

Block B

Operational Amplifiers

G. Rizzoni Ch 8.2

Summary

If an output of a sensor was 1mV, which is the smallest your DMM could measure, you will need to amplify the voltage for it to be detectable by the DMM. The effect you want to achieve is illustrated in Fig 1. You can see that through the amplifier, the signal is now stronger.

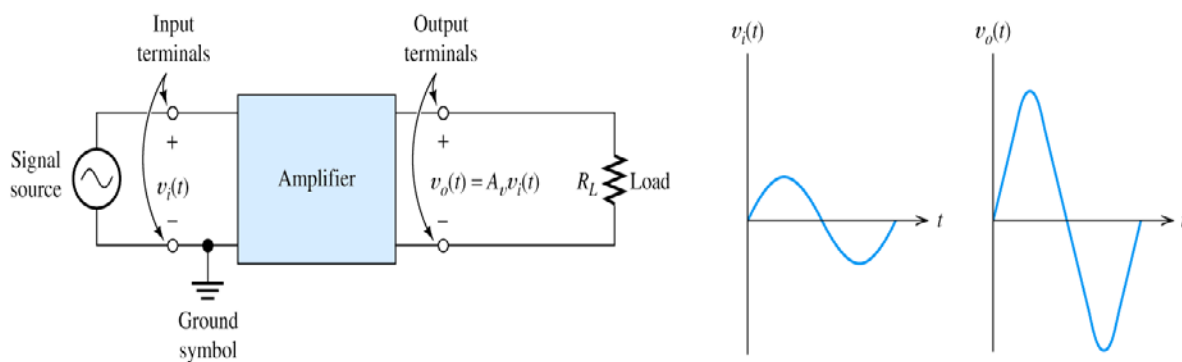


Figure 11.1 Electronic amplifier.

This entire block is dedicated to building circuits for amplifying weak signals. These circuits are based upon operational amplifiers (Op amps for short). If you are interested to know what is inside an op amp, this will have to wait till Block D. In this block, we focus on how to use the op amp and apply them to build practical circuits first.

I. The Ideal Operational Amplifier

Terminals

The ideal op amp has 3 terminals that you should notice. Fig 1 shows the symbol for an op amp. The 3 terminals to notice are:

- Non-inverting input terminal (+)
- Inverting input terminal (-)
- Output terminal (V_{out})

In other words, it has two inputs and one output.

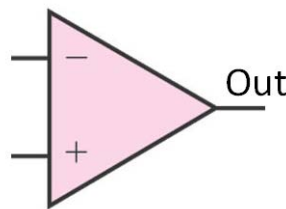


Figure 1: Symbol of an op amp

Open loop gain (A)

The op amp sees the voltage difference across the two inputs and multiplies it by a factor A. This factor A is known as the open loop gain:

$$V_{out} = A(V_{+} - V_{-})$$

Input resistance (R_{in})

In an op amp, there is a resistance that lies between the two inputs. This resistance is thus predictably known as the input resistance.

Output resistance (R_{out})

To see the effect of the output resistance, we refer to the model for the op amp shown in Fig 2. It contains a voltage controlled voltage source. Only when R_{out} is 0, do we obtain the above relationship for V_{out} .

