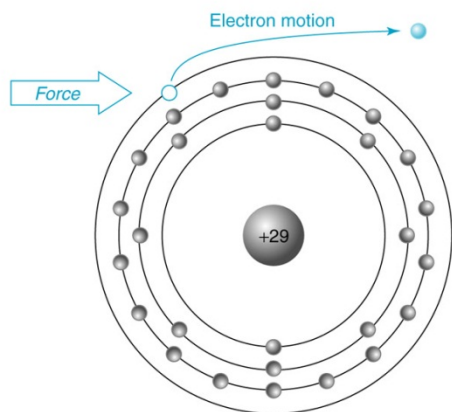


Circuits I Outline

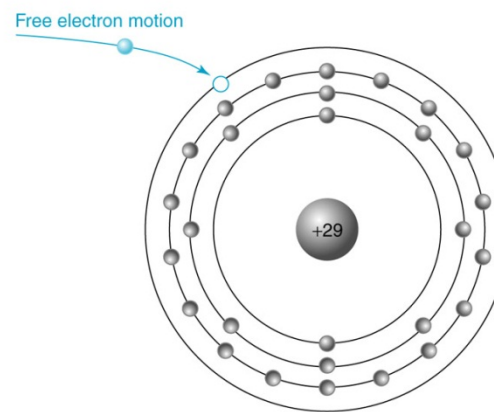
- Electrical materials
- Basic electrical quantities
 - > Current, voltage, resistance, power
- Terminologies in circuits
 - > Branch, Mesh, Node
- Laws applied in circuit analysis
 - > Ohm's law; Kirchhoff's current and voltage laws
 - > Resistive networks
 - > Parallel and series; current and voltage divider rule

Electric Charge

- Charge is a fundamental electric quantity, measured by the unit Coulomb (C)
- The smallest amount of charge that exists is the charge that is carried by 1 electron = $-1.602 \times 10^{-19} \text{ C}$
 - > Note the negative sign that symbolizes negative charge
 - > Protons have positive charge (found in the nucleus of an atom)
- Therefore charge quantities in real life occur in integral multiples of an electron's charge
- Typically denoted by the symbol Q or q



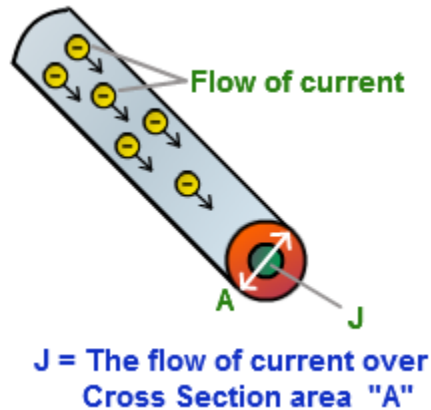
Note: The circle in the valence shell represents the gap left by the lost electron.



Note: The circle in the valence shell represents the gap left by a lost electron.

No. of protons (in nucleus) must be balanced by no. of electrons to be electrically neutral

Current and charge



Current is simply the flow of electric charges through a surface at the rate of 1 C/s
Mathematically:

$$I = dQ/dt$$

Unit: Ampere (A)

Typical symbol: I or i

More fundamental to current is the current density, typically symbolized by J :

$$J = nqv \quad \text{Note that } J \text{ is independent of geometry}$$

where n : number of charged particles per unit volume (i.e. concentration),
 q : charge on each particle, v : drift velocity of the charged particles

In relation to I : $J = I/A$

In the case of electrons (where the charge is negative), note that J and thus I are negative as a consequence.

Electrical materials

For current to flow, the charged particles must be free to move in the first place
What are the conditions that govern if the charged particles in a material is free to move when a force is applied? That depends on the electrical properties!

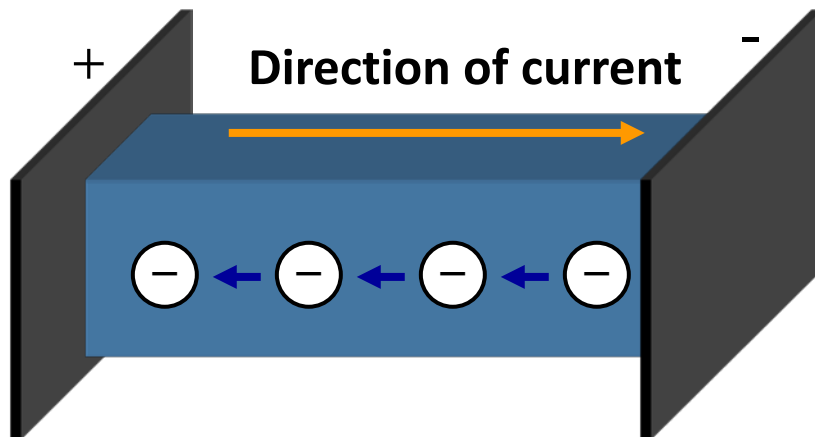
Conductors:

Charged particles are free to move under the influence of a force

Charged particles are electrons (negatively charged)

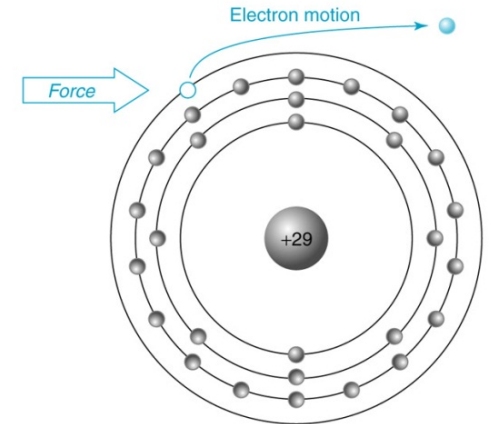
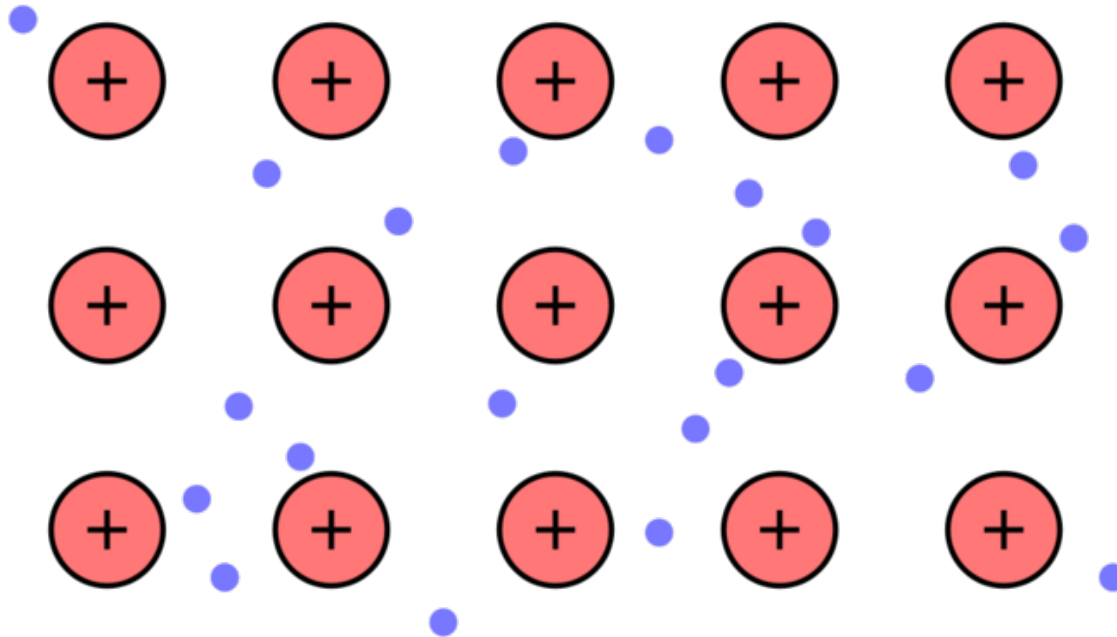
Negative charge: J and I are negative relative to direction of charge flow

Conventional current: Opposite to the direction of charge flow



But where do the free/mobile electrons come from?

Conductors: Electron sea model



Note: The circle in the valence shell represents the gap left by the lost electron.

Free electrons shared among metal atoms

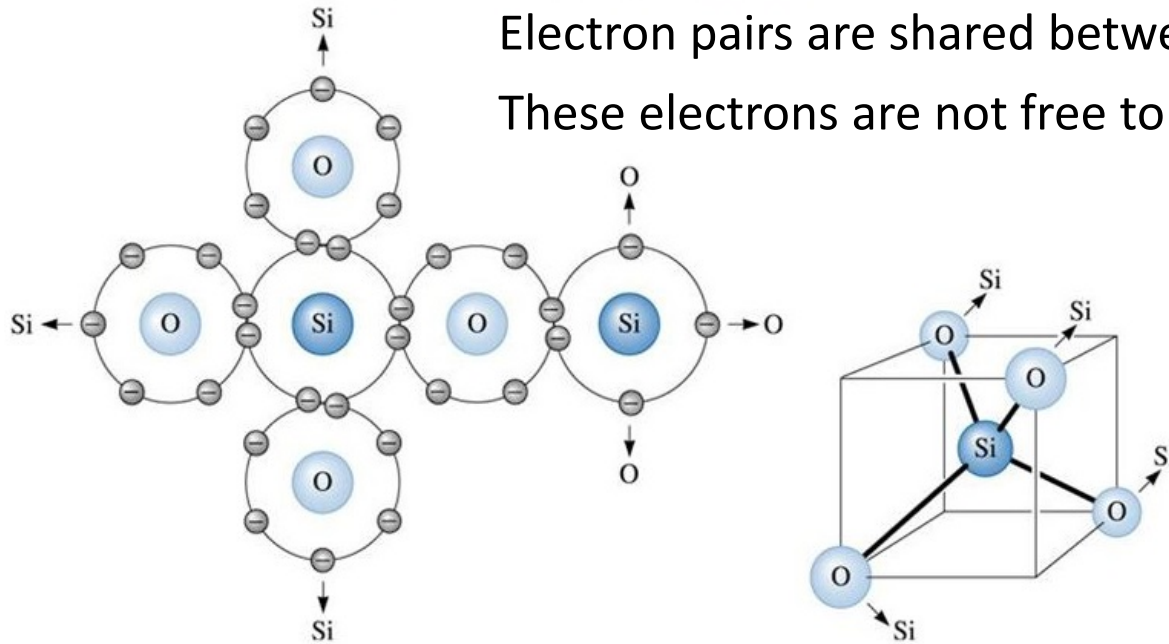
“Sea of electrons” surrounding the nucleus of each atom

Unbound electrons are free to move through the spaces between the nuclei

Note: This is one of several ways to understand conductors. This framework is known as the Electron Sea Model.

Insulators

Electron pairs are shared between atoms (covalent bond)
These electrons are not free to move



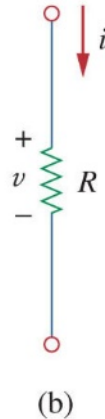
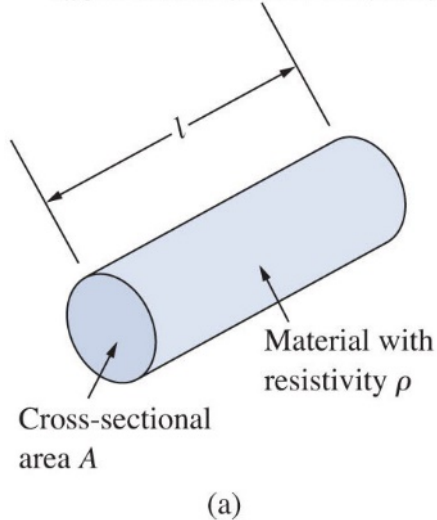
Due to a lack of free/mobile electrons, there is no flow of charges when a force is applied

When charges cannot flow, current therefore cannot flow: material does not conduct electrical current

Examples: SiO_2 (silicon dioxide: extensively used in the MOS – more of this when we get to DEVICES)

Resistivity and Resistance

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Electrical resistivity

Quantifies how strongly a given material opposes the flow of current

Fundamental property (i.e. **generally** does not depend on the shape and size of the structure)

Unit: Ωm , Symbol: ρ (Greek symbol *rho*)

Electrical resistance

$$R = \frac{\rho l}{A}$$

Quantifies how strongly an object/structure opposes the flow of current

Higher resistance: Less current for the same force applied

Unit: Ohm (Ω – Greek symbol Omega), Symbol: R

Long and narrow channel will result in a **higher** resistance (though the actual value will depend on the resistivity of the material).

