

# Tutorial 6

## Capacitance

# Practice Questions: Capacitance

Chapter 25

Questions: \*\*14, \*\*22, \*\*28, \*\*37, \*\*45, \*\*49

# \*\*14

••14 In Fig. 25-30, the battery has a potential difference of  $V = 10.0 \text{ V}$  and the five capacitors each have a capacitance of  $10.0 \mu\text{F}$ . What is the charge on (a) capacitor 1 and (b) capacitor 2?

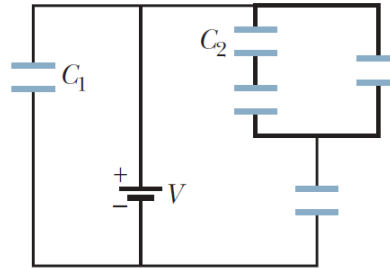
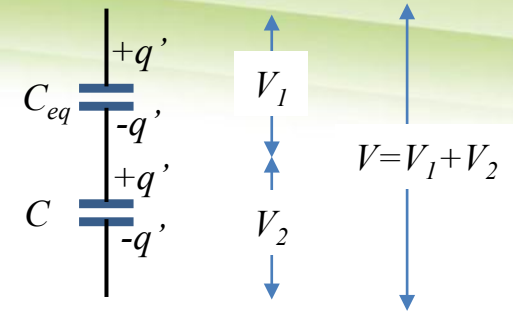
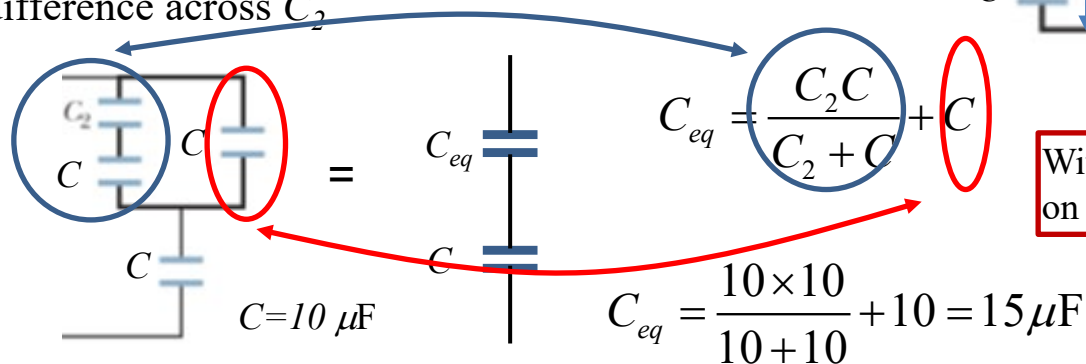


Fig. 25-30 Problem 14.

(a) With  $q = CV$ , we can find  $q$  if we know the values of  $C$  and  $V$ . Since the potential difference across  $C_1$  is  $V = 10.0 \text{ V}$ . Thus,

$$q_1 = C_1 V = (10.0 \mu\text{F})(10.0 \text{ V}) = 1.0 \times 10^{-4} \text{ C}.$$

(b) To find  $q$  on  $C_2$ , we need to find the potential difference across  $C_2$

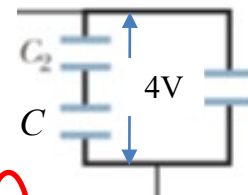


$$V = V_1 + V_2 = q' \left( \frac{1}{C_{eq}} + \frac{1}{C} \right)$$

$$10\text{V} = q' \left( \frac{1}{15\mu\text{F}} + \frac{1}{10\mu\text{F}} \right) = 0.167 \times 10^6 q'$$

$$q' = 6 \times 10^{-5} \text{ C}$$

$$V_1 = \frac{q'}{C_{eq}} = \frac{6 \times 10^{-5} \text{ C}}{15 \times 10^{-6} \text{ F}} = 4\text{V}; \quad V_2 = \frac{q'}{C} = \frac{6 \times 10^{-5} \text{ C}}{10 \times 10^{-6} \text{ F}} = 6\text{V}$$



The voltage across  $C_2$  and  $C$  is  $4\text{V}$ , since the capacitance of  $C_2$  and  $C$  are the same, therefore the voltage across the individual capacitor is half the total which is equal to  $2\text{V}$ .

