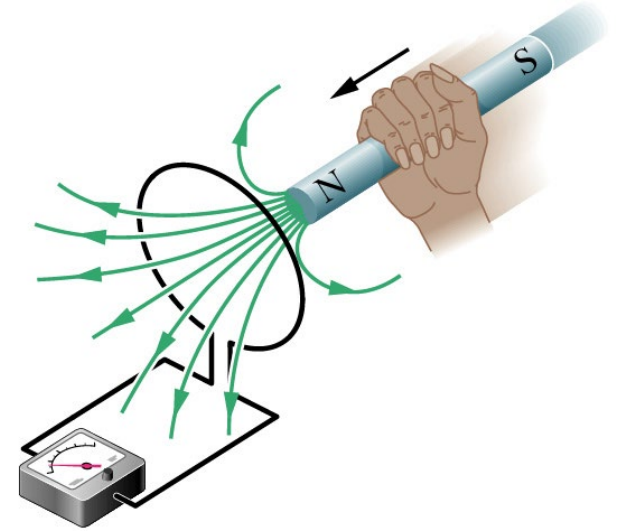


Tutorial 11

Induction and Inductance

Induced Current involving Relative Motion Between a Magnet and a Coil

1. A **current** appears only if there is relative motion between the loop and the magnet (one must move relative to the other); the current disappears when the relative motion between them ceases.
2. Faster motion produces a greater **current**.
3. If moving the magnet's north pole toward the loop causes, say, clockwise **current**, then moving the north pole away causes counter clockwise current. Moving the south pole toward or away from the loop also causes currents, but in the reversed directions.



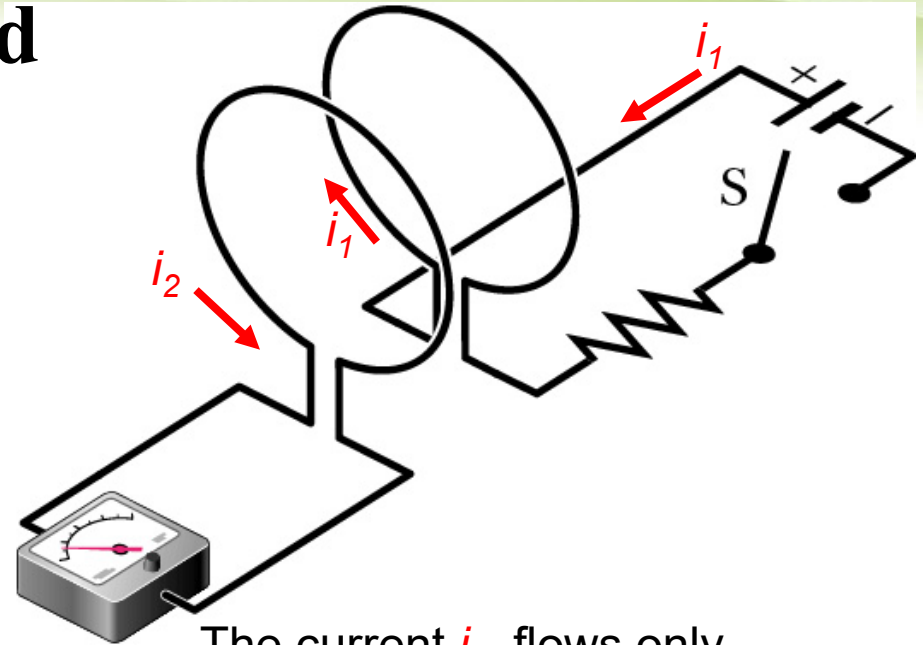
Note:

- A magnetic field can produce an electric field that can drive an (induced) current!
- The key observation is that when the "amount" of magnetic field passing through the loop changes, this will induce a current.
- The work done per unit charge to produce that current is called an induced "electromotive force" (emf $\epsilon = dW/dq$ unit: $1 \text{ J/C} = 1 \text{ V}$)
- The process of producing the current and the emf is called induction.
- The emf is equivalent to a virtual electrical potential which drives the current in the coil.