Conductors vs Insulators

Work out the resistance for each of the following cases (assume $\rho = 10^{-5} \Omega\text{m}$)

Case 1: $L = 10 \text{ cm}$ (0.1 m), $A = 1 \text{ cm by } 1 \text{ cm}$ (10^{-4} m^2)

Case 2: $L = 1 \text{ mm}$ (10^{-3} m), $A = 0.001 \text{ mm by } 0.001 \text{ mm}$ (10^{-12} m^2)

Case 1: $R = 10^{-2} \Omega$;

Case 2: $R = 100000 \Omega$

We can see how the geometry makes a huge difference to the resistance even when the material is the same. So we **cannot use resistance** to determine if something is a conductor or insulator, or even how good it conducts current.



We will look into semiconductors when we get to DEVICES

Prefixes

From the previous example, we can already see that in EE we can expect to work with values that cover a wide range. Instead of using power indexes (e.g. 10^{-7} or 10^4) or using lots of zeroes, we use PREFIXES to keep the answers short and elegant!

You don't here people say 16 billion Bytes

Examples:

$$2.15 \times 10^{-3} \text{ A} =$$

$$4.7 \times 10^4 \Omega =$$

Prefixes: Memorize and **apply** them!

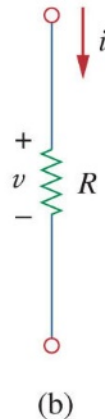
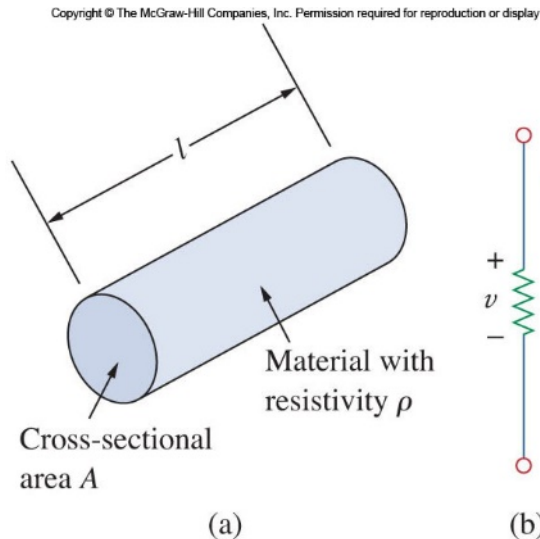
tera	T	10^{12}
giga	G	10^9
mega	M	10^6
kilo	k	10^3
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}

Electromotive force (EMF)

The force we have been talking about that pushes charges to move is referred to as the electromotive force (EMF)

EMF is often interchangeably used with voltage or potential difference

Voltage: Symbol (V), Unit – Volt (V)



Increasing the force will increase the drift velocity of the charged particles, thereby increasing current

$$V = IR$$

Ohm's law

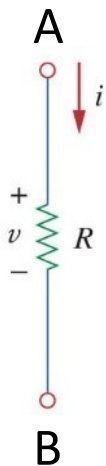
Note:

V refers to the voltage **across** the element

I refers to the current **through** that element

I defined to flow from higher to lower voltage

Voltage sign convention



Look at how the + and – signs are arranged around the symbol v

It means that the point A is higher than B by a voltage of v

If $v = 1\text{V}$, point A is higher than B by 1V

If $v = -1\text{V}$, point A is higher than B by -1V

If we assigned voltage points to A and B:

$$V_A - V_B = iR$$

$$V_{AB} = V_A - V_B$$

V_{AB} is the voltage of point A relative to point B (stand at point B and look at A). If voltage at A is lower than B, then $V_{AB} < 0$.

What is wrong with writing: $V_B - V_A = iR$ for the diagram above?

Electrical Power

Does mixing up the signs for the voltages and current matter?

Significance of direction of current and voltage difference:

When V **drops** in the direction of the arrow for I: Power consumed

When V **rises** in the direction of the arrow for I: Power generated

Electrical Power:

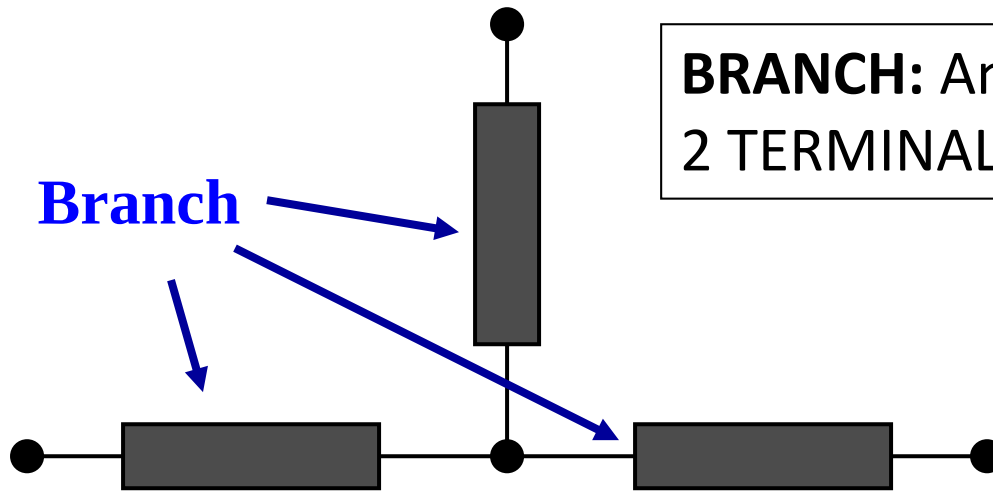
Energy supplied or provided per unit second (i.e. rate)

For the same power consumed, the energy consumed increases proportionally over time

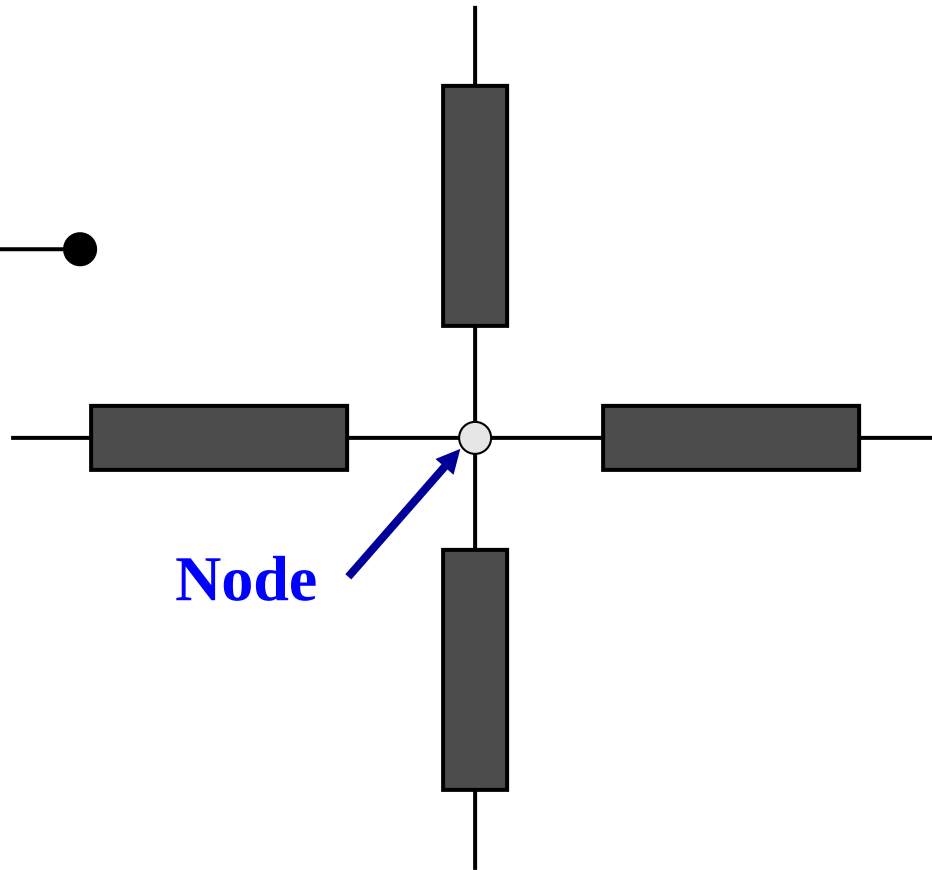
Symbol – P, Unit – Watt (W)

$$P = VI$$

Terminology: Branch and Node

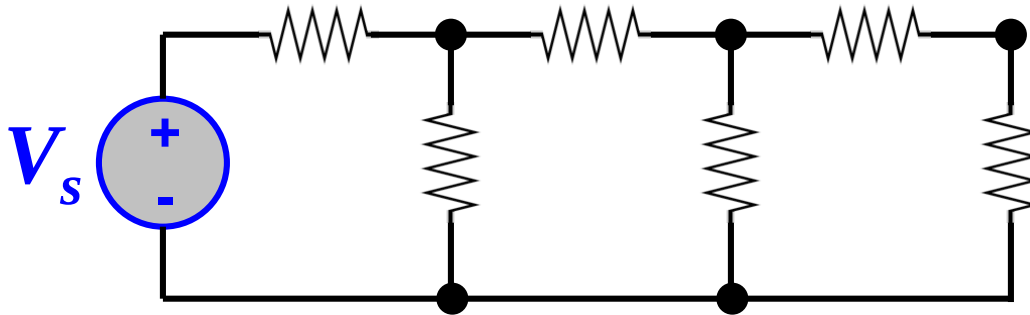


BRANCH: Any path of a circuit with 2 TERMINALS connected to it



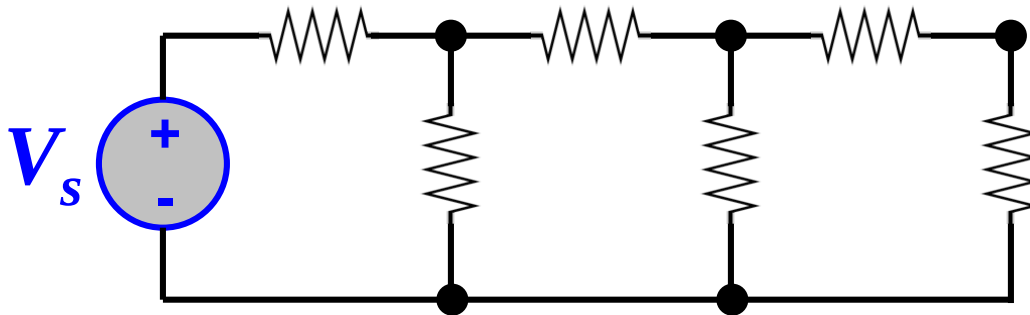
NODE: Junction of 2 or more branches

Terminology: Loop and Mesh



LOOP: Any closed connection of branches

In the above circuit, there are 6 loops in total



Mesh: Loop that does not contain other loops

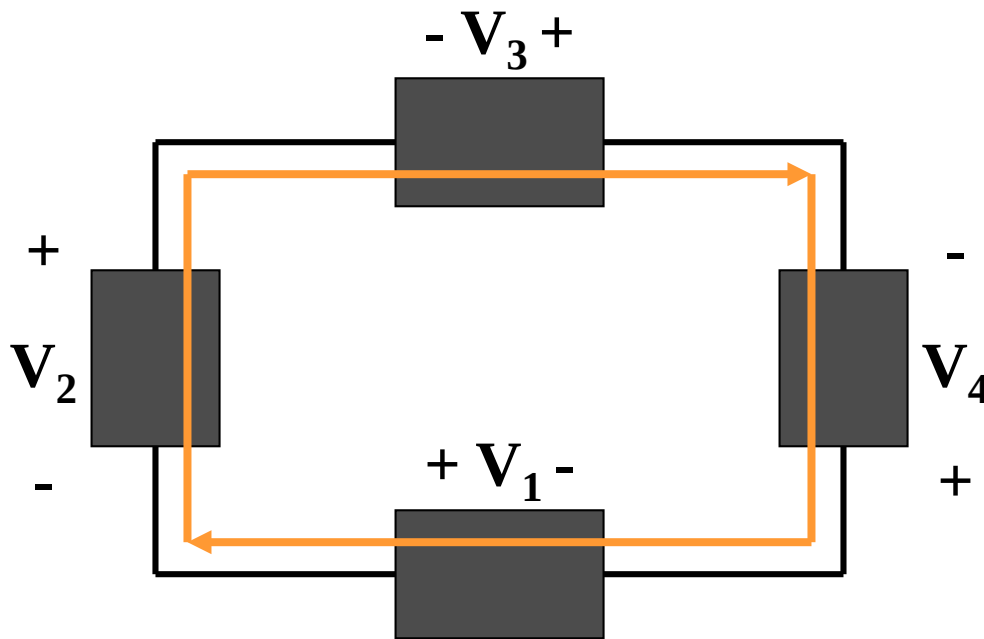
In the above circuit, there are 3 meshes in total

