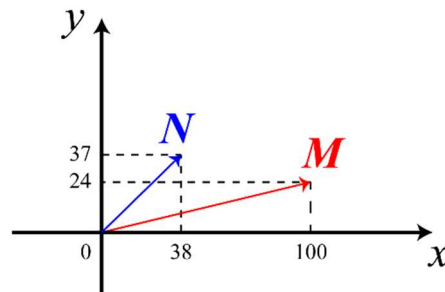


EE2104 Introduction to Electromagnetics
Assignment 1 Solution

Deadline (for uploading): 11:59 pm, 16 Sep 2021
Please put down your name and university ID.

1. Given the vectors $\mathbf{M} = 100\hat{x} + 24\hat{y}$ and $\mathbf{N} = 38\hat{x} + 37\hat{y}$. Draw the two vectors and then find:
- the unit vector pointing from $\mathbf{M}$ to $\mathbf{N}$;
 - $\mathbf{M} \cdot \mathbf{N}$;
 - the angle between $\mathbf{M}$ and $\mathbf{N}$;
 - $\mathbf{M} \times \mathbf{N}$; and
 - an unit vector perpendicular to both $\mathbf{M}$ and $\mathbf{N}$.

Solution:



- (a) The vector pointing from $\mathbf{M}$ to $\mathbf{N}$ can be expressed as

$$\mathbf{R} = \mathbf{N} - \mathbf{M} = -62\hat{x} + 13\hat{y}$$

We have

$$|\mathbf{R}| = \sqrt{62^2 + 13^2} = 63$$

Therefore, the unit vector pointing from $\mathbf{M}$ to $\mathbf{N}$ is

$$\frac{\mathbf{R}}{|\mathbf{R}|} = \frac{-62\hat{x} + 13\hat{y}}{63} = -0.98\hat{x} + 0.21\hat{y}$$

- (b)

$$\mathbf{M} \cdot \mathbf{N} = (100\hat{x} + 24\hat{y}) \cdot (38\hat{x} + 37\hat{y}) = 3800 + 888 = 4688$$

- (c) The angle between $\mathbf{M}$ and $\mathbf{N}$ is

$$\theta = \cos^{-1} \left(\frac{\mathbf{M} \cdot \mathbf{N}}{|\mathbf{M}||\mathbf{N}|} \right) = \cos^{-1} \left(\frac{4688}{\sqrt{100^2 + 24^2} \times \sqrt{38^2 + 37^2}} \right) = 0.54 \text{ (rad)} = 31^\circ$$

- (d)

$$\mathbf{M} \times \mathbf{N} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 100 & 24 & 0 \\ 38 & 37 & 0 \end{vmatrix} = 2788\hat{z}$$

- (e) The unit vector perpendicular to both $\mathbf{M}$ and $\mathbf{N}$ is

$$\frac{\mathbf{M} \times \mathbf{N}}{|\mathbf{M} \times \mathbf{N}|} = \frac{2788\hat{z}}{\sqrt{2788^2}} = \hat{z}$$

2. A field is given as

$$\mathbf{G}(x, y) = \frac{y\hat{x} - x\hat{y}}{x^2 + y^2}$$

Find:

- (a) the unit vector in the direction of $\mathbf{G}$ at $P(3, 3)$ and
- (b) the angle between $\mathbf{G}(x, y)$ and $\hat{x}$ at P .

Solution:

- (a) At point $P(3, 3)$, we have

$$\mathbf{G} = \frac{3\hat{x} - 3\hat{y}}{3^2 + 3^2} = 0.17\hat{x} - 0.17\hat{y}, \quad |\mathbf{G}| = \sqrt{0.17^2 + 0.17^2} = 0.24$$

Therefore, the unit vector in the direction of $\mathbf{G}$ at $P(3, 3)$ is

$$\frac{\mathbf{G}}{|\mathbf{G}|} = 0.71\hat{x} - 0.71\hat{y}$$

- (b) At point $P(3, 3)$, the angle between $\mathbf{G}$ and $\hat{x}$ is

$$\theta = \cos^{-1} \left(\frac{\mathbf{G} \cdot \hat{x}}{|\mathbf{G}| |\hat{x}|} \right) = \cos^{-1} \left[\frac{(0.17\hat{x} - 0.17\hat{y}) \cdot \hat{x}}{0.24 \times 1} \right] = 0.78 \text{ (rad)} = 45^\circ$$