Induced Current Between Two Coils with No Motion Involved

The current meter registers a current in the left-hand wire loop just as switch S is closed (to turn on the current in the right-hand wire loop) or opened (to turn off the current in the right-hand loop).

No motion of the coils is involved.

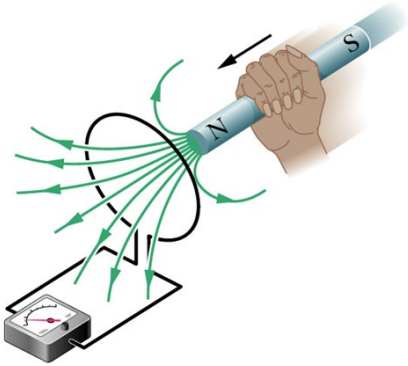


- The current i_2 flows only for a **very short time** after current i_1 is switched on!
- The direction of i_2 is opposite to i_1 !

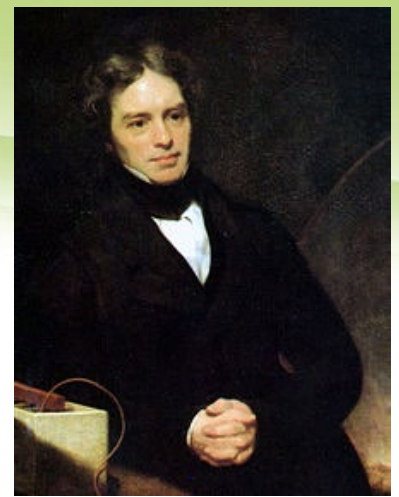
Note: A current in a circuit can transmit information into a secondary circuit via a magnetic field!

(current 1 $\Rightarrow$ magnetic field $\Rightarrow$ current 2)

Faraday's Law of Induction



An emf is induced in a loop when the number of magnetic field lines that pass through the loop is changing.

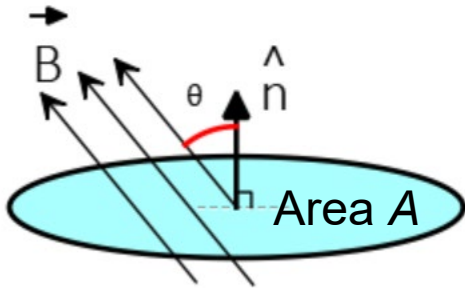


Michael Faraday
(22 September 1791 – 25 August 1867)
English scientist
https://en.wikipedia.org/wiki/Michael_Faraday

The actual number of field lines passing through the loop does not matter; the values of the induced emf and induced current are determined by the *rate* at which that number *changes*.

Faraday's law does not explain *why* a current and an emf are induced in either experiment; it is just a statement that helps us visualize the induction.

The Magnetic Flux Φ_B Through an Area A



$$\Phi_B = \int \mathbf{B} \cdot d\mathbf{A}$$

Unit of magnetic flux:

$$1 \text{ weber} = 1 \text{ Wb} = 1 \text{ Tm}^2$$

If the magnetic field $\mathbf{B}$ is uniform, then $\mathbf{B}$ can be brought out in front of the integral sign:

$$\Phi_B = BA \quad (\vec{B} \perp \text{area } A, \vec{B} \text{ uniform}).$$

The magnitude of the **emf** (electromotive force) induced in a conducting loop is equal to the rate at which the **magnetic** flux Φ_B through that loop changes with time.

Faraday's Law:

$$\mathcal{E} = - \frac{d\Phi_B}{dt}$$

$$\begin{aligned} \text{Units: } 1 \text{ Wb/s} &= 1 \text{ Tm}^2/\text{s} \\ &= 1 \text{ J/C} = 1 \text{ V} \end{aligned}$$

The minus sign indicates that the induced emf tends to oppose the flux change.