Kirchhoff's voltage law (KVL)

$\sum V_j = 0$ Sum of voltages in a loop is 0

Applying KVL to the example below:

$$V_1 + V_2 + V_3 + V_4 = 0$$



If we go around the loop in the direction indicated here (clockwise):

V_1 : Rise (+ sign)

V_2 : Rise (+ sign)

V_3 : Rise (+ sign)

V_4 : Rise (+ sign)

Kirchhoff's voltage law

$\Sigma V_j = 0$ Sum of voltages in a loop is 0

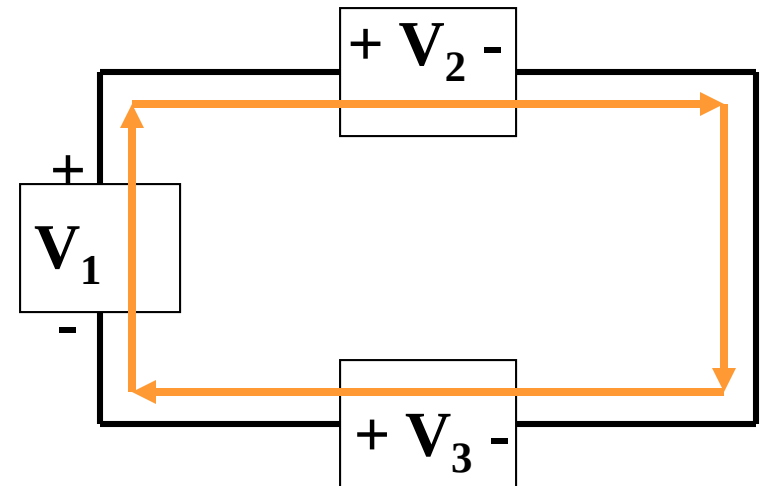
Applying KVL to the example below: $V_1 - V_2 + V_3 = 0$

If we go around the loop in the direction indicated here (clockwise):

V_1 : Rise (+ sign)

V_2 : Drop (- sign)

V_3 : Rise (+ sign)



If we went around the loop anti-clockwise:

V_1 :

V_2 :

V_3 :

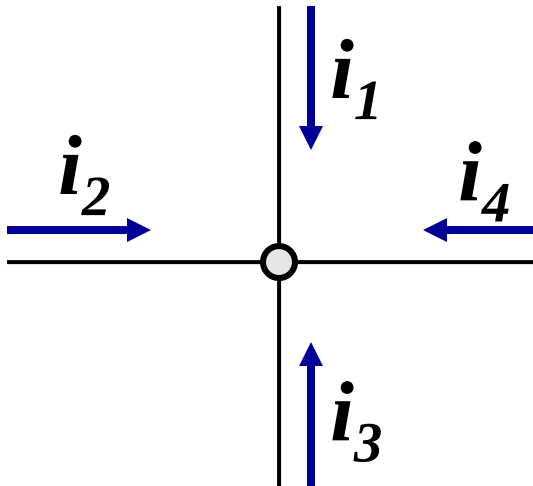
$$-V_1 + V_2 - V_3 = 0$$

Kirchhoff's current law (KCL)

$\Sigma I_j = 0$ Sum of currents flowing into a node is 0

Applying KCL to the example below:

$$i_1 + i_2 + i_3 + i_4 = 0$$



i_1 : In flow (+ sign)

i_2 : In flow (+ sign)

i_3 : In flow (+ sign)

i_4 : In flow (+ sign)

Implication:

At any given node,

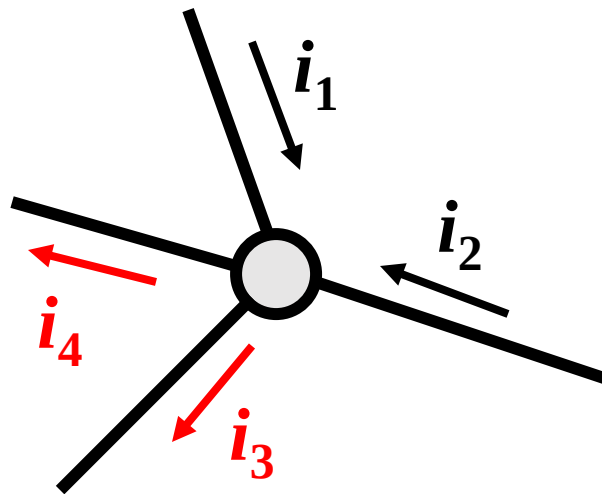
$$\Sigma I (\text{leaving}) = \Sigma I (\text{enter})$$

Kirchhoff's current law

Sign convention when applying KCL

Currents flowing OUT of node (- sign)

Currents flowing INTO node (+ sign)



i_1 : In flow (+ sign)

i_2 : In flow (+ sign)

i_3 : Out flow (- sign)

i_4 : Out flow (- sign)

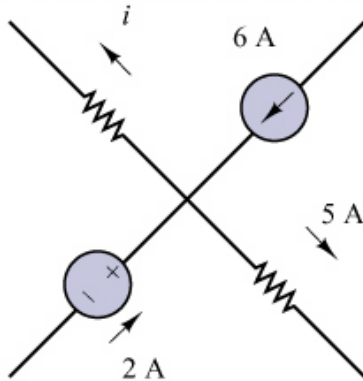
$$i_1 + i_2 - i_3 - i_4 = 0$$

KCL example

Problem 2.14 and 2.15

Find the unknown current using KCL

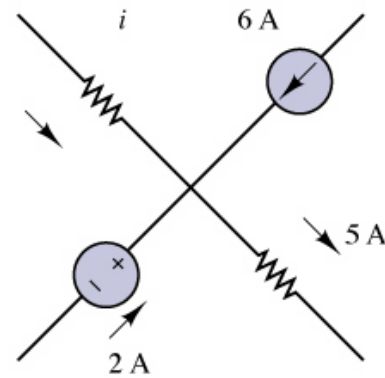
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$$6\text{A} - 5\text{A} + 2\text{A} - i = 0$$

$$\Rightarrow i = 3\text{A}$$

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$$6\text{A} - 5\text{A} + 2\text{A} + i = 0$$

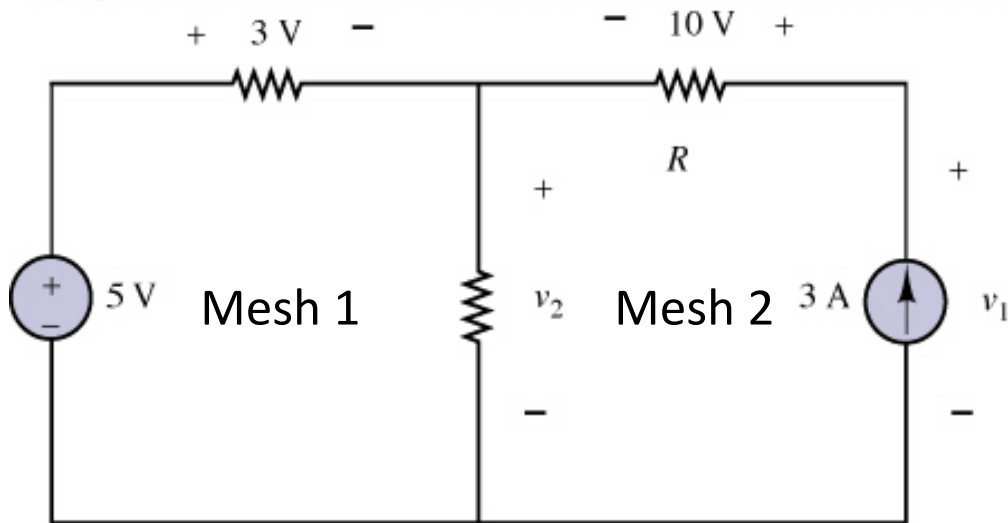
$$\Rightarrow i = -3\text{A}$$

KVL example

Problem 2.16

Apply KVL to find voltage V_1 and V_2

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Mesh 1 (Clockwise):

$$5V - 3V - V_2 = 0$$

$$\Rightarrow V_2 = 2V$$

Loop 2 (Anticlockwise):

$$V_1 - 10V - V_2 = 0$$

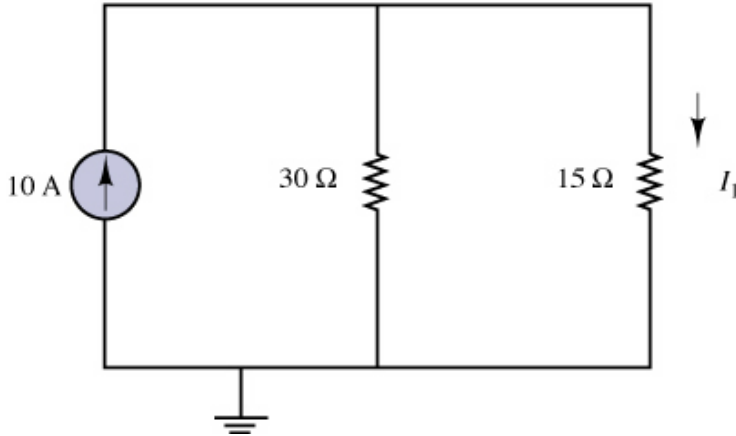
$$\Rightarrow V_1 = 12V$$

Example: KCL + Ohm's

Problem 2.17

Use Ohm's law + KCL to find the current through the 15Ω resistor

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$$\text{KCL: } I_1 + I_2 = 10\text{A}$$

$$\text{Ohm's: } 15I_1 = V_{15\Omega} \text{ (1); } 30I_2 = V_{30\Omega} \text{ (2)}$$

$$\text{KVL: } V_{15\Omega} = V_{30\Omega}$$

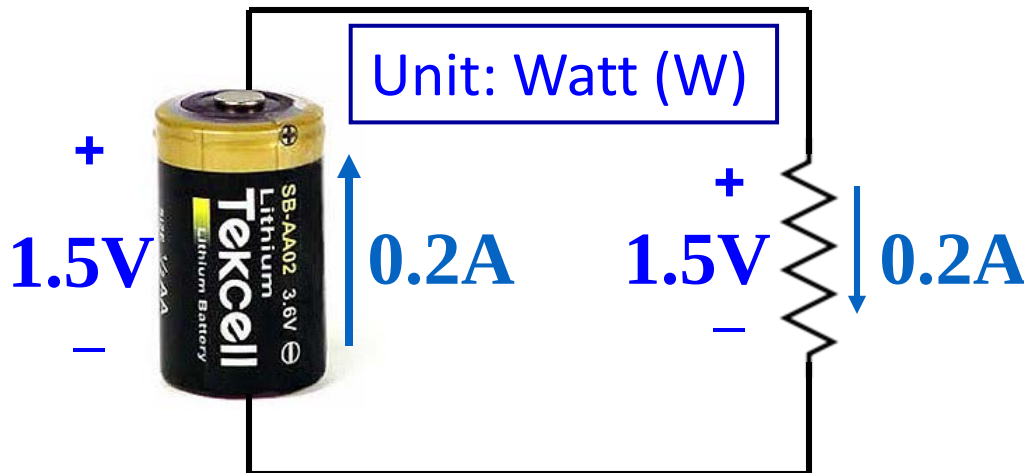
$$\text{Therefore, } 2I_2 = I_1$$

Solving for the variables:

$$I_2 = 3.33 \text{ A, } I_1 = 6.67 \text{ A}$$

Electrical Power

Electrical power generated/dissipated in a given element is defined by the product of the voltage across that element and the current through it



$$P = VI$$
$$P = I^2 R$$
$$P = V^2 / R$$

Source: $P = 1.5V \times 0.2A = 0.3W$

Load: $P = 1.5V \times 0.2A = 0.3W$

Power generated by source

MUST EQUAL

Power dissipated in the load

- In a source, power is generated
- In a load (eg. resistor), power is dissipated/consumed