With the voltage across  $C_2$  is  $2\text{V}$ , the charge on  $C_2$  is  $q_2 = CV_2 = 20\mu\text{C}$  or  $2.0 \times 10^{-5} \text{ C}$

## \*\*22

•22 In Fig. 25-37,  $V = 10 \text{ V}$ ,  $C_1 = 10 \mu\text{F}$ , and  $C_2 = C_3 = 20 \mu\text{F}$ . Switch  $S$  is first thrown to the left side until capacitor 1 reaches equilibrium. Then the switch is thrown to the right. When equilibrium is again reached, how much charge is on capacitor 1?

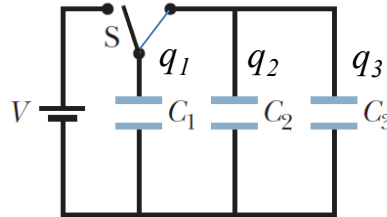


Fig. 25-37 Problem 22.

Charge is conserved; therefore, if  $Q = C_1 V_{\text{bat}} = 10\text{V} \times 10\mu\text{F} = 100 \mu\text{C}$ , and  $q_1$ ,  $q_2$  and  $q_3$  are the charges on  $C_1$ ,  $C_2$  and  $C_3$  after the switch is thrown to the right and equilibrium is reached, then

$$Q = q_1 + q_2 + q_3.$$

Since the parallel pair  $C_2$  and  $C_3$  are identical, it is clear that  $q_2 = q_3$ . They are in parallel with  $C_1$  so that  $V_1 = V_3$ , or

$$\frac{q_1}{C_1} = \frac{q_3}{C_3} \rightarrow q_1 = \frac{C_1}{C_3} q_3 = \frac{10}{20} q_3 = \frac{q_3}{2}$$

which leads to  $q_1 = q_3/2$ . Therefore,

$$Q = (q_3 / 2) + q_3 + q_3 = 5q_3 / 2$$

which yields

$$q_3 = 2Q/5 = 2(100 \mu\text{C})/5 = 40 \mu\text{C}$$

and consequently  $q_1 = q_3/2 = 20 \mu\text{C}$ .

# \*\*\*28

\*\*\*28  Figure 25-43 displays a 12.0 V battery and 3 uncharged capacitors of capacitances  $C_1 = 4.00 \mu\text{F}$ ,  $C_2 = 6.00 \mu\text{F}$ , and  $C_3 = 3.00 \mu\text{F}$ . The switch is thrown to the left side until capacitor 1 is fully charged. Then the switch is thrown to the right. What is the final charge on (a) capacitor 1, (b) capacitor 2, and (c) capacitor 3?

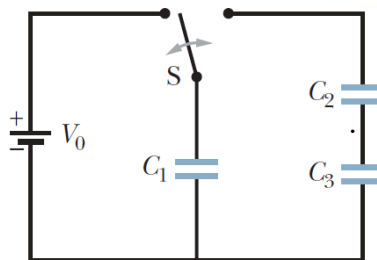


**Fig. 25-43** Problem 28.

(a) Solving the two equations

$$\frac{q_1}{C_1} = \frac{q_2}{C_{\text{eq}}} \quad q_1 + q_2 = C_1 V_0$$

for  $q_1$  and  $q_2$ , we obtain

$$q_1 = \frac{C_1^2 V_0}{C_{\text{eq}} + C_1} = \frac{C_1^2 V_0}{\frac{C_2 C_3}{C_2 + C_3} + C_1} = \frac{C_1^2 (C_2 + C_3) V_0}{C_1 C_2 + C_1 C_3 + C_2 C_3}.$$

With  $V_0 = 12.0 \text{ V}$ ,  $C_1 = 4.00 \mu\text{F}$ ,  $C_2 = 6.00 \mu\text{F}$  and  $C_3 = 3.00 \mu\text{F}$ , we find  $C_{\text{eq}} = 2.00 \mu\text{F}$  and  $q_1 = 32 \mu\text{C}$ .

