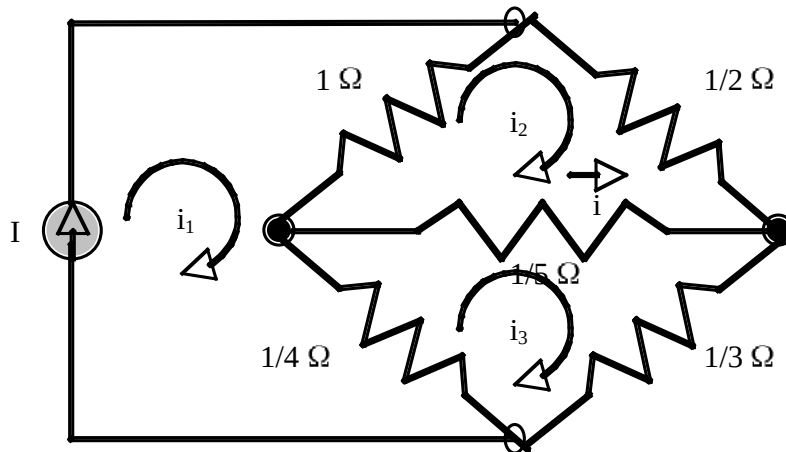
Now we solve equation (1) and (2):

$$i_1 = -1/11\text{A}, i_2 = -5/11\text{A}$$

$$I_A = i_1 - i_2 = 4/11\text{A}$$

Worked Example on MCA 1

Use mesh current analysis to find the current through the $1/5\Omega$ resistor, given that $I = 1\text{A}$.



We see 3 meshes so it looks like there are 3 unknown mesh currents. But note that i_1 is already given by the source current I , so only 2 unknown mesh currents.

Apply KVL around mesh 2:

$$i_2(1 + 1/2 + 1/5) - i_3(1/5) - i_1 = 0 \quad (1)$$

Apply KVL around mesh 3:

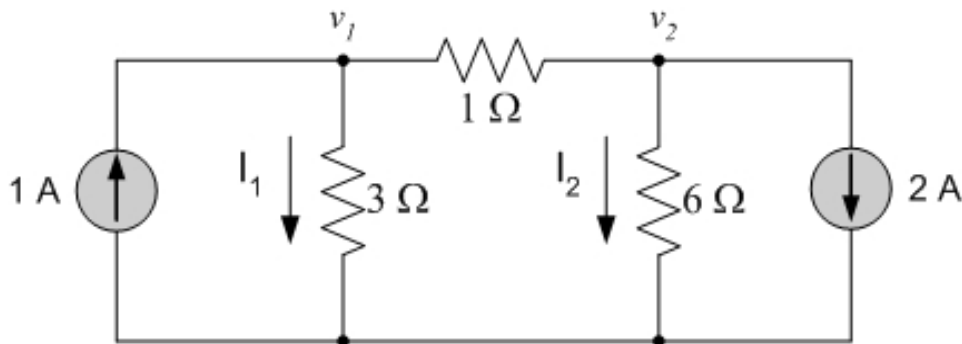
$$i_3(1/4 + 1/2 + 1/5) - i_2(1/5) - i_1(1/4) = 0 \quad (2)$$

Solve (1) and (2): $i_2 = 20/31\text{ A}$, $i_3 = 15/31\text{ A}$

Finally, $i = i_3 - i_2 = 5/31\text{ A}$

Worked Example on MCA 2

Use mesh current analysis to find currents I_1 and I_2



We see 3 meshes, but only the middle mesh is unknown, the other 2 are defined by the current sources. For the sake of simplicity, define the direction of known mesh currents in the same sense as the sources.

Apply KVL around the middle mesh:

$$I(3 + 1 + 6) - (1)(3) - (2)(6) = 0$$

$$I = 1.5\text{A}$$

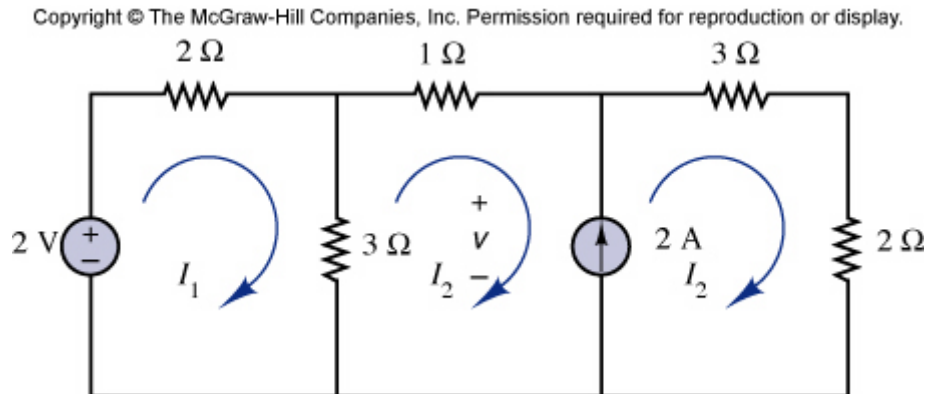
Now all the mesh currents are known and we can use them to find the branch currents I_1 and I_2 .