Characteristics of an ideal op amp

- Infinite open loop gain
- Infinite input resistance
- Zero output resistance

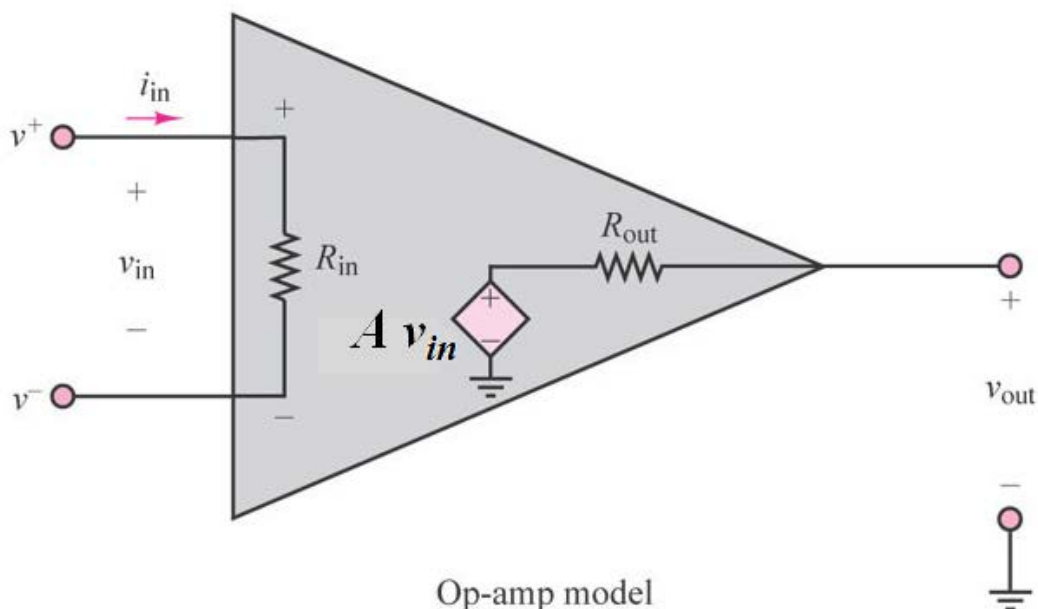


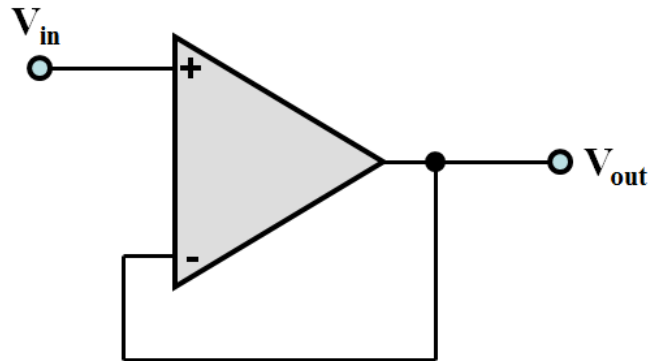
Figure 2: Op amp model

In the next section, we will analyze a number of circuits based on op amps. You will find there are a very great number of different combinations (Hint: memorizing all the possible combinations is futile).

II. Op amp circuits

Source follower

Closed loop gain (V_{out}/V_{in})



- (1) Gain: $\frac{V_{out}}{V_{in}} = 1$
- (2) Input resistance: Infinite

V_{out} is connected to V_-

$$\Rightarrow V_{out} = V_-$$

$V_+ = V_-$ (Infinite loop gain with negative feedback)

$$\Rightarrow V_{out} = V_+$$

$$V_{in} = V_+$$

$$\Rightarrow V_{out} = V_{in}$$

Negative feedback: Connect the output to the inverting input of the op amp. The effect is to reduce the voltage difference seen between the non-inverting and inverting input of the op amp.

Open loop gain (A) is specific to the **op amp** and is usually very large such that we can assume it to be infinite.

Closed loop gain (G) refers to the ratio of the output to the input specific to the **circuit** when the circuit is connected in negative feedback. The closed loop gain is always much lower than the open loop gain.

Circuit input resistance (R_{in})

Input resistance is defined by ratio of the input voltage over the input current.

In the case of the source follower, the input current is the sees the infinite input resistance of the op amp. So the input current is zero.

⇒ Circuit input resistance is therefore infinite.

This is not always the case as will see for the inverting amplifier.