(b) The charge on capacitors 2 is

$$q_2 = C_1 V_0 - q_1 = (4.00 \mu\text{F})(12.0 \text{ V}) - 32 \mu\text{C} = 16 \mu\text{C}$$

(c) The charge on capacitor 3 is the same as that on capacitor 2:

$$q_3 = C_1 V_0 - q_1 = (4.00 \mu\text{F})(12.0 \text{ V}) - 32 \mu\text{C} = 16 \mu\text{C}$$

The charges on capacitors 2 and 3 are the same, so these capacitors may be replaced by an equivalent capacitance determined from

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_2} + \frac{1}{C_3} = \frac{C_2 + C_3}{C_2 C_3}.$$

Thus,  $C_{\text{eq}} = C_2 C_3 / (C_2 + C_3)$ . The charge on the equivalent capacitor is the same as the charge on either of the two capacitors in the combination, and the potential difference across the equivalent capacitor is given by  $q_2 / C_{\text{eq}}$ . The potential difference across capacitor 1 is  $q_1 / C_1$ , where  $q_1$  is the charge on this capacitor. The potential difference across the combination of capacitors 2 and 3 must be the same as the potential difference across capacitor 1, so  $q_1 / C_1 = q_2 / C_{\text{eq}}$ . Now some of the charge originally on capacitor 1 flows to the combination of 2 and 3. If  $q_0$  is the original charge, conservation of charge yields  $q_1 + q_2 = q_0 = C_1 V_0$ , where  $V_0$  is the original potential difference across capacitor 1.

## \*\*37

**•37 SSM ILW WWW** The parallel plates in a capacitor, with a plate area of  $8.50 \text{ cm}^2$  and an air-filled separation of  $3.00 \text{ mm}$ , are charged by a  $6.00 \text{ V}$  battery. They are then disconnected from the battery and pulled apart (without discharge) to a separation of  $8.00 \text{ mm}$ . Neglecting fringing, find (a) the potential difference between the plates, (b) the initial stored energy, (c) the final stored energy, and (d) the work required to separate the plates.

(a) Capacitance of a parallel plates capacitor  $C = \epsilon_0 A/d_i$

Charge on a capacitor with potential difference of  $V_i$

$$q = CV = \epsilon_0 AV_i/d_i$$

After the plates are pulled apart, their separation is  $d_f$  and the potential difference is  $V_f$ . Then

$$q = \epsilon_0 AV_f/d_f$$

and

$$V_f = \frac{d_f}{\epsilon_0 A} q = \frac{d_f}{\epsilon_0 A} \frac{\epsilon_0 A}{d_i} V_i = \frac{d_f}{d_i} V_i$$

$$V_f = \frac{8.00 \times 10^{-3}}{3.00 \times 10^{-3}} 6 = 16 \text{ V}$$

(b) The initial energy stored in the capacitor is

$$\begin{aligned} U_i &= \frac{1}{2} CV_i^2 \\ &= \frac{\epsilon_0 AV_i^2}{2d_i} \\ &= \frac{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(8.50 \times 10^{-4} \text{ m}^2)(6.00 \text{ V})^2}{2(3.00 \times 10^{-3} \text{ m})} \\ &= 4.51 \times 10^{-11} \text{ J}. \end{aligned}$$

(c) The final energy stored is

$$\begin{aligned} U_f &= \frac{1}{2} \frac{\epsilon_0 A}{d_f} V_f^2 = \frac{1}{2} \frac{\epsilon_0 A}{d_f} \left( \frac{d_f}{d_i} V_i \right)^2 \\ &= \frac{d_f}{d_i} \left( \frac{\epsilon_0 AV_i^2}{2d_i} \right) = \frac{d_f}{d_i} U_i \\ &= \left( \frac{8}{3} \right) 4.51 \times 10^{-11} = 1.20 \times 10^{-10} \text{ J}. \end{aligned}$$

(d) The work done to pull the plates apart is the difference in the energy:

$$W = U_f - U_i = 7.49 \times 10^{-11} \text{ J}.$$



# \*\*45

•45 A certain parallel-plate capacitor is filled with a dielectric for which  $\kappa = 5.5$ . The area of each plate is  $0.034 \text{ m}^2$ , and the plates are separated by  $2.0 \text{ mm}$ . The capacitor will fail (short out and burn up) if the electric field between the plates exceeds  $200 \text{ kN/C}$ . What is the maximum energy that can be stored in the capacitor?