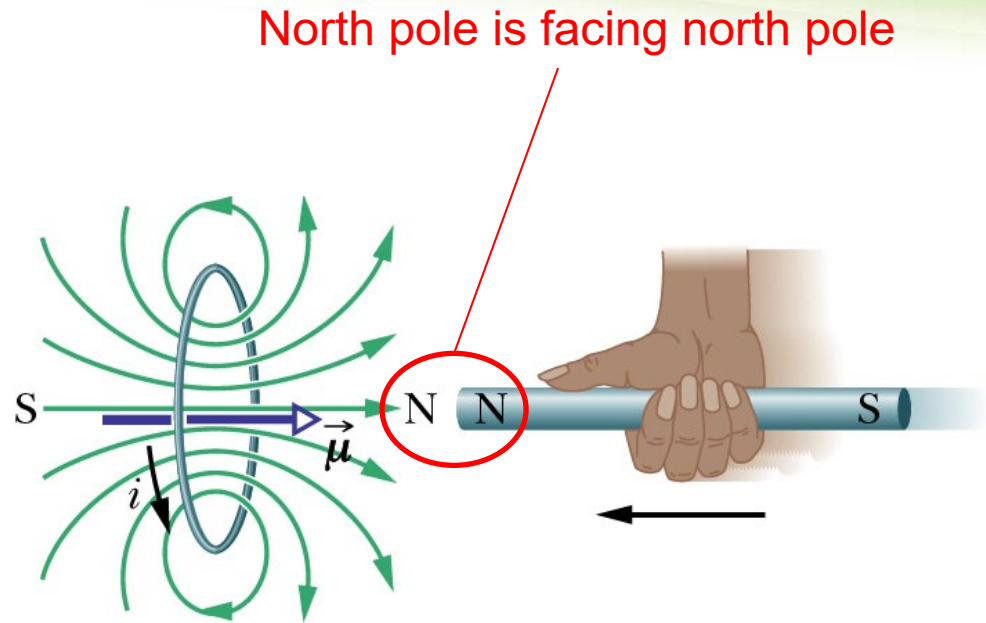
Lenz's Law

(gives direction of induced currents)



Heinrich Friedrich Emil Lenz
(12 February 1804
– 10 February 1865)
Russian physicist
- Baltic German ethnicity

An **induced current** has a direction such that the **magnetic field** due to *the current* **opposes** the change in the **magnetic flux** that induces the current.



Lenz's law at work. As the magnet is moved toward the loop, a **current** is induced in the loop.

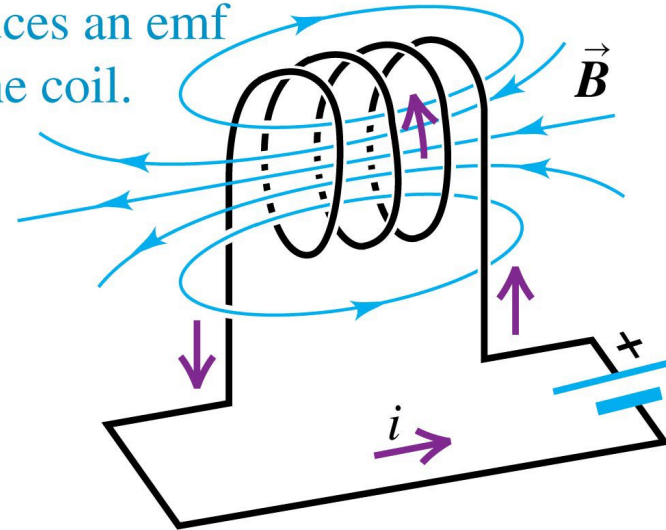
The current produces its own **magnetic field**, with **magnetic dipole moment** $\vec{\mu}$ oriented so as to **oppose** the motion of the magnet.

Thus, the induced current must be counter clockwise as shown.

Self-inductance

- Any circuit with a coil that carries a **varying current** has a **self-induced emf**.
- We define the self-inductance L of the circuit as:

Self-inductance: If the current i in the coil is changing, the changing flux through the coil induces an emf in the coil.