$$I_1 = 1 - I = -0.5\text{A}$$

$$I_2 = I - 2 = -0.5\text{A}$$

Worked Example on MCA 3

Use mesh current analysis to find the voltage across the current source



Apply KVL around mesh 1:

$$2 = I_1(2+3) - I_2(3)$$

$$\Rightarrow 5I_1 - 3I_2 = 2 \quad (1)$$

Apply KVL around mesh 2:

$$-V = I_2(3+1) - I_1(3)$$

$$\Rightarrow 4I_2 - 3I_1 = -V \quad (2)$$

Apply KVL around mesh 3:

$$V = I_3(3+2)$$

$$\Rightarrow 5I_3 = V \quad (3)$$

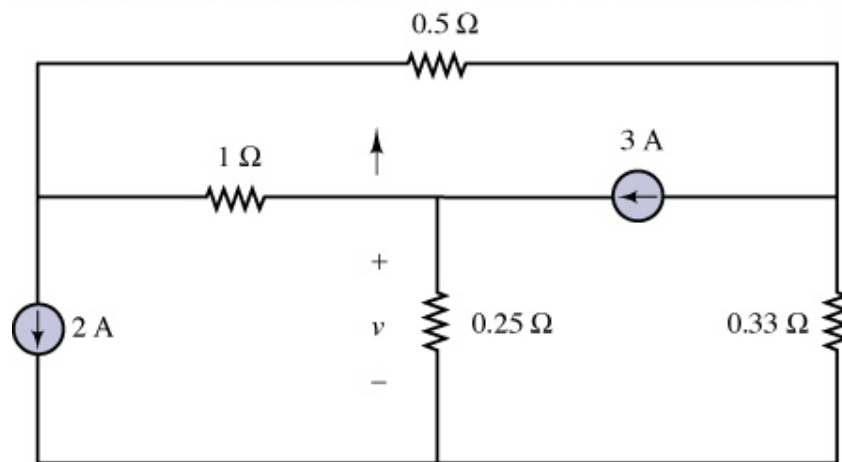
In the process of deriving equations (2) and (3), we had to introduce a new variable V . Hence we now need one more equation. We obtain the final equation needed by considering the how I_2 and I_3 are related through the 2A current source:

$$I_3 - I_2 = 2A \quad (4)$$

Finally, we solve equations (1) – (4): $V = 35/9V$

Worked Example on MCA 4

Using mesh current analysis, find the voltage across the 0.25Ω resistor.



Objective: Find voltage across 0.25Ω resistor

Strategy: Find the current through 0.25Ω resistor first, and then use this to find the voltage.

Although there are 3 meshes in the circuit, the bottom left mesh is already known (2A going anti-clockwise). So this leaves use with 2 unknown meshes to solve.

Apply KVL around mesh 1:

$$V_s = I_1(0.25 + 0.33) - (2)(0.25)$$
$$\Rightarrow V = 0.58I_1 - 0.5 \quad (1)$$

Apply KVL around mesh 2:

$$V_s = I_2(0.5 + 1) + (2)(1)$$
$$\Rightarrow V = 1.5I_2 + 2 \quad (2)$$

Consider the current source:

$$I_1 + I_2 = 3 \quad (3)$$

Solve (1) – (3):

$$I_1 = 175/72\text{A}, I_2 = -19/52\text{A}$$

$$\text{Current through } 0.25\Omega = I_1 - 2 = 71/52\text{A}$$

Pay attention to the defined direction of this current

$$\Rightarrow V = (71/52)(0.25) = 0.34\text{V}$$

If we had defined the directions of the mesh currents differently, we would get different answers to I_1 and I_2 but the final answer for V should be the same.

Quick note on choosing between methods

How do you choose between NVA and MCA? Do not choose the method because you are more comfortable with it. Choose the method that makes solving a given circuit the easiest. For example, having fewer equations certainly makes solving easier.

III. Superposition

The method

This method applies only to circuits that have multiple sources. In such case, it can come in handy (or not). In a circuit with multiple sources, superposition considers the current or voltage associated with a given branch for one of the sources, while turning the rest of.