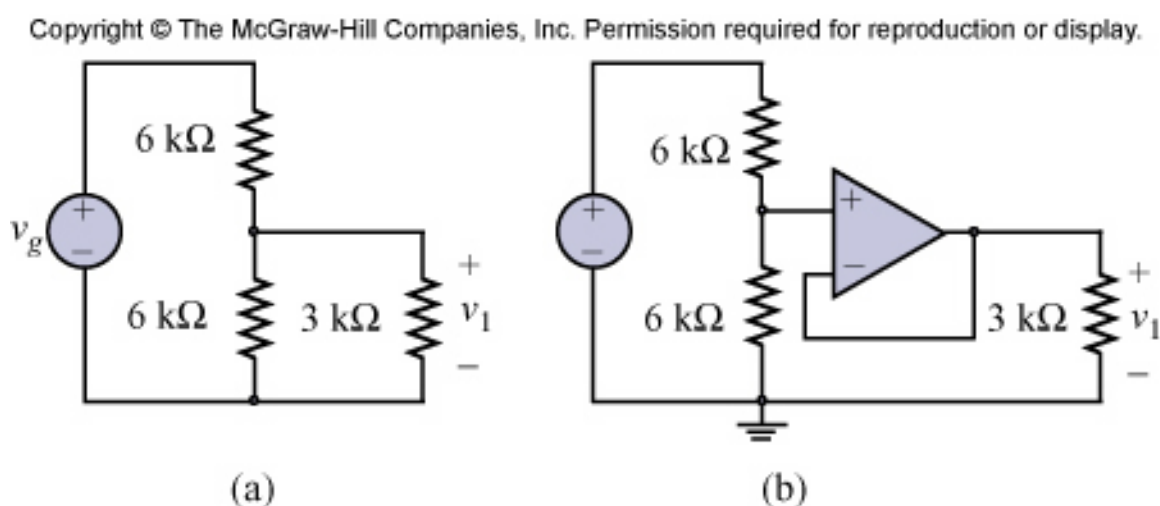
Features of Source Follower

Large input resistance

Small output resistance

Gain of close to one

Application: Buffer between a large source resistance and small load resistance. See example below.



Find the v_1 for the above 2 circuits where the $3\text{k}\Omega$ is the output load

Without the $3\text{k}\Omega$ load, the open circuit voltage across the $6\text{k}\Omega$ resistor = $v_g/2$ (This is the ideal case with the load is actually infinite – i.e. we get an open circuit).

For circuit (a), when $3\text{k}\Omega$ is connected at the output:

$6\text{k}\Omega$ and $3\text{k}\Omega$ are in parallel = $2\text{k}\Omega$

So $v_1 = [2/(2+6)] * v_g = v_g/4$

Note that the output voltage has now dropped by half due to the $3\text{k}\Omega$ load compared to the no load case.

So the $3\text{k}\Omega$ load actually has an effect on the output. This is not a good thing!

This problem of isolating the output load from the input resistance can be solved with a source follower like in circuit (b).

For circuit (b), when $3\text{k}\Omega$ is connected at the output:

Voltage across $6\text{k}\Omega = v_g/2$ (note that the 2 resistors are **in series since no current is flowing into the op amp inputs**)

Since the gain is 1, $v_1 = v_g/2$

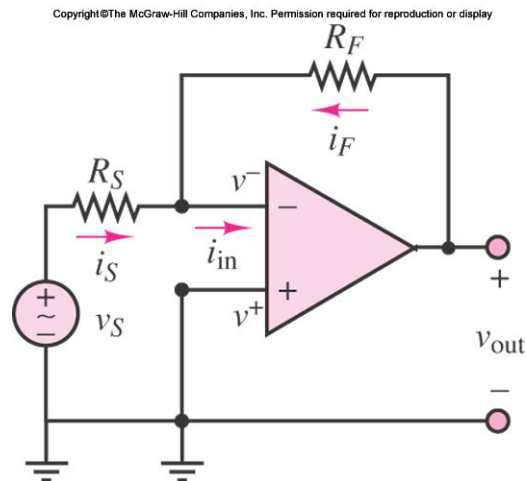
SUMMARY:

With the source follower, the $3\text{k}\Omega$ load has no effect on the circuit on the left hand side.

In other words, the source follower works as a buffer. It isolates the input from the output load as the output resistance of the op amp is zero.