Self-inductance (or inductance) of a coil

$$L = \frac{N\Phi_B}{i}$$

Number of turns in coil

Flux due to current through each turn of coil

Current in coil

Self-Induction:



An induced emf $\mathcal{E}_L$ appears in any coil in which the current is changing.

This process (see Fig. 30-13) is called **self-induction**, and the emf that appears is called a **self-induced emf**. It obeys Faraday's law of induction just as other induced emfs do.

Example for varying current circuit:

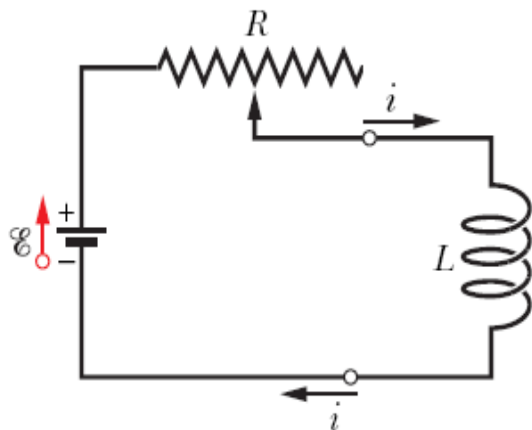


Fig. 30-13 If the current in a coil is changed by varying the contact position or a variable resistor, a self-induced emf $\mathcal{E}_L$ will appear in the coil *while the current is changing*.

$$N\Phi_B = Li.$$

$$\mathcal{E}_L = - \frac{d(N\Phi_B)}{dt}.$$

$$\mathcal{E}_L = -L \frac{di}{dt} \quad (\text{self-induced emf}).$$

increasing current: emf_L counteracts emf
decreasing current: emf_L adds to emf
 $\Rightarrow$ emf_L tends to delay current increases or decreases

The energy for this behavior is needed for (or comes from) the build-up (or decay) of magnetic field energy of the coil

Practice Questions:

Chapter 30 - Induction and Inductance

Questions: *3, *7, **10, *29, *37, *45

*3

•3 SSM WWW In Fig. 30-32, a 120-turn coil of radius 1.8 cm and resistance 5.3Ω is coaxial with a solenoid of 220 turns/cm and diameter 3.2 cm. The solenoid current drops from 1.5 A to zero in time interval $\Delta t = 25$ ms. What current is induced in the coil during Δt ?

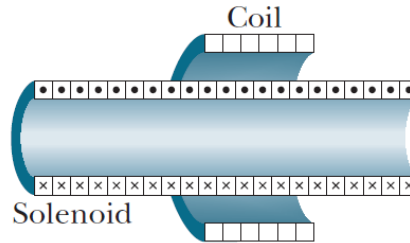


Fig. 30-32 Problem 3.

The B field in the solenoid is related to the current i , by

$$B = \mu_0 n i$$

n is the number of turn per length of the solenoid, therefore

Coaxial means the two structure both having a common axis.

The induced emf of in the coil depends on the number of the variation of the total magnetic flux in the coil as function of time.

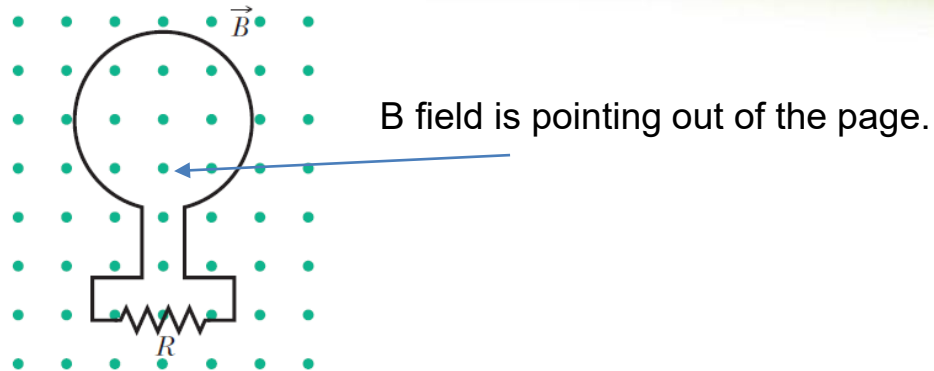
$$\mathcal{E} = -N \frac{d\Phi_B}{dt}$$

Instead of the time rate of change of the magnetic flux, we are given the rate of change of the current. Therefore, we will need to equate the expression from the magnetic flux rate of change to the rate of change of the current.