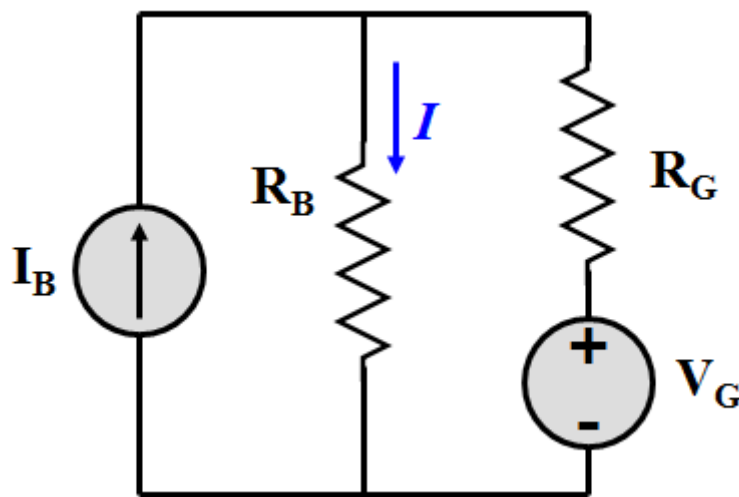


Fig 1: Basic circuit with 2 sources

Refer to the above figure to see how superposition is applied. Say we want to find the current through R_B . In superposition, we find the current through R_B when only I_G is present (V_G is thus removed). We call this current I_1 . Then we find the current through R_B again when only V_G is present (I_B is removed). We call this current I_2 . The net current I is then the sum of I_1 and I_2 .

But how does one “remove” a current source or voltage source?

Disabling sources

Voltage source: The function of voltage source is to fix the voltage across its terminals. Hence if it does not exist, the voltage across the terminals would be zero. It then looks like a short circuit. As such, to disable a voltage source we replace it with a **short circuit** (see Fig 2a).

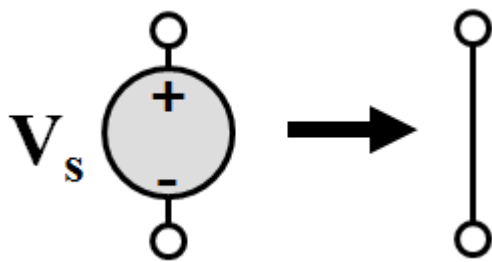


Fig 2a: Disable a voltage source by replacing with a short circuit

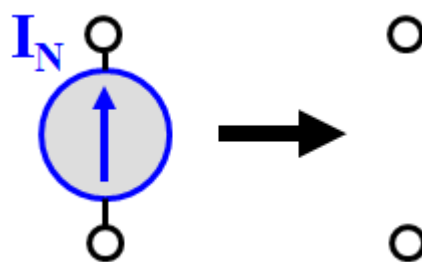
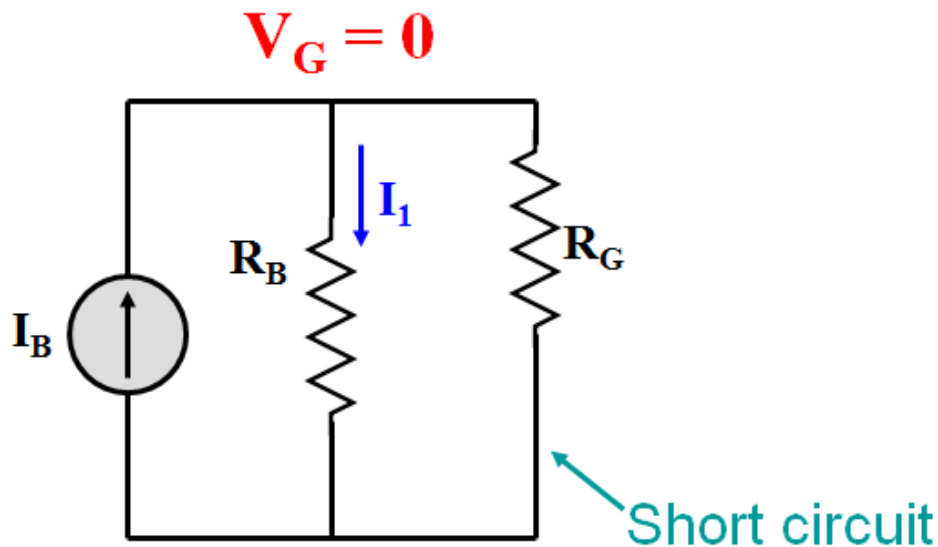