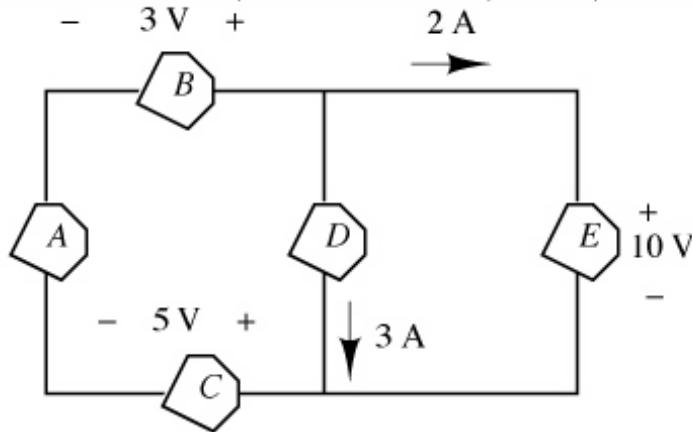
Power example

Problem 2.22

Determine which components are absorbing power and which are delivering power

Is conservation of power observed in this example?

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First find the voltage drops across each element

Apply KVL,

Loop 2: $D = 10V$

Loop 1: $10V - 3V - V_A + 5V = 0 \Rightarrow V_A = 12V$

Next find all branch currents

Apply KCL: $I = 5A$

Power example solution

Finally calculate power through each element

$$A: (12V)(5A) = 60W \text{ [generating]}$$

$$B: (3V)(5A) = 15W \text{ [generating]}$$

$$C: (-5V)(5A) = -25W \text{ [absorbing]}$$

$$D: (-10V)(3A) = -30W \text{ [absorbing]}$$

$$E: (-10V)(2A) = -20W \text{ [absorbing]}$$

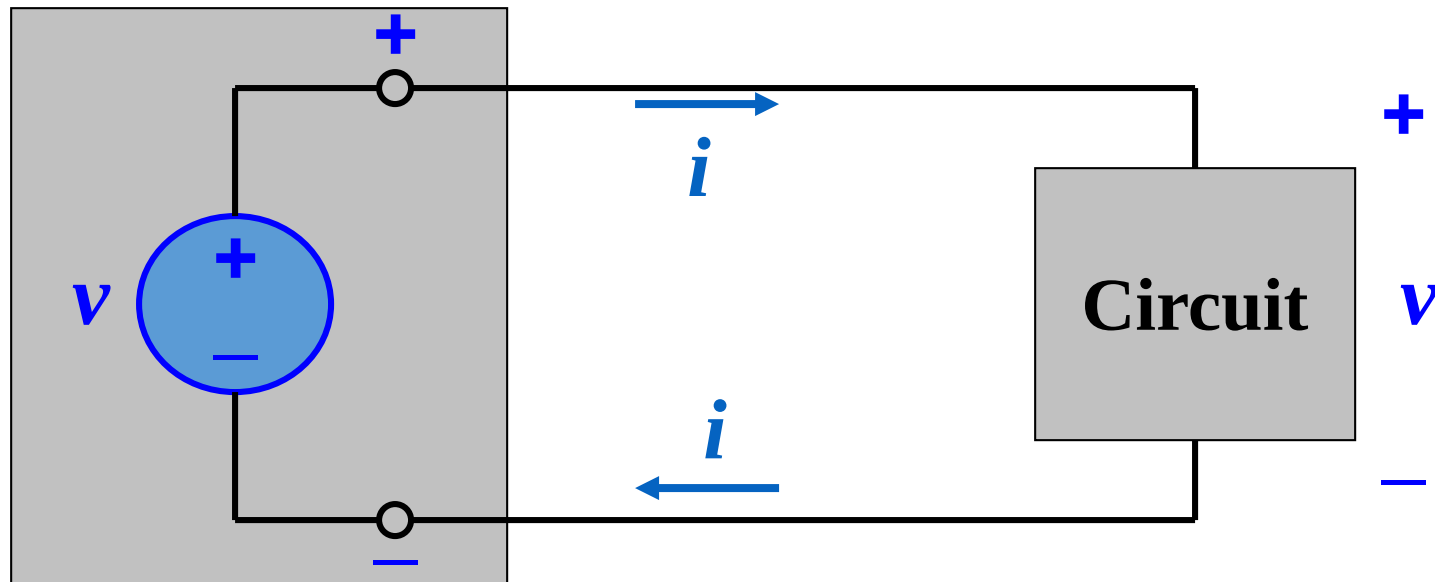
Last part: “Is power conserved?”

$$\text{Generating: } 60W + 15W = 75W$$

$$\text{Absorbing: } 25W + 30W + 20W = 75W$$

YES!

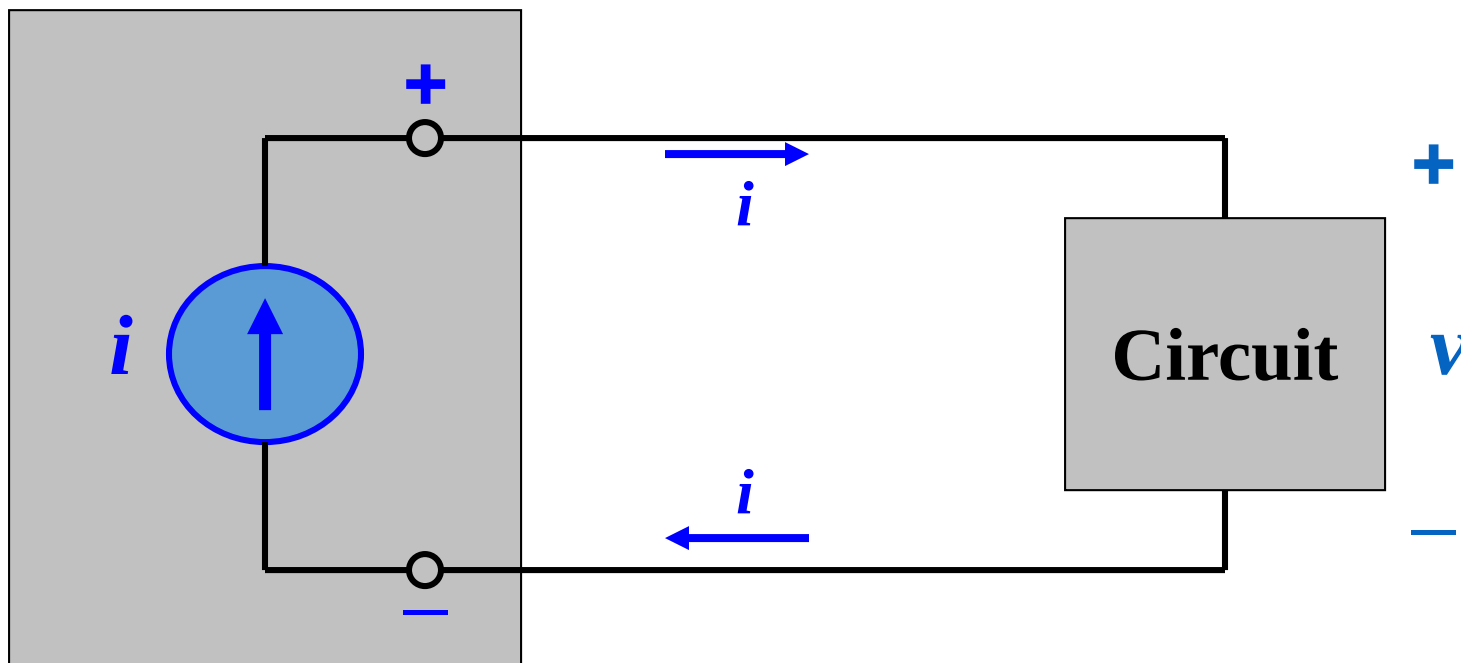
Independent Voltage Source



Voltage across the source is fixed

Current through the source can take any value (depending on the circuit connected across the source)

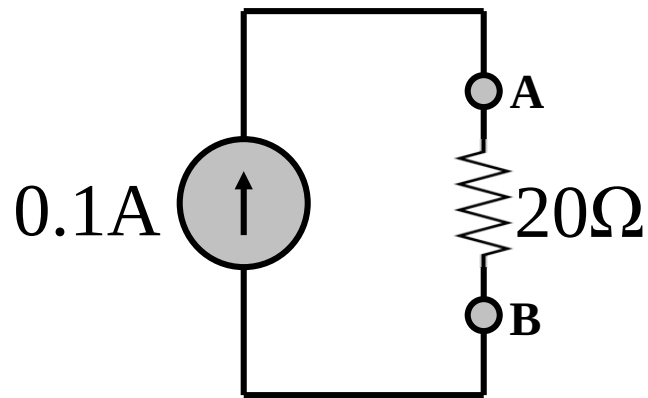
Independent Current Source



Current through the source is fixed

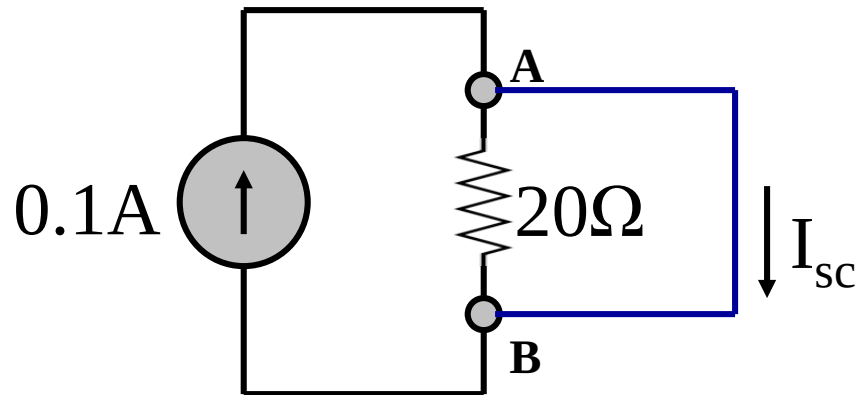
Voltage across the source can take any value (depending on the circuit connected across the source)

Short Circuit



Short circuit: Connect 2 or more terminals so that the voltage between each of them is the same
Typically associated with current, eg. short-circuit current

Short circuit A & B

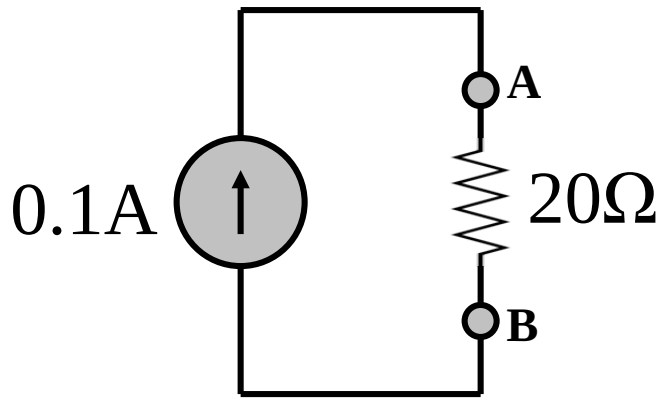


20Ω resistor is now bypassed

Current from source flows through the short-circuit to give a short-circuit current (I_{sc})