$$\begin{aligned} \mathcal{E} &= -N \frac{d\Phi_B}{dt} = -NA \left(\frac{dB}{dt} \right) \\ &= -NA \frac{d}{dt} (\mu_0 n i) = -N \mu_0 n A \frac{di}{dt} \\ &= -N \mu_0 n (\pi r^2) \frac{di}{dt} \\ &= -(120)(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(22000/\text{m}) \pi (0.016\text{m})^2 \left(\frac{1.5 \text{ A}}{0.025 \text{ s}} \right) \\ &= 0.16 \text{ V.} \end{aligned}$$

*7

•7 In Fig. 30-36, the magnetic flux through the loop increases according to the relation $\Phi_B = 6.0t^2 + 7.0t$, where Φ_B is in milliwebers and t is in seconds. (a) What is the magnitude of the emf induced in the loop when $t = 2.0$ s? (b) Is the direction of the current through R to the right or left?



(a) The magnitude of the induced emf equals to the time rate of change of the magnetic flux through the loop.

$$\begin{aligned} |\mathcal{E}| &= \left| \frac{d\Phi_B}{dt} \right| \\ &= \frac{d}{dt}(6.0t^2 + 7.0t) \\ &= 12t + 7.0 = 12(2.0) + 7.0 \\ &= 31\text{mV}. \end{aligned}$$

(b) Appealing to Lenz's law (especially Fig. 30-5(a)) we see that the current flow in the loop is clockwise. Thus, the current is to the left through R .

**10

•10 Figure 30-37 shows a closed loop of wire that consists of a pair of equal semicircles, of radius 3.7 cm, lying in mutually perpendicular planes. The loop was formed by folding a flat circular loop along a diameter until the two halves became perpendicular to each other. A uniform magnetic field $\vec{B}$ of magnitude 76 mT is directed perpendicular to the fold diameter and makes equal angles (of 45°) with the planes of the semicircles. The magnetic field is reduced to zero at a uniform rate during a time interval of 4.5 ms. During this interval, what are the (a) magnitude and (b) direction (clockwise or counterclockwise when viewed along the direction of $\vec{B}$) of the emf induced in the loop?

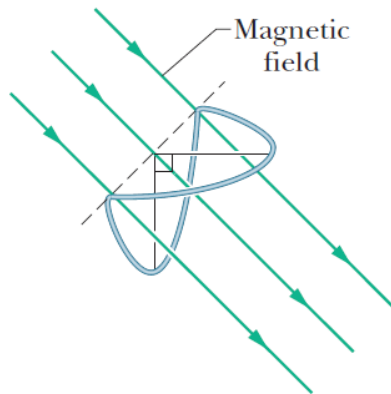


Fig. 30-37 Problem 10.

(a) The magnetic flux Φ_B through the loop is given by

$$\Phi_B = 2B\left(\pi r^2/2\right)(\cos 45^\circ) = \pi r^2 B/\sqrt{2}$$

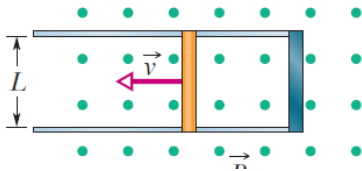
Thus,

$$\begin{aligned}\mathcal{E} &= -\frac{d\Phi_B}{dt} \\ &= -\frac{d}{dt}\left(\frac{\pi r^2 B}{\sqrt{2}}\right) \\ &= -\frac{\pi r^2}{\sqrt{2}}\left(\frac{\Delta B}{\Delta t}\right) \\ &= -\frac{\pi(3.7 \times 10^{-2} \text{ m})^2}{\sqrt{2}}\left(\frac{0 - 76 \times 10^{-3} \text{ T}}{6.0 \times 10^{-3} \text{ s}}\right) \\ &= 3.9 \times 10^{-2} \text{ V}.\end{aligned}$$

(b) The direction of the induced current is clockwise when viewed along the direction of $\vec{B}$.