Fig 2b: Disable a current source by replacing with an open circuit

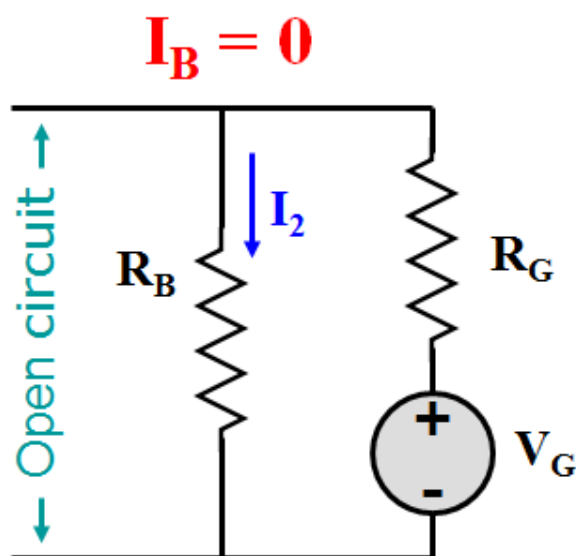
Current source: The function of a current source is to fix the current flowing between its terminals. Hence if it does not exist, the current through it would be zero. It then looks like an open circuit. Therefore, to disable a current source we replace it with an **open circuit** (see Fig 2b).

If we apply these tips to Fig 1, disabling V_G so that I_B is the only source in the circuit we get:



We then find the resulting current through R_B due only to I_B , which is denoted as I_1 .

Next, we replace V_G back into the circuit and disable I_B so that V_G is now the only source in the circuit:



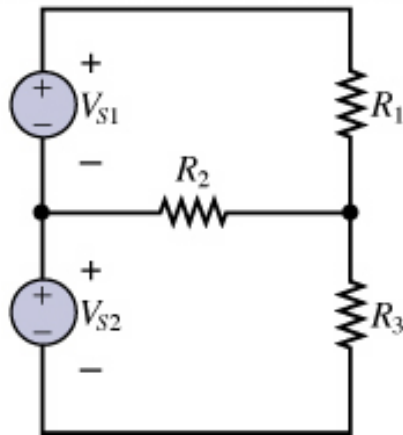
Finally, we sum the currents: $I = I_1 + I_2$

Worked Example on Superposition 1

Determine the current through R_1 using superposition.

Given $R_1 = 560\Omega$, $R_2 = 3.5k\Omega$, $R_3 = 810\Omega$, $V_{S2} = V_{S1} = 90V$

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Short V_{S1} to give the following circuit:

Voltage across R_1 ,

$$\begin{aligned} V_{R1a} &= \frac{R_1 \parallel R_2}{(R_1 \parallel R_2) + R_3} V_{S1} \\ &= \left(\frac{560 \parallel 3500}{(560 \parallel 3500) + 810} \right) 90 \\ &= 33.61V \end{aligned}$$

Current through R_1 :

$$\begin{aligned} I_{R1a} &= V_{R1a} / R_{1a} \\ &= 33.61 / 560 \\ &= 0.06A \end{aligned}$$

Next, short V_{S2} to give the following circuit:

Current through R_1 :

$$\begin{aligned} I_{R1b} &= \frac{V_{S1}}{R_1 + (R_2 \parallel R_3)} = \frac{90}{560 + (3500 \parallel 810)} \\ &= 0.074A \end{aligned}$$

Once again, take careful note of the directions defined for each of these currents.

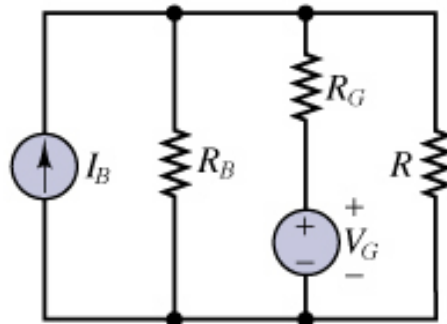
Finally, $I_{R1} = I_{R1a} + I_{R1b} = 0.134A$

Worked Example on Superposition 2

Determine the voltage across R using superposition

$$I_B = 12\text{A}, R_B = 1\Omega, V_G = 12, R_G = 0.3\Omega, R = 0.23\Omega$$

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Short I_B :

Note that R_B is now in parallel with R . We can find the voltage across R using voltage divider rule:

$$\begin{aligned} V_{RG} &= \frac{R \parallel R_B}{R_G + (R \parallel R_B)} V_G = \frac{0.23 \parallel 1}{0.3 + (0.23 \parallel 1)} 12 \\ &= 4.6077\text{V} \end{aligned}$$

Short V_G :

Now all 3 resistors are in parallel. We simply find the equivalent resistance to find V_R .

Resistance seen by I_B :

$$\begin{aligned} R_P &= R \parallel R_B \parallel R_G = 0.23 \parallel 1 \parallel 0.3 \\ &= 0.1152\Omega \end{aligned}$$

$$\begin{aligned} V_{RB} &= I_B R_P = (12)(0.1152) \\ &= 1.3823V \end{aligned}$$

Finally, we sum up the voltages (take careful note of the polarity of these voltages):

$$V_R = V_{RG} + V_{RB} = 5.99V$$