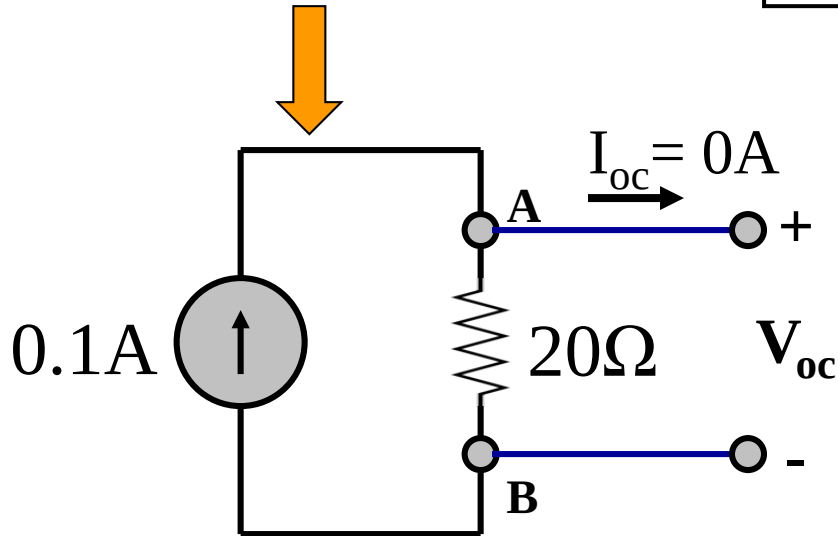
Short circuit current $I_{sc} = 0.1A$

Open Circuit



Open circuit: Leave 2 terminals unconnected **externally**

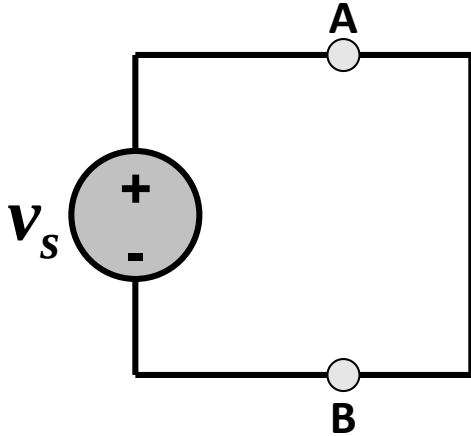
Typically associated with voltage, eg. open-circuit voltage



Open circuit voltage

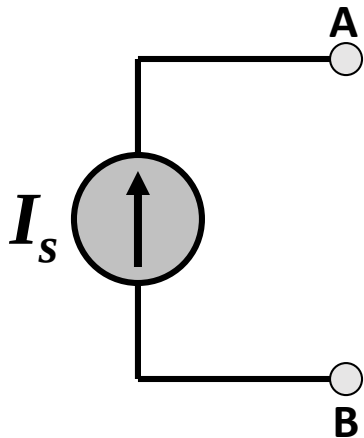
$$V_{oc} = 0.1A \times 20\Omega = 2V$$

Self-contradictory circuits



What is the voltage across A and B?

Voltage sources in parallel?



What is the current arriving at A?

Current sources in series?

Parallel resistor network

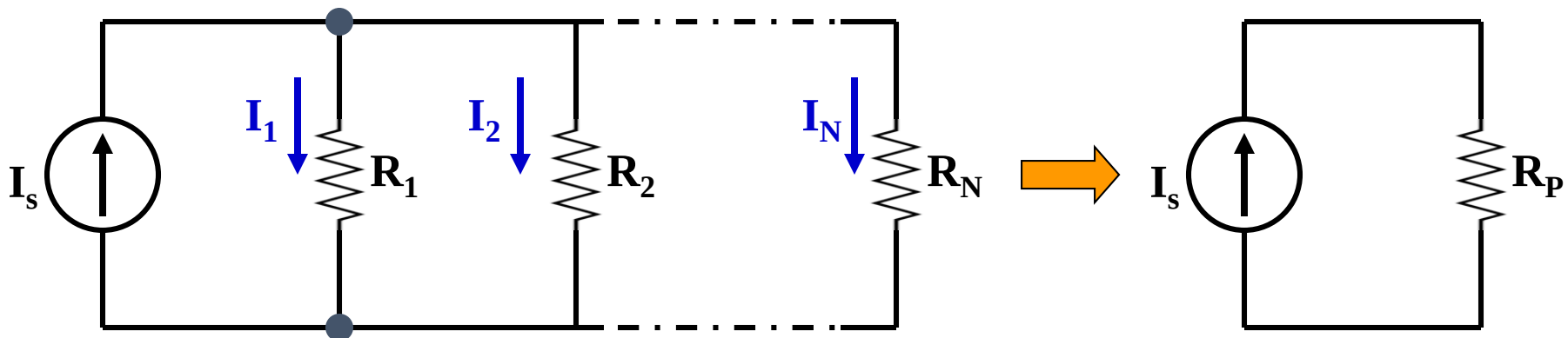


Figure above shows N resistors arranged in parallel

Resistors in parallel start at the same node (node A in the figure) and end at the same node (node B in the figure)

Notice that I_s divides itself among the resistors when it flows into node A but the divided currents will meet again node B to give the same I_s .

Parallel resistor network tricks

We can merge all the parallel resistors and replace them with one effective resistor (R_p) that represents that net effect:

Equivalent Resistance

$$1/R_p = 1/R_1 + 1/R_2 + \dots + 1/R_N$$

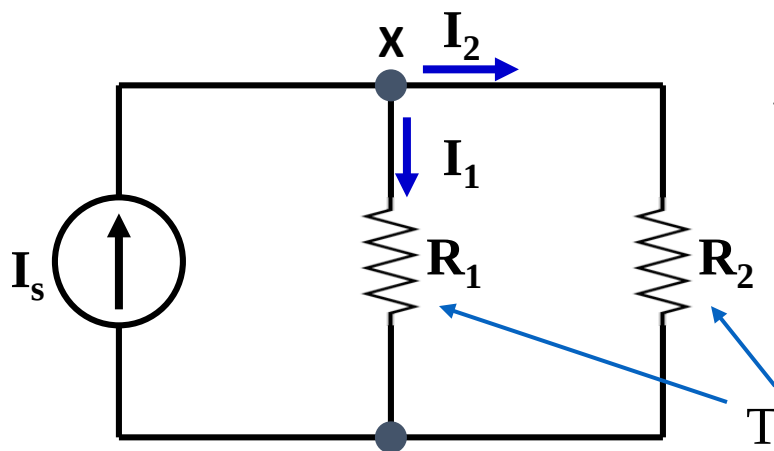
The distribution of currents between each resistor is determined by the resistor values governed by the current divider rule:

Current divider rule

$$I_N = \frac{1/R_N}{1/R_p} I_S$$

The smallest resistor gets the largest share of current among all the resistors in parallel

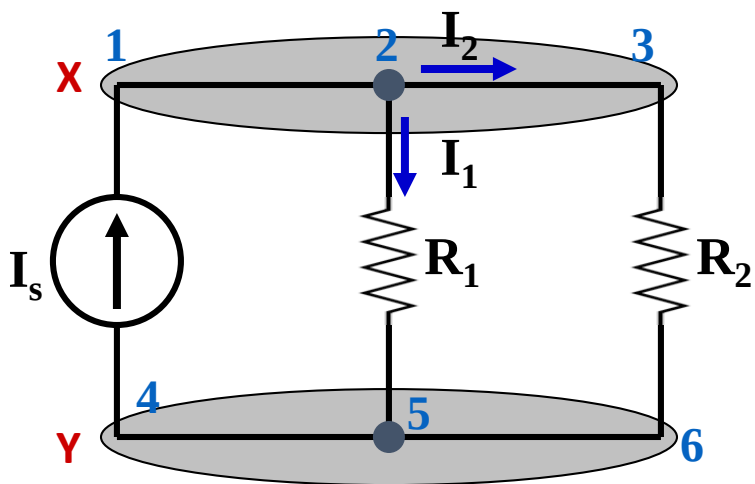
Parallel Network: Proof



Apply Kirchhoff's current law (KCL) at X:

$$I_s = I_2 + I_1$$

This can be seen as I_s is split into the 2 branches

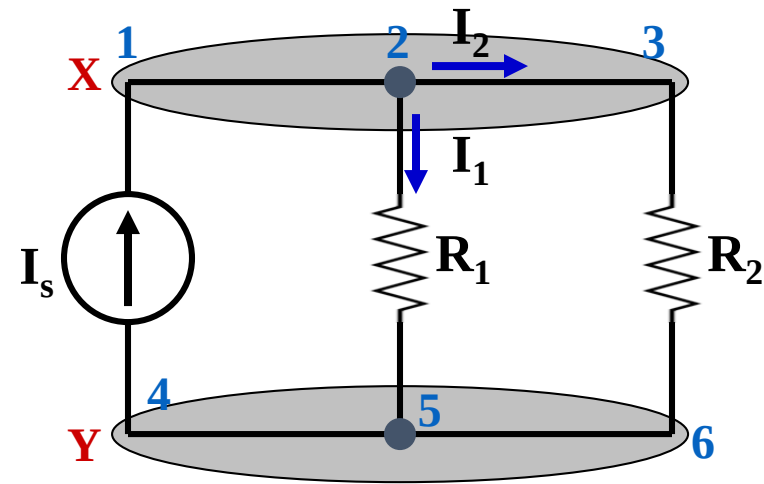


Note that points 1, 2, 3 are all at the same voltage, therefore same node (X)

Points 4, 5, 6 are all at the same voltage, therefore same node (Y)

Voltage across R_1 = Voltage across R_2 = V_{xy}

Parallel network: Proof



Apply Ohm's to both R_1 and R_2 :

$$V_{XY} = I_1 R_1 = I_2 R_2$$

$$I_1 = V_{XY}/R_1; I_2 = V_{XY}/R_2$$

Adding I_2 & I_1 up according to KCL:

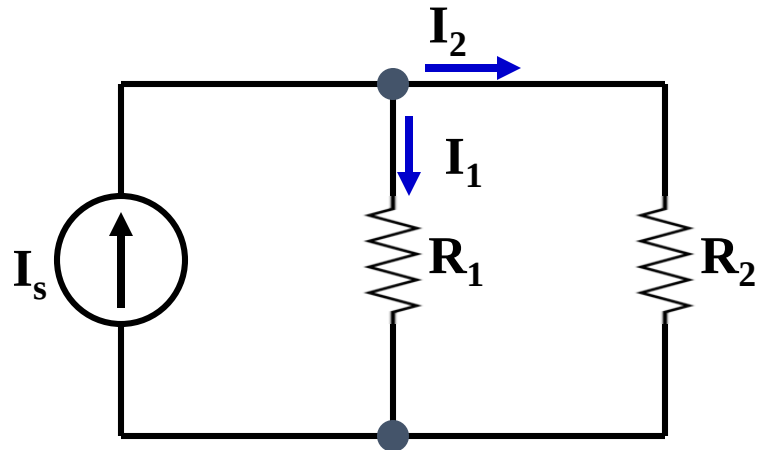
$$I_s = V_{XY}(1/R_1 + 1/R_2)$$

We can now find how much of I_s is distributed between the 2 branches

$$I_1 = \left(\frac{1/R_1}{1/R_1 + 1/R_2} \right) I_s \quad I_2 = \left(\frac{1/R_2}{1/R_1 + 1/R_2} \right) I_s$$