Inverting amplifier *Lab 3

Derive the gain



$$(3) \text{ Gain: } \frac{v_{out}}{v_S} = -\frac{R_F}{R_S}$$

$$(4) \text{ Input resistance} = R_S$$

Apply KCL at inverting input (V-):

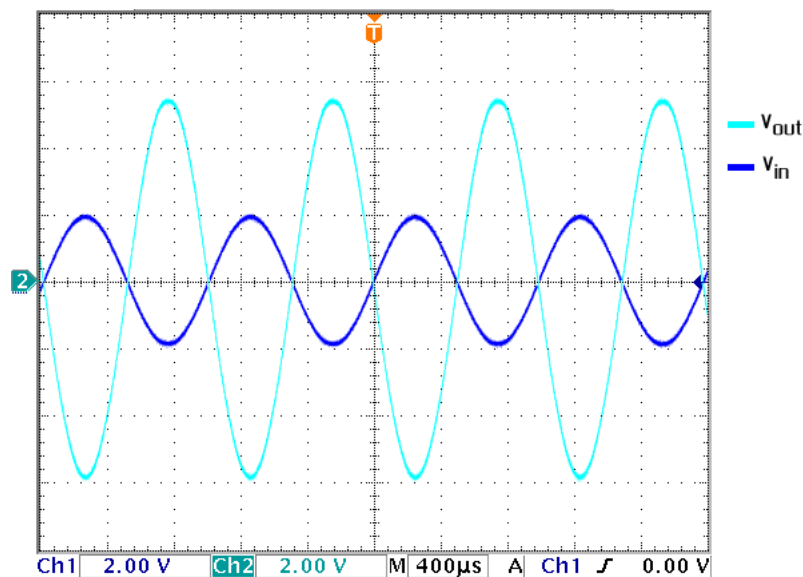
Note that $v^- = v^+ = 0$, assume that _____

Apply KCL at node v-, assuming that _____

$$\frac{v_s - 0}{R_S} + \frac{v_{out} - 0}{R_F} = 0$$

$$\frac{V_{out}}{V_S} = -\frac{R_F}{R_S}$$

But what does the minus sign mean?



Comparison of input to output for inverting amp

The minus sign indicates that the gain is inverting. This means that the direction of the signal is reversed by 180° .

The minus sign does not mean that the output is less than the input.

If the modulus of the gain (i.e. $|G|$) is more than 1, that means the output is larger than the input. The value of $|G|$ determines the amplification factor.

Circuit input resistance (R_{in})

As mentioned in the example of the source follower, the circuit input resistance is defined as the ratio of the input voltage over input current.

$$I_{in} = (V_S - 0)/R_S$$

$$\Rightarrow V_S/I_{in} = R_S$$

$$\Rightarrow \text{Input resistance} = R_S$$

You can see now that R_{in} is not infinite in the case of the inverting amplifier, unlike the source follower. R_{in} is a circuit-related parameter that depends on the actual connections in the circuit.

Since R_{in} is not infinite, the consequence is that any sort of internal resistance from the source (R_a) would have an effect on the output:

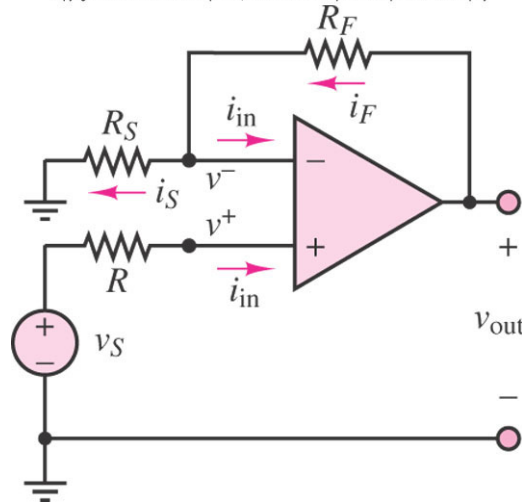
Verify: R_a from the source appears in series with R_S . $\Rightarrow$
Replace R_S with $R_S + R_a$:

$$V_{out} = -\frac{R_F}{R_S + R_a} V_S = \left(\frac{R_S}{R_S + R_a} \right) \left(-\frac{R_F}{R_S} V_S \right)$$

Hence the output is reduced to a factor of $R_S/(R_S+R_a)$ of the value if $R_a = 0$.