*29

•29 In Fig. 30-50, a metal rod is forced to move with constant velocity $\vec{v}$ along two parallel metal rails, connected with a strip of metal at one end. A magnetic field of magnitude $B = 0.350 \text{ T}$ points out of the page. (a) If the rails are separated by $L = 25.0 \text{ cm}$ and the speed of the rod is 55.0 cm/s , what emf is generated? (b) If the rod has a resistance of 18.0Ω and the rails and connector have



negligible resistance, what is the current in the rod? (c) At what rate is energy being transferred to thermal energy?

(a) From L11 slide 16

$$\begin{aligned}\varepsilon &= \frac{d\Phi_B}{dt} = \frac{d}{dt} BLx = BL \frac{dx}{dt} = BLv \\ &= (0.350 \text{ T})(0.250 \text{ m})(0.55 \text{ m/s}) = 0.0481 \text{ V}\end{aligned}$$

(b) By Ohm's law, the induced current is

$$i = V/R = 0.0481 / 18.0 = 0.00267 \text{ A}$$

By Lenz's law, the current is clockwise in the figure.

(c) From the relationship of power P , current i and resistance R $P = i^2 R = 0.000129 \text{ W}$.

*37

•37 SSM ILW A long solenoid has a diameter of 12.0 cm. When a current i exists in its windings, a uniform magnetic field of magnitude $B = 30.0$ mT is produced in its interior. By decreasing i , the field is caused to decrease at the rate of 6.50 mT/s. Calculate the magnitude of the induced electric field (a) 2.20 cm and (b) 8.20 cm from the axis of the solenoid.

The radial r dependency of the induced E field is linear if the point is located inside the solenoid and inversely proportional to $1/r$ if it is outside of the solenoid.

(a) The point at which we are evaluating the field is inside the solenoid, so Eq. 30-25 applies. The magnitude of the induced electric field is

$$\begin{aligned} E &= \frac{1}{2} \frac{dB}{dt} r \\ &= \frac{1}{2} (6.5 \times 10^{-3} \text{ T/s}) (0.0220 \text{ m}) \\ &= 7.15 \times 10^{-5} \text{ V/m.} \end{aligned}$$

(b) Now the point at which we are evaluating the field is outside the solenoid and Eq. 30-27 applies. The magnitude of the induced field is

$$\begin{aligned} E &= \frac{1}{2} \frac{dB}{dt} \frac{R^2}{r} \\ &= \frac{1}{2} (6.5 \times 10^{-3} \text{ T/s}) \frac{(0.0600 \text{ m})^2}{(0.0820 \text{ m})} \\ &= 1.43 \times 10^{-4} \text{ V/m.} \end{aligned}$$

*45

•45 At a given instant the current and self-induced emf in an inductor are directed as indicated in Fig. 30-57. (a) Is the current increasing or decreasing? (b) The induced emf is 17 V, and the rate of change of the current is 25 kA/s; find the inductance.

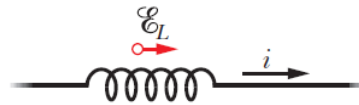


Fig. 30-57 Problem 45.

(b) From Eq. 30-35 (in absolute value) we get

$$L = \left| \frac{\varepsilon}{di/dt} \right| = \frac{17 \text{ V}}{2.5 \text{ kA/s}} = 6.8 \times 10^{-4} \text{ H}.$$

(a) From L11 slide 25, the self-induced *emf* is negative if di/dt is positive.

Note that the direction of the induced emf is defined in the direction of the current.

In this case, the current i is already flowing to the right, however the induced emf is also pointing rightward, indicating the induced emf is positive with respect to the current direction. Therefore, the current i must be decreasing.