This is referred to as the **current divider rule**

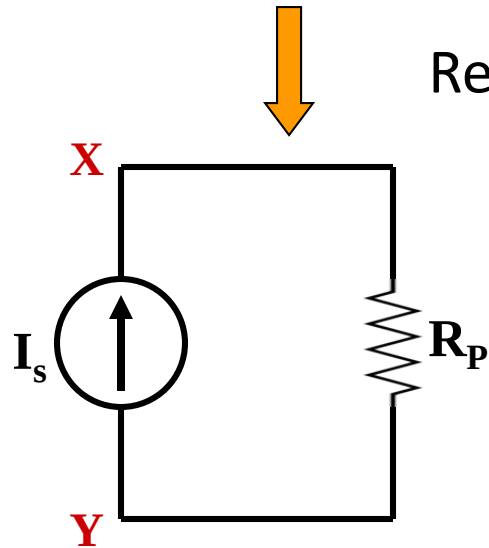
Parallel network: Proof



$$I_s = V_{XY}(1/R_1 + 1/R_2)$$

Replace the parallel network of resistors with a **single equivalent resistor**

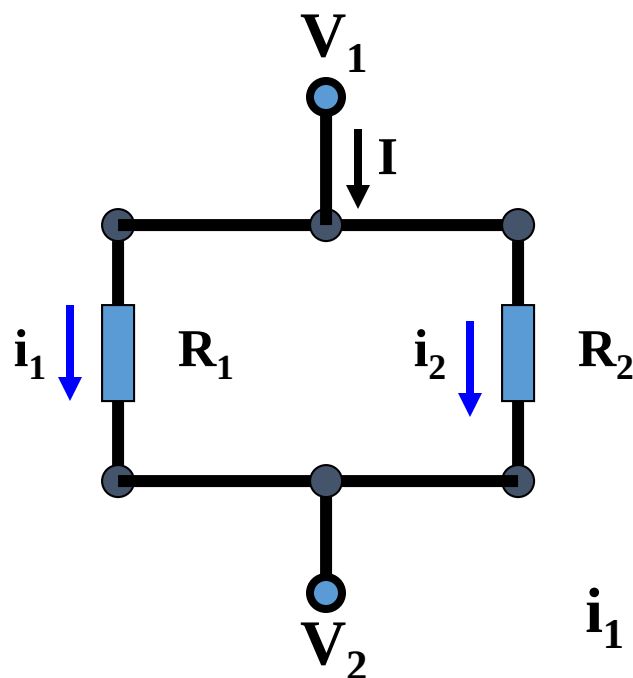
Remember that the voltage across R_p is still V_{XY} !



$$V_{XY}/R_p = V_{XY}(1/R_1 + 1/R_2)$$

$$1/R_p = 1/R_1 + 1/R_2$$

Current divider: Example 1



Find i_1 and i_2 in terms of I

If $R_1 = R_2$, find the ratio between i_1 and i_2

If $R_2 = 3R_1$, find the ratio between i_1 and i_2

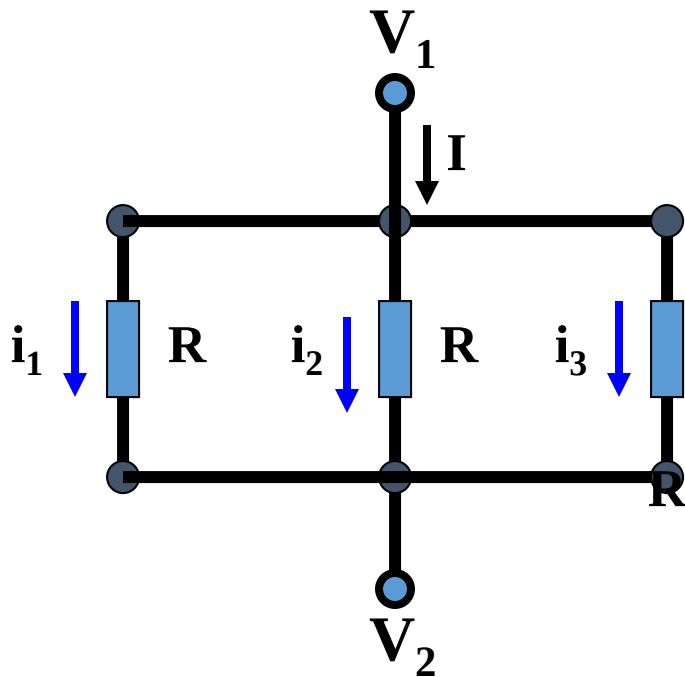
$$i_1 = [R_2 / (R_1 + R_2)]I; i_2 = [R_1 / (R_1 + R_2)]I$$

If $R_1 = R_2$, then $i_1 = i_2 = I/2$

If $R_2 = 3R_1$, then $i_1 = 3I/4$, $i_2 = I/4$

Therefore $i_1 = 3i_2$

Current divider: Example 2



Find the current through each resistor in terms of I

Find the current through each resistor in terms of I if a 4th resistor was added in parallel

How many resistor are required to reduce the current in each resistor to 1% of I (ie $1/100$)

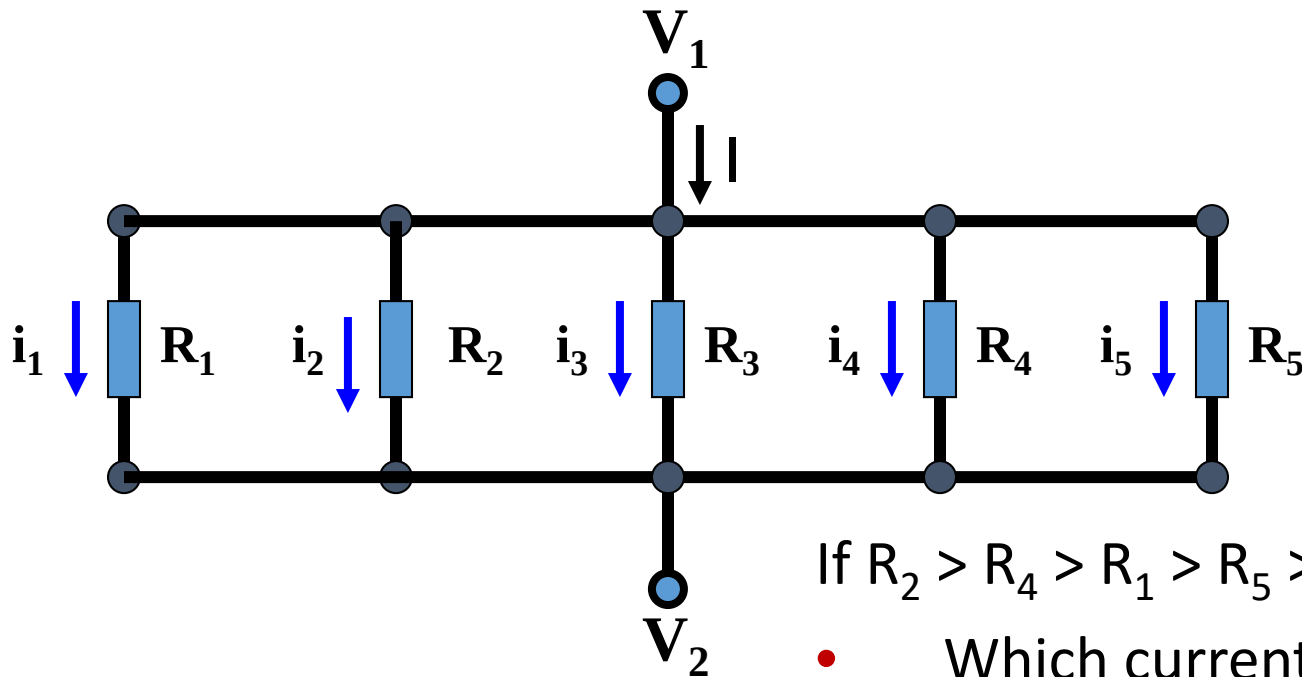
$$i_1 = i_2 = i_3 = I/3$$

$$i_1 = i_2 = i_3 = i_4 = I/4$$

Consider that, $i_n = I/n$

Hence $n = 100$ for $i_n = 0.01I$

Current divider: Example 3



If $R_2 > R_4 > R_1 > R_5 > R_3$

- Which current is the largest?
- Which current is the smallest?
- Rank the currents from largest to smallest

$$I_3 > I_5 > I_1 > I_4 > I_2$$

Series resistor network

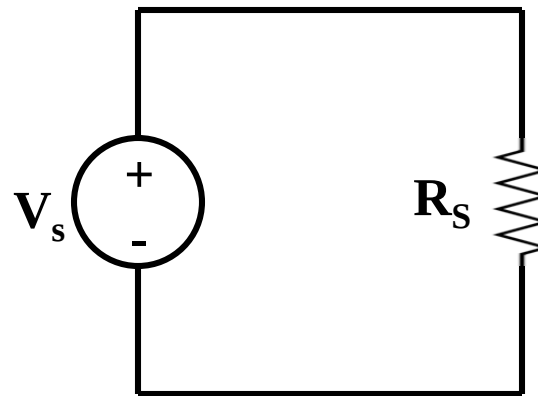
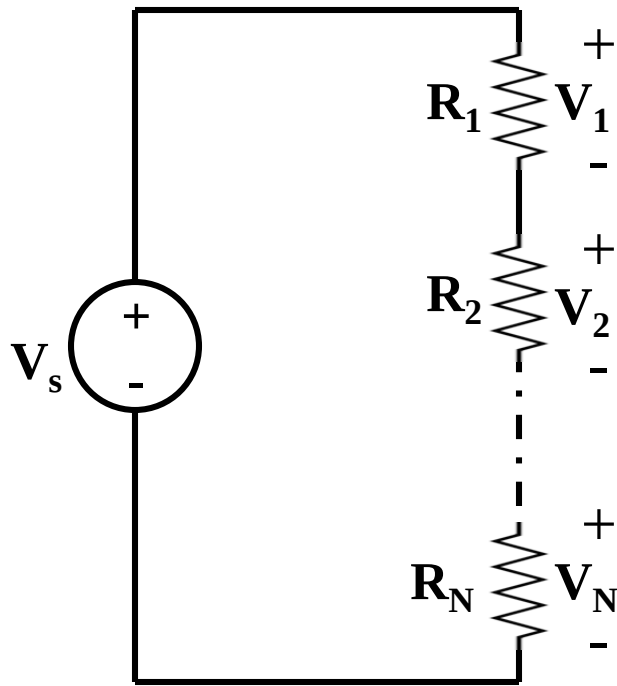


Figure on left shows N resistors arranged in series

For resistors in series, the current in one resistor must be the same as the next resistor

Notice that V_s divides itself among the resistors

Series resistor network

We can merge all the series resistors and replace them with one effective resistor (R_S) that represents that net effect:

Equivalent Resistance

$$R_S = R_1 + R_2 + \dots + R_N$$

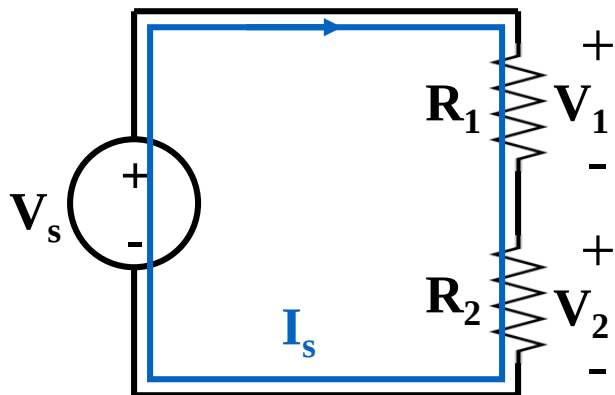
The distribution of EMF between each resistor is determined by the resistor values governed by the voltage divider rule:

Voltage divider rule

$$V_N = V_S(R_N/R_S)$$

The largest resistor gets the largest share of voltage among all the resistors in series

Series network: Proof



Apply Ohm's law to both resistors:

$$V_1 = I_s R_1; V_2 = I_s R_2$$

Adding up V_1 and V_2 according to KVL:

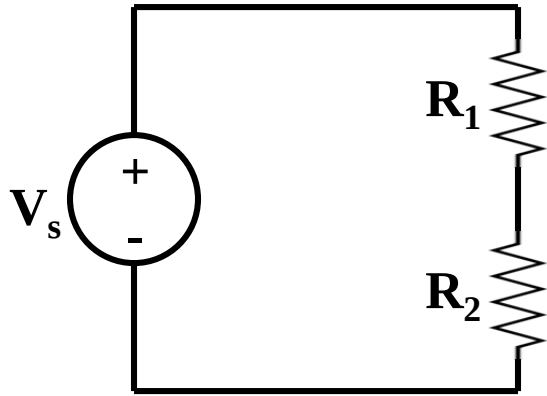
$$V_s = I_s(R_1 + R_2)$$

We can now find how much of V_s is distributed between the 2 resistors

$$V_1 = \left(\frac{R_1}{R_1 + R_2} \right) V_s \quad V_2 = \left(\frac{R_2}{R_1 + R_2} \right) V_s$$

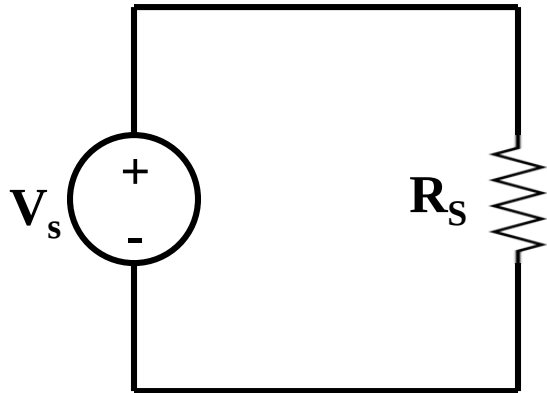
This is referred to as the **voltage divider rule**

Series network: Proof



$$V_s = I_s(R_1 + R_2)$$

Replace the series network of resistors with a **single equivalent resistor**

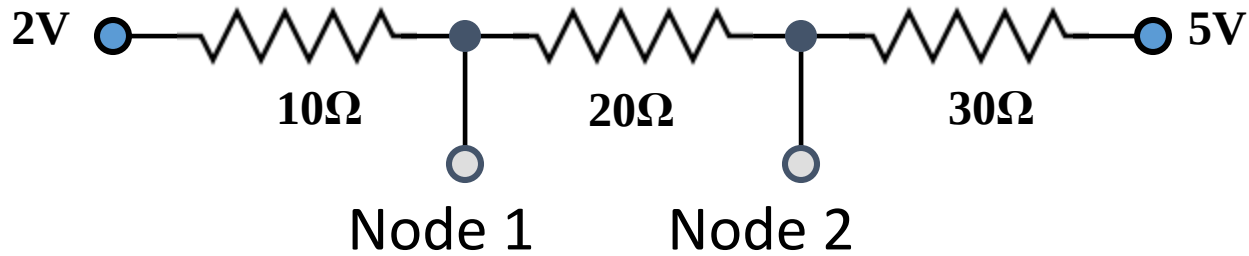


Remember that the current going into R_s is still I_s !

$$I_s R_s = I_s(R_1 + R_2)$$

$$R_s = R_1 + R_2$$

Voltage Divider: Example 1



- Find the voltage across each resistor
- Find the voltage at nodes 1 and 2

Total resistance (in series) $R_T = 10 + 20 + 30 = 60\Omega$

$$V_{10\Omega} = (1/6)(3) = 0.5V; V_{20\Omega} = (1/3)(3) = 1V; V_{30\Omega} = (1/2)(3) = 1.5V$$

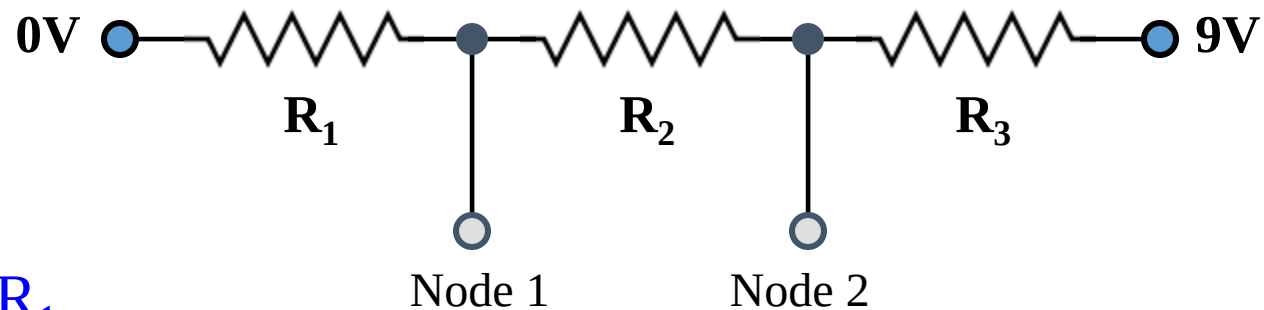
At node 1:

$$V_1 = 2 + 0.5 = 2.5V$$

At node 2:

$$V_2 = 5 - 1.5 = 3.5V$$

Voltage Divider: Example 2



If $R_3 = 3R_1$ and $R_2 = 2R_1$

- Find the voltage across each resistor
- Find the voltage at nodes 1 and 2

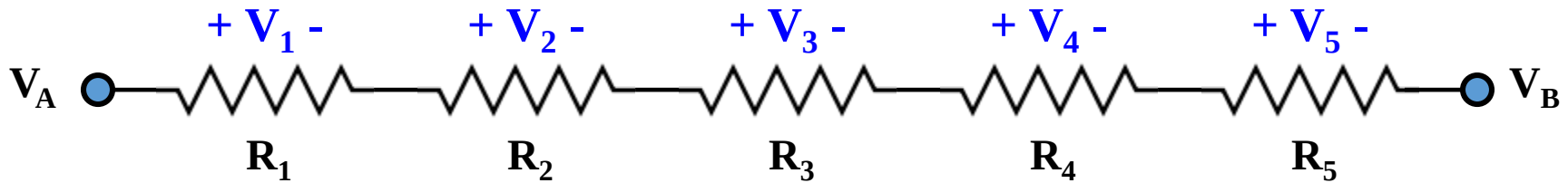
$$V_{R1} = (R_1/R_T)(V_B - V_A) = 9/6 = 1.5V; V_{R2} = (R_2/R_T)(V_B - V_A) = 9/3 = 3V$$

$$V_{R3} = (R_3/R_T)(V_B - V_A) = 9/2 = 4.5V$$

$$\text{At node 1: } V_1 = V_{R1} + V_A = V_B/6$$

$$\text{At node 2: } V_2 = V_B - V_{R3} = V_B/2$$

Voltage Divider: Example 3



If $R_2 > R_4 > R_1 > R_5 > R_3$

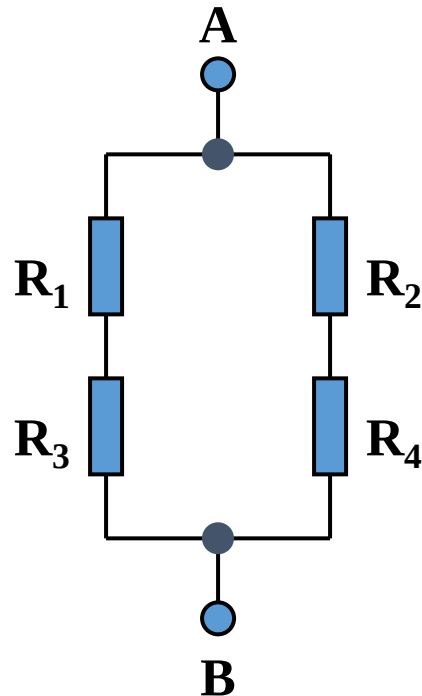
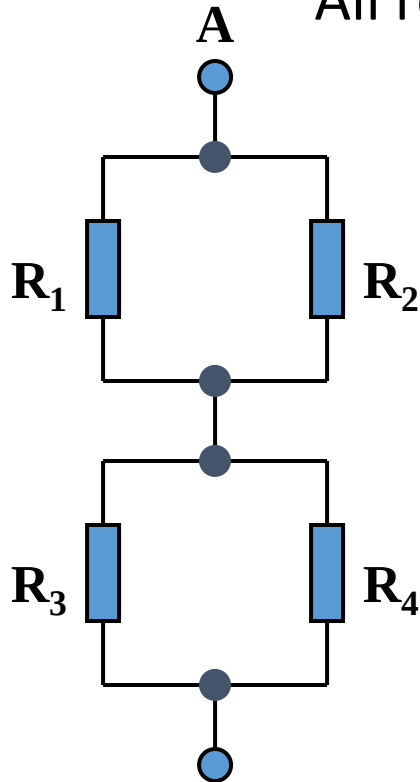
- Which voltage difference is the largest?
- Which voltage difference is the smallest?
- Rank the voltage differences from largest to smallest

$$V_2 > V_4 > V_1 > V_5 > V_3$$

Series-Parallel combo: Example 1

Find the total resistance as seen across A and B

All resistors have value of 1Ω



$$R_{AB} = (R_1 \parallel R_2) + (R_3 \parallel R_4) \\ = 1 \parallel 1 + 1 \parallel 1 = 1\Omega$$

$$R_{AB} = (R_1 + R_3) \parallel (R_2 + R_4) \\ = 2 \parallel 2 = 1\Omega$$

B

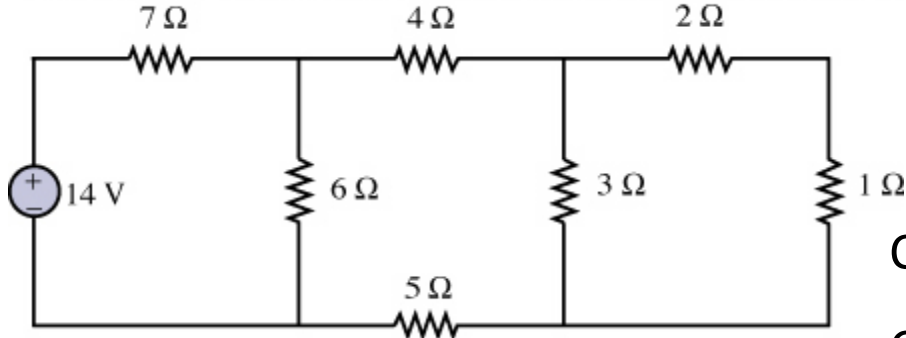
Combo example 2

Problem 2.48

Find the equivalent resistance seen by the source

How much power is delivered by the source?

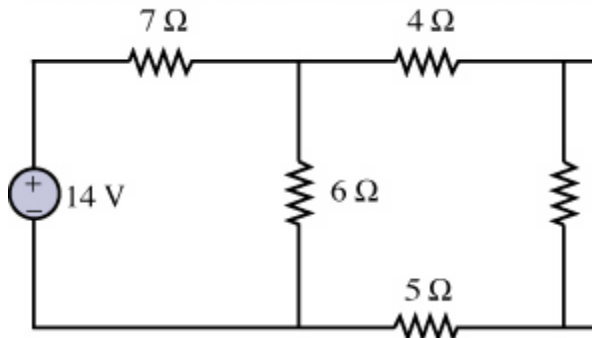
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Combine 1Ω and 2Ω in series: $R_1 = 3\Omega$

Combine R_1 with 3Ω in parallel: $R_2 = 1.5\Omega$

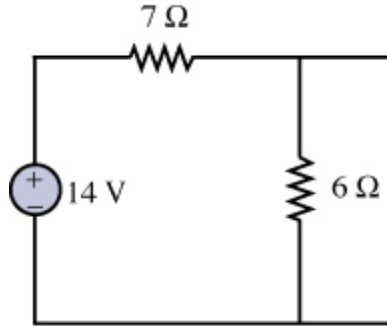
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Combine R_2 with 4Ω & 5Ω in series: $R_3 = 10.5\Omega$

Combo example 2

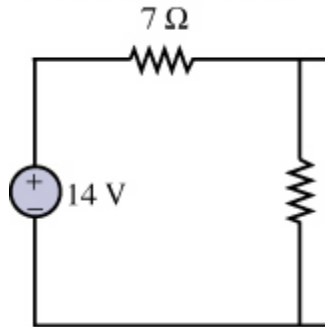
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Combine R_3 with 6Ω in parallel: $R_4 = 3.818\Omega$

