

EE1001 Test 1 Sem B, Exercise

Q1

Determine the following statements whether they are true or false.

- (i) “Oxygen is one of the basic elements of life” is a valid and sound argument.
- (ii) A valid argument can go from all false premises to a true conclusion.
- (iii) Given the statement of “If you study hard, you get an A in this course”, “Studying hard” is a necessary condition for “getting an A in this course”.
- (iv) “If you study hard, you get an A in this course” is logically equivalent to “If you do not get an A in this course, you did not study hard”.
- (v) Given $A = \{a_1, a_2, a_3\}$. $\exists x \in A, T(x)$ is logically equivalent to $T(a_1) \wedge T(a_2) \wedge T(a_3)$.

Q2

Determine whether the following statement is a tautology, a contradiction, or neither.

$$\sim[(a \rightarrow b) \wedge \sim b \wedge (a \vee c) \rightarrow c]$$

Q3

Given two types of devices, namely normal and hacked devices, the normal devices always give **true** (“**T**”) messages, whereas the hacked devices always give **false** (“**F**”) messages.

There are THREE (3) devices, and their messages are as follows:

Device A: “All of us are normal.”

Device B: “If A is hacked, then B is normal and C is hacked.”

Device C: “I’m normal, and at least one of the others is hacked”

Let a , b , and c be

a = “Device A is normal”

b = “Device B is normal”

c = “Device C is normal”

Q3(i) Formulate the statements of Devices A, B, and C, respectively.

Q3(ii) Using truth table to determine whether Devices A, B, and C are normal or hacked, respectively.

(Hint: There are THREE (3) possible situations)

Q4

A series of experiments is carried out that involves the use of increasing number of non-reusable capacitors (i.e., the capacitors used in each round of experiment cannot be re-used in the next round).

In the first experiment, the testbed involves 50 capacitors. In the second experiment, 65 capacitors are involved.

- (i) It is given that the numbers of the capacitors used form an arithmetic progression. Determine the total number of the capacitors in the first 25 experiments.

- (ii) Instead, it is given that the number of the capacitors used form a geometric progression. There are a total of 23,000 capacitors available for the testbed. Determine the greatest number of possible experiment (N).

Q5.

Analysis the following situations and find the answer for each question with numbers or permutation/combination expression (e.g. ${}^4C_1$, ${}^7P_3$...).

- (i) Lucy plans to wear a shirt, pants, socks, and shoes to join a party. She owns 8 pairs of socks, 3 pairs of shoes, 5 pairs of pants, and 4 shirts.

How many different outfits can John make?

- (ii) Martin claims that he can read minds. To test Martin's abilities, Kelvin draws 5 cards without replacement from a standard deck of 52 playing cards. Kelvin then asks Martin to identify in any order which 5 cards he drew without looking.

Assume that Luka has no special abilities and is randomly guessing the cards.

- (a) What is the probability that Martin correctly identifies all 5 cards in any order?

- (b) If the order the matters, what is the probability that Martin correctly identifies all 5 cards in the exactly right order?

- (iii) Leo is 1 of 30 students in a class. Every month, Leo's teacher randomly selects 4 students from their class to act as class president, vice president, secretary, and treasurer. No one student can hold two positions.

In a given month, what is the probability that Leo is chosen as president?

- (iv) AC3 restaurant is planning to offer a special price on pizzas with 2 toppings. They offer the toppings below:

Pepperoni; Chicken; Mushroom; Sausage; Green pepper; Pineapple; Ham; Onion

Suppose that your favorite is *Chicken* and *Mushroom*. Your classmate Steven don't know this, so he is going to randomly choose 2 different toppings.

What is the probability that Stefen chooses your favorite topping? With this question, discuss why it is better to make a clear order when ask others to help you buy food.

Q6.

If Set A contains all even numbers larger than 0 and less than 45, and Set B contains all numbers larger than 0 and less than 45 that are multiples of 5.

- (a) Find the set C which is the intersection of Set A and Set B? (2marks)

- (b) What is $A-B$?

- (c) What is $B-A$?

- (d) What is $B-A \cup B$?

Q7.

- (i) The coefficient of x^2 in the expansion of $(1+2x)^n$ is 40. Given that n is a positive integer, find the value of n .
- (ii) In the binomial expansion of $(1 + 3x)^n$ ($n>0$), If the coefficient of x^2 is six times the coefficient of x , find the value of n .

Q8.

There are 125 lights with each light labelled by a serial number in the following sequence:
1, 2, 3, ..., 125.

Each light has a button, which can switch/change (toggle) the state of the light between “on” and “off”. All lights are turned off at the beginning. To examine the function of these lights. An engineer clicks the buttons of the lights with serial numbers which are multiple of 4. Secondly, he clicks the buttons of the lights with serial numbers which are multiple of 6.

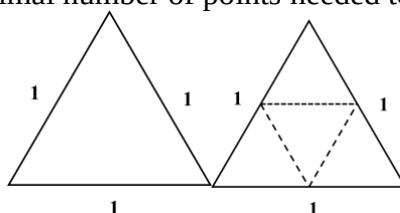
(i) How many lights are on?

(ii) In the third time, the engineer clicks the buttons of lights with the serial numbers which are multiple of 4, but not multiple of 6. How many lights are on?

Q9.

It is required to put several points (more than 3) at random in a triangular with side length 1. In these points, any two of them can form a short-line. The more points you place, the more lines can be formed. The formed lines may have different lengths, depending on the point positions.

Now, you need to guarantee that there is at least one formed line with a length equal to or less than $1/2$. What is the minimal number of points needed to be placed?



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