3. For two vector fields

$$\begin{aligned}\mathbf{A}(\rho, \phi) &= \rho^{\frac{3}{2}} \cos\phi \hat{\rho} + \rho^{\frac{3}{2}} \sin\phi \hat{\phi}, \\ \mathbf{B}(\rho, \phi) &= \rho^{\frac{1}{2}} \cos\phi \hat{\rho} + \rho^{\frac{1}{2}} \sin\phi \hat{\phi},\end{aligned}$$

is there any point in space with $\mathbf{A}$ and $\mathbf{B}$ being perpendicular? Please provide clear reasoning using mathematics.

Solution:

We find

$$\mathbf{A} \cdot \mathbf{B} = (\rho^2 \cos^2\phi + \rho^2 \sin^2\phi) = \rho^2 (\cos^2\phi + \sin^2\phi) = \rho^2 = |\mathbf{A}||\mathbf{B}|\cos\theta$$

where θ is the angle between $\mathbf{A}$ and $\mathbf{B}$.

When $\mathbf{A}$ and $\mathbf{B}$ is perpendicular, we have

$$\theta = 90^\circ \Rightarrow \cos\theta = 0 \Rightarrow \rho^2 = |\mathbf{A}||\mathbf{B}|\cos\theta = 0 \Rightarrow \rho = 0$$

But when $\rho = 0$, $\mathbf{A} = \mathbf{0}$ and $\mathbf{B} = \mathbf{0}$.

So there is no point in space with $\mathbf{A}$ and $\mathbf{B}$ being perpendicular.

4. Consider a line at $y = L$ to $5L$. It contains electric charge of density $\rho_L = \rho_{L0}(y/L)^{-20}$ for some constant ρ_{L0} in C/m. Find the total charge of this line.

Solution:

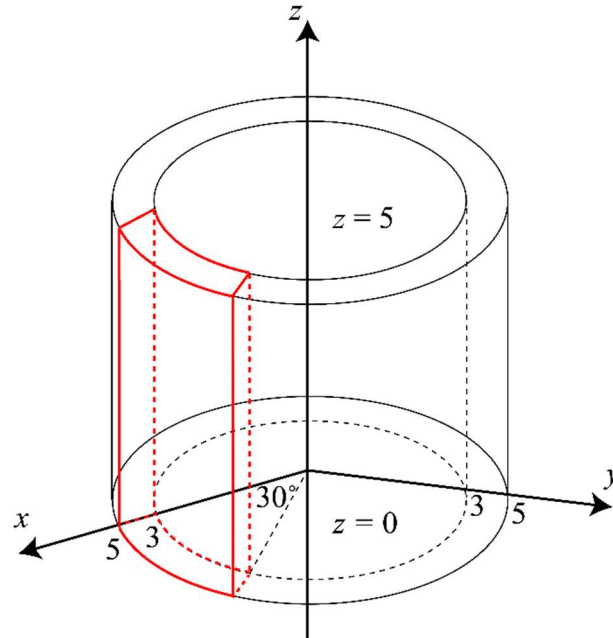
The total charge is

$$\begin{aligned} Q &= \int_L^{5L} \rho_L dy \\ &= \int_L^{5L} \rho_{L0} (y/L)^{-20} dy \\ &= \rho_{L0} \cdot L^{20} \int_L^{5L} y^{-20} dy \\ &= \rho_{L0} \cdot L^{20} \left[\frac{L^{-19}(1 - 5^{-19})}{19} \right] \\ &= \frac{L \cdot \rho_{L0}}{19} \text{ (C)} \end{aligned}$$

5. In the cylindrical coordinate system, the six surfaces: $\rho = 3$, $\rho = 5$, $\phi = 0^\circ$, $\phi = 30^\circ$, $z = 0$, and $z = 5$ define a closed surface.
- (a) Sketch the volume enclosed by the six surfaces as a 3D picture.
- (b) Calculate the enclosed volume.

Solution:

- (a) The volume enclosed by the six surfaces is plotted as the figure below.



- (b) The enclosed volume is

$$\text{Vol} = \int_{z=0}^5 \int_{\phi=0^\circ}^{30^\circ} \int_{\rho=3}^5 \rho d\rho d\phi dz = 8 \times 0.52 \times 5 = 21$$