Combine R_4 with 7Ω in series: $R_p = 10.818\Omega$

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Note that V_s is dropped across R_p
(i.e. the voltage across R_p is V_s)

Power delivered by source must be
equal to power consumed by R_p

Power delivered by source: $P = V_s^2/R_p = 18.1W$

Combo example 3

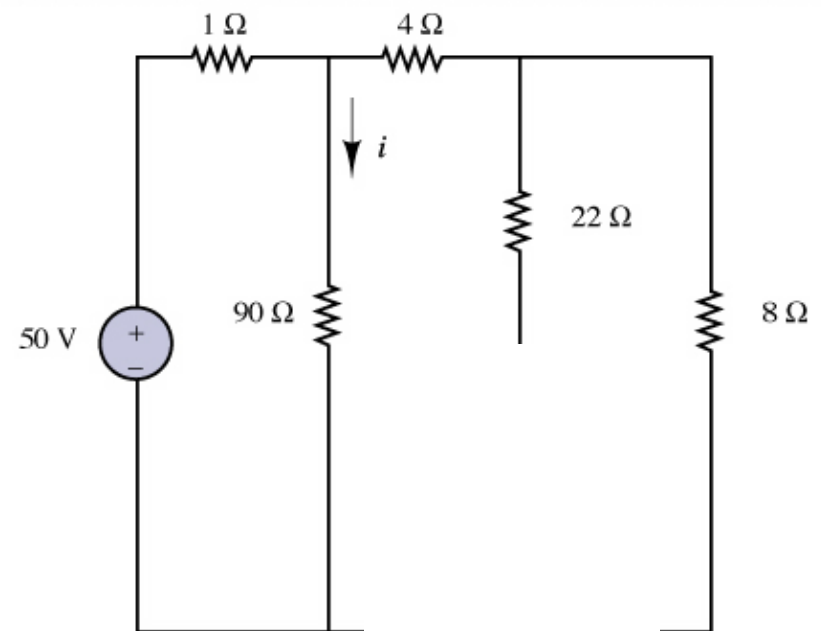
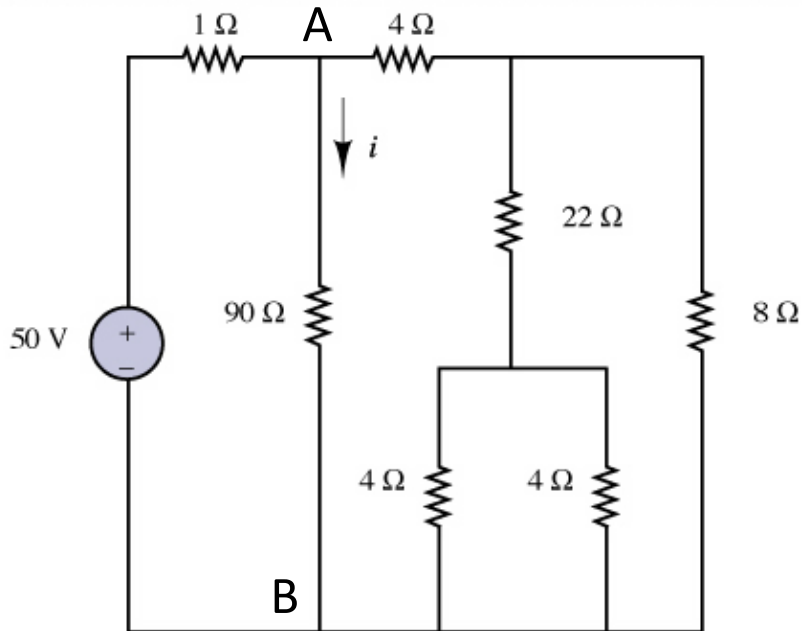
Problem 2.48

Find the equivalent resistance seen by the source

Find the current through the 90Ω resistor

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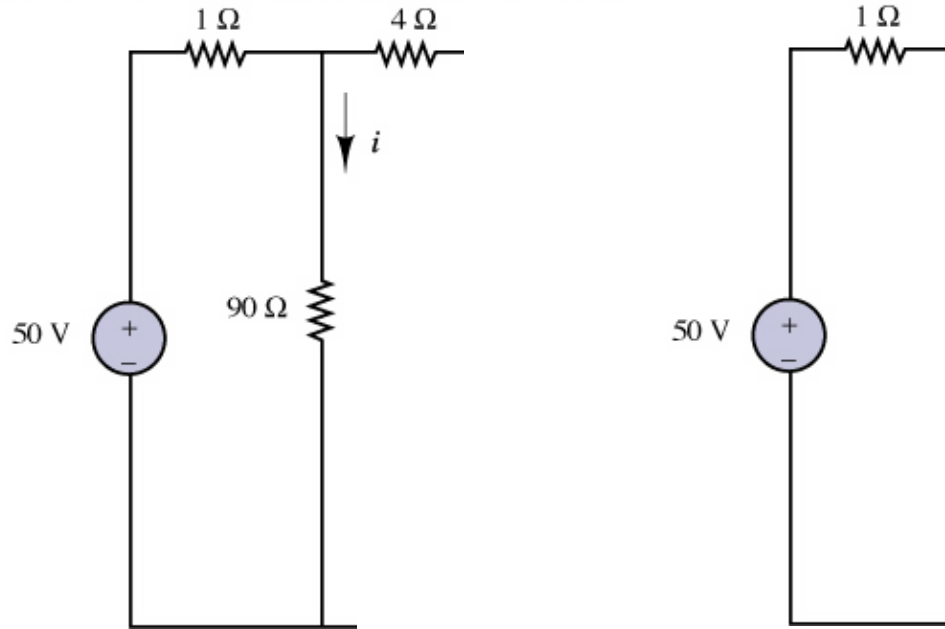
Combine the two 4Ω in parallel: $R_1 = 2\Omega$

Combine R_1 with 22Ω in series: $R_2 = 24\Omega$

Combine R_2 with 8Ω in parallel: $R_3 = 6\Omega$

Combo example 3

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What is the relationship between the voltage across 90Ω and R_X ?

Combine R_3 with 4Ω in series: $R_4 = 10\Omega$

Combine R_4 with 90Ω in parallel: $R_5 = 9\Omega$

Combine R_5 with 1Ω in series: $R_X = 10\Omega$

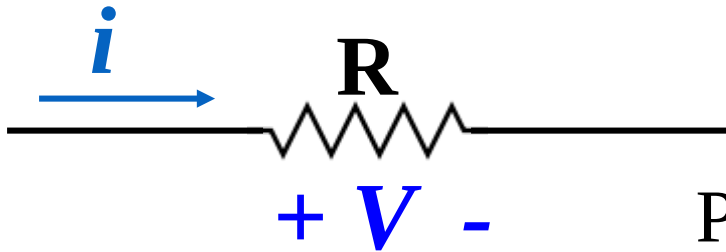
Voltage across 90Ω , $V_{AB} = (9/10) \cdot 50 = 45V$

Finally,

$$I = 45/90 = 0.5A$$

Power dissipation

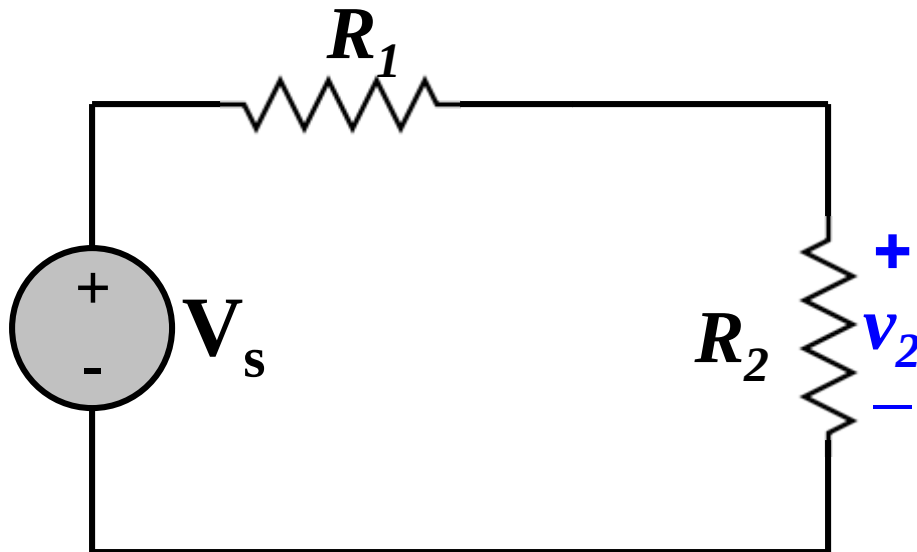
Power: $P = V I$



Power dissipated in resistor R

$$P = I^2 R = V^2 / R$$

Power dissipated in R_2 : $P = v_2^2 / R_2$



$$P = \frac{R_2}{(R_1 + R_2)^2} V_s^2$$

For Max Power Transfer:

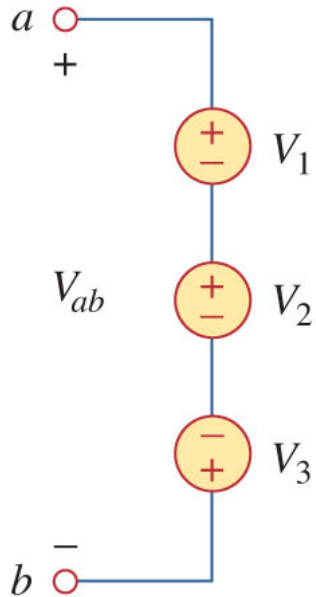
$$R_1 = R_2$$

Find condition for $dP/dR_2 = 0$ (max pt)

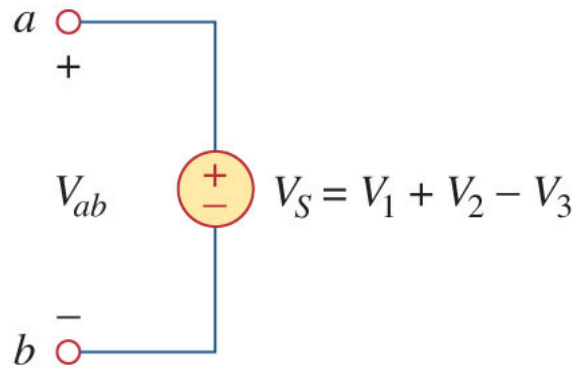
Sources in series and parallel

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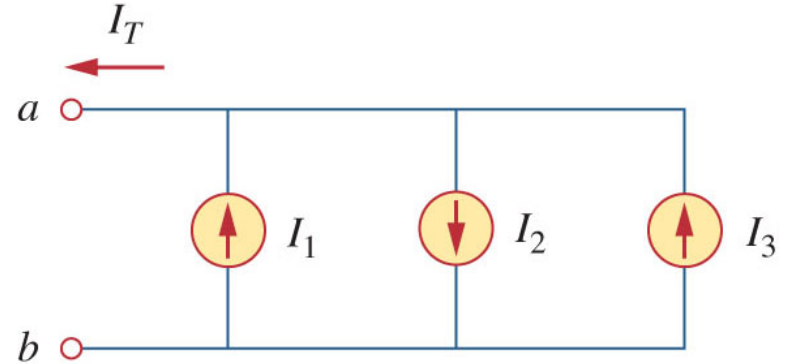


(a)

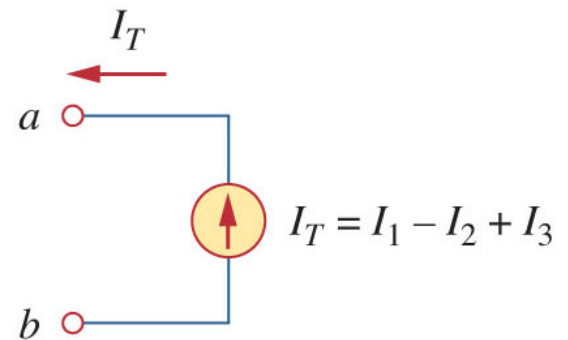


(b)

Apply KVL



(a)



(b)

Apply KCL