The relationship between the charge, electric field and the plate area  $A$  of the air-filled parallel-plate capacitor is

$$q = \epsilon_0 EA$$

The relationship between the charge, electric field and the plate area  $A$  of the dielectric-filled parallel-plate capacitor with dielectric constant  $\kappa$  is

$$q = \kappa \epsilon_0 EA \rightarrow E = \frac{q}{\kappa \epsilon_0 A}$$

The maximum electric field between the plates cannot exceeds  $E_{\max} = 200 \text{ kN/C}$ , thus the maximum charge on the plates is

$$\begin{aligned} q_{\max} &= \kappa \epsilon_0 E_{\max} A \\ &= (5.5)(8.85 \times 10^{-12})(200 \times 10^3)(0.034) = 3.3 \times 10^{-7} \text{ C} \end{aligned}$$

therefore lead to

$$\begin{aligned} U &= \frac{q^2}{2C} \\ &= \frac{q^2 d}{2\kappa \epsilon_0 A} \\ &= \frac{(3.3 \times 10^{-7})^2 (2 \times 10^{-3})}{2(5.5)(8.85 \times 10^{-12})(0.034)} \\ &= 6.6 \times 10^{-5} \text{ J} . \end{aligned}$$

# \*\*49

•49 Figure 25-48 shows a parallel-plate capacitor with a plate area  $A = 7.89 \text{ cm}^2$  and plate separation  $d = 4.62 \text{ mm}$ . The top half of the gap is filled with material of dielectric constant  $\kappa_1 = 11.0$ ; the bottom half is filled with material of dielectric constant  $\kappa_2 = 12.0$ . What is the capacitance?

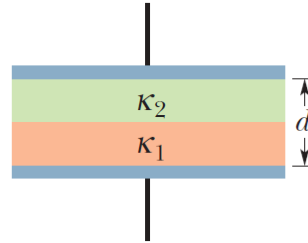


Fig. 25-48  
Problem 49.

This expression is exactly the same as that for  $C_{\text{eq}}$  of two capacitors in series, one with dielectric constant  $\kappa_1$  and the other with dielectric constant  $\kappa_2$ . Each has plate area  $A$  and plate separation  $d/2$ . Also we note that if  $\kappa_1 = \kappa_2$ , the expression reduces to  $C = \kappa_1 \epsilon_0 A/d$ , the correct result for a parallel-plate capacitor with plate area  $A$ , plate separation  $d$ , and dielectric constant  $\kappa_1$ .

We assume there is charge  $q$  on one plate and charge  $-q$  on the other. The electric field in the lower and upper halves of the region between the plates are

$$E_1 = \frac{q}{\kappa_1 \epsilon_0 A}, \quad E_2 = \frac{q}{\kappa_2 \epsilon_0 A}.$$

where  $A$  is the plate area. Let  $d/2$  be the thickness of each dielectric. Since the field is uniform in each region, the potential difference between the plates is

$$V = \frac{E_1 d}{2} + \frac{E_2 d}{2} = \frac{q d}{2 \epsilon_0 A} \left[ \frac{1}{\kappa_1} + \frac{1}{\kappa_2} \right] = \frac{q d}{2 \epsilon_0 A} \frac{\kappa_1 + \kappa_2}{\kappa_1 \kappa_2},$$

and

$$C = \frac{q}{V} = \frac{2 \epsilon_0 A}{d} \frac{\kappa_1 \kappa_2}{\kappa_1 + \kappa_2}.$$

$$A = 7.89 \times 10^{-4} \text{ m}^2$$

$$d = 4.62 \times 10^{-3} \text{ m}$$

$$\kappa_1 = 11.0, \quad \kappa_2 = 12.0$$

$$C = \frac{2(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(7.89 \times 10^{-4} \text{ m}^2)}{4.62 \times 10^{-3} \text{ m}} \frac{(11.0)(12.0)}{11.0 + 12.0} = 1.73 \times 10^{-11} \text{ F}.$$