

## EE3101 Tutorial 6 Solutions

(1) (a) For Continuous-phase FSK (CFSK) :  $f_1 - f_2 = 1/T_b$ ; i.e.  $\omega_1 - \omega_2 = 2\pi/T_b$

Takes  $\omega_c = (\omega_1 + \omega_2)/2$ , we have  $\omega_1 = \omega_c + \pi/T_b$ ;  $\omega_2 = \omega_c - \pi/T_b$

i.e. for binary 1:  $s_1(t) = E \sqrt{\frac{2}{T_b}} \cos(\omega_c t + \frac{\pi t}{T_b})$

for binary 0:  $s_2(t) = E \sqrt{\frac{2}{T_b}} \cos(\omega_c t - \frac{\pi t}{T_b})$

→  $s(t) = E \sqrt{\frac{2}{T_b}} \cos(\omega_c t \pm \frac{\pi t}{T_b})$  where + and - are for binary 1 and 0 respectively

(b) The transmitted signal for **CFSK** can be represented as:

$$E \sqrt{\frac{2}{T_b}} [\cos(\omega_c t) \cos(\pm \frac{\pi t}{T_b}) - \sin(\omega_c t) \sin(\pm \frac{\pi t}{T_b})] \quad \text{where + and - represent '1' and '0' respectively}$$

The psd of  $\cos(\frac{\pi t}{T_b})$  is :  $\frac{1}{4} [\delta(f - \frac{1}{2T_b}) + \delta(f + \frac{1}{2T_b})]$

The psd of  $\mp \sin(\frac{\pi t}{T_b})$  is :  $\frac{4T_b \cos^2(\pi T_b f)}{\pi^2 (4T_b^2 f^2 - 1)^2}$

The overall **psd** at the modulator output is (equal to the summation of individual psd):

$$S(f) = \frac{E^2}{8T_b} \left[ \delta(f - f_c - \frac{1}{2T_b}) + \delta(f + f_c - \frac{1}{2T_b}) + \delta(f - f_c + \frac{1}{2T_b}) + \delta(f + f_c + \frac{1}{2T_b}) \right] \\ + \frac{2E^2 \cos^2[\pi T_b (f - f_c)]}{\pi^2 [4T_b^2 (f - f_c)^2 - 1]^2} + \frac{2E^2 \cos^2[\pi T_b (f + f_c)]}{\pi^2 [4T_b^2 (f + f_c)^2 - 1]^2}$$

The overall **baseband equivalent psd** is:

$$S(f) = \frac{E^2}{2T_b} \left[ \delta(f - \frac{1}{2T_b}) + \delta(f + \frac{1}{2T_b}) \right] + \frac{8E^2 \cos^2(\pi T_b f)}{\pi^2 (4T_b^2 f^2 - 1)^2}$$

{Note: In a lot of applications, baseband equivalent **psd** is used because it may be quite tedious to express the **psd** in the passband. }

Additional Notes:

The psd of  $\sin(2\pi f_c t)$  or  $\cos(2\pi f_c t)$  is  $\frac{1}{4} [\delta(f - f_c) + \delta(f + f_c)]$

→ The psd of  $\sin(xt)$  or  $\cos(xt)$  is  $\frac{1}{4} [\delta(f - x/2\pi) + \delta(f + x/2\pi)]$

(2) For the **MSK** signal, the transmitted signal is represented as:

$$E\sqrt{\frac{2}{T}}[\pm \cos(\omega_c t) \cos(\frac{\pi t}{2T_b}) \mp \sin(\omega_c t) \sin(\frac{\pi t}{2T_b})] \quad \text{where - and + represent '1' and '0' respectively}$$

The **I**-channel is:

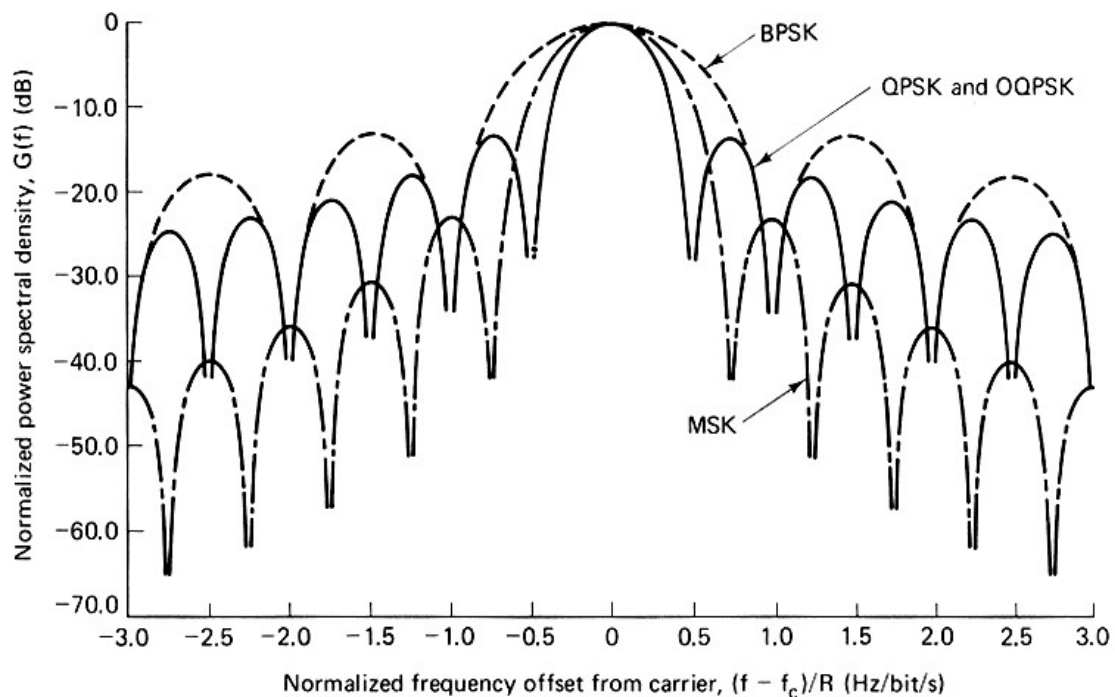
$$\pm E\sqrt{\frac{2}{T}} \cos(\omega_c t) \cos(\frac{\pi t}{2T_b}) \quad \text{for } -T_b \leq t < T_b$$

The **Q**-channel is:

$$\mp E\sqrt{\frac{2}{T}} \sin(\omega_c t) \sin(\frac{\pi t}{2T_b}) \quad \text{for } 0 \leq t < 2T_b$$

The baseband equivalent **psd** of the **I** or **Q** channel is:  $\frac{2E^2}{2T_b} \frac{4(2T_b) \cos^2[\pi(2T_b)f]}{\pi^2[4(2T_b)^2 f^2 - 1]^2}$

i.e the overall baseband equivalent **psd** of **MSK** modulation is thus:  $\frac{16E^2 \cos^2(2\pi T_b f)}{\pi^2(16T_b^2 f^2 - 1)^2}$



Normalized power spectral density for BPSK, QPSK, OQPSK, and MSK. (Reprinted with permission from F. Amoroso, "The Bandwidth of Digital Data Signals," *IEEE Commun. Mag.*, vol. 18, no. 6, Nov. 1980, Fig. 2A, p. 16. © 1980 IEEE.)

(3) (a)  $S\left(\frac{3}{4T_b}\right) = 0$

(b) Since both denominator and numerator approach to zero when  $f = \frac{1}{4T_b}$ , we apply L'Hospital's Rule here

$$S(f)|_{f=\frac{1}{4T_b}} = \frac{16E^2}{\pi^2} \frac{2 \cos(2\pi T_b f) [-\sin(2\pi T_b f)] 2\pi T_b}{2(16T_b^2 f^2 - 1) 32T_b^2 f} = \frac{-E^2 \sin(4\pi T_b f)}{2\pi T_b (16T_b^2 f^3 - f)}$$

The denominator and numerator still approach to zero, apply L'Hospital's Rule again,

$$S(f)|_{f=\frac{1}{4T_b}} = \frac{-E^2 \cos(4\pi T_b f) 4\pi T_b}{2\pi T_b (48T_b^2 f^2 - 1)} = -\frac{2E^2 \cos(4\pi T_b f)}{48T_b^2 f^2 - 1} = E^2$$

The maximum psd occurs at  $f = 0$ ,  $S(0) = \frac{16E^2}{\pi^2}$

i.e.  $S\left(\frac{1}{4T_b}\right)$  is  $\frac{E^2}{\frac{16E^2}{\pi^2}} = 0.6168$  of the maximum value, or 2.1 dB below the maximum value.

(4) No, it isn't.

It is because the shape of the waveforms is different, so it is very unlikely that they have the same psd.

(5)  $\frac{1}{\sqrt{2}} = \exp[-2(\pi\sigma BT_b)^2]$   
 $2 = \exp[(2\pi\sigma BT_b)^2]$   
 $\Rightarrow BT_b = \frac{\sqrt{\log_e 2}}{2\pi\sigma}$