Non-inverting amplifier

Copyright © The McGraw-Hill Companies, Inc. Permission required for reproduction or display



(1) Infinite input resistance

(2) Gain: $\frac{v_{out}}{v_S} = 1 + \frac{R_F}{R_S}$

Closed-loop gain (G)

Note that $i_{in} = 0$ since it is going into the op amp input

$$\Rightarrow v_S = v^+ = v^- \quad (1)$$

Note there is also no current running into inverting input of op amp as well ($i_{in} = 0$ for either inputs)

$\Rightarrow R_S$ and R_F are thus in series

Apply voltage divider rule:

$$v^- = \frac{R_S}{R_S + R_F} v_{out} \quad (2)$$

Substitute (1) into (2):

$$v_s = \frac{R_S}{R_S + R_F} v_{out} \quad (3)$$

After rearranging (3):

$$G = \frac{v_{out}}{v_s} = 1 + \frac{R_F}{R_S}$$

Circuit input resistance (R_{in})

As mentioned already, $i_{in} = 0$ since the current from the sources sees the infinite resistance of the op amp (R_i).

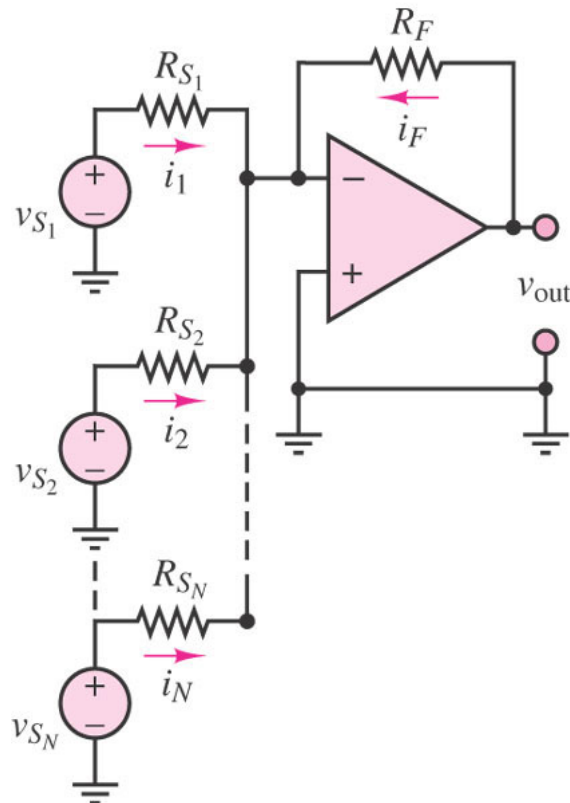
Hence if $i_{in} = 0$, then:

$$R_{in} = \frac{v_s}{i_{in}} \rightarrow \infty$$

So just like the source follower, any internal resistance at the source has no effect on the output.

Summing amplifier *Lab 3

Copyright © The McGraw-Hill Companies, Inc. Permission required for reproduction or display



$$v_{out} = - \left(\frac{R_F}{R_{S1}} v_{S1} + \frac{R_F}{R_{S2}} v_{S2} + \dots + \frac{R_F}{R_{SN}} v_{SN} \right)$$

This circuit is very similar to the inverting amplifier.

Note that since $v^+ = 0$, therefore $v^- = 0$.

Therefore:

$$i_1 = \frac{v_{S1}}{R_{S1}}, i_2 = \frac{v_{S2}}{R_{S2}}, \dots, i_N = \frac{v_{SN}}{R_{SN}}$$

Applying KCL at v^- :

$$i_F + i_1 + i_2 + \dots + i_N = 0$$

Hence by nodal voltage analysis:

$$\frac{v_{out}}{R_F} + \frac{1}{R_{S1}} v_{S1} + \frac{1}{R_{S2}} v_{S2} + \dots + \frac{1}{R_{SN}} v_{SN} = 0$$

Rearrange the above equation, we obtain:

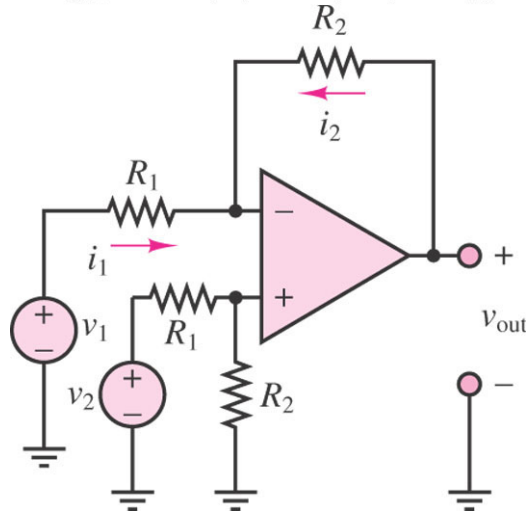
$$v_{out} = - \left(\frac{R_F}{R_{S1}} v_{S1} + \frac{R_F}{R_{S2}} v_{S2} + \dots + \frac{R_F}{R_{SN}} v_{SN} \right)$$

Note that as the name suggests, the function of this amplifier circuit is to amplify each input voltage and add them all up at the output. This sum however is inverted relative to the input as indicated by the negative sign.

Differential amplifier

This circuit is very useful for building instruments that make fine measurements. You will have the chance to build your own temperature sensing instrument in Lab 3.

Copyright © The McGraw-Hill Companies, Inc. Permission required for reproduction or display



Apply KCL at inverting terminal:

Assuming no current into/out of op amp inverting input:

$$i_1 + i_2 = 0$$

$$\Rightarrow \frac{v_1 - v^-}{R_1} + \frac{v_{out} - v^-}{R_2} = 0$$

$$\left(\frac{v_1}{R_1} + \frac{v_{out}}{R_2} \right) / \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = v^-$$

$$\left(\frac{R_2}{R_1 + R_2} \right) v_1 + \left(\frac{R_1}{R_1 + R_2} \right) v_{out} = v^- \quad (1)$$

At the non-inverting input of op amp,

Assuming no current into/out of non-inverting input of op amp

⇒ R_1 and R_2 appear in series

⇒ Apply voltage divider rule to R_1 and R_2 :

$$v^+ = \left(\frac{R_2}{R_1 + R_2} \right) v_2 \quad (2)$$

Finally, assume $v^- = v^+$ due to infinite open loop gain in negative feedback.

Combine (1) and (2):

$$\left(\frac{R_2}{R_1 + R_2} \right) v_2 = \left(\frac{R_2}{R_1 + R_2} \right) v_1 + \left(\frac{R_1}{R_1 + R_2} \right) v_{out}$$

$$\left(\frac{R_1}{R_1 + R_2} \right) v_{out} = \left(\frac{R_2}{R_1 + R_2} \right) (v_2 - v_1)$$

$$v_{out} = \left(\frac{R_2}{R_1} \right) (v_2 - v_1)$$

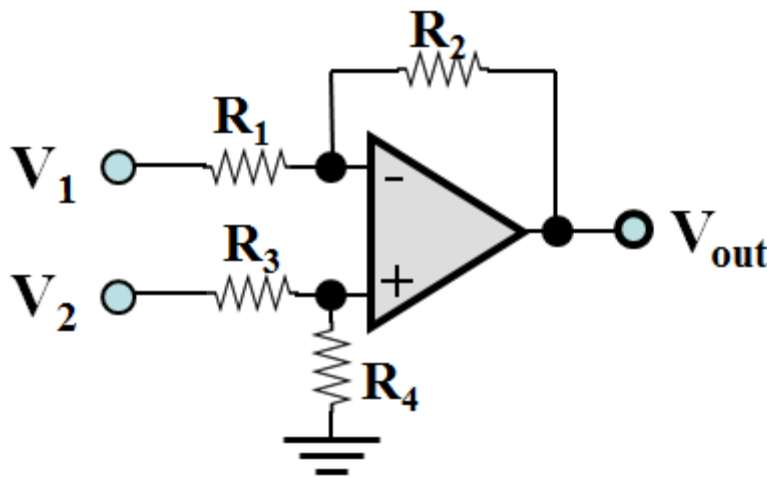
The gain expression takes the form:

$$V_{out} = G V_{diff}, \text{ where } V_{diff} = V_2 - V_1$$

Note that the function of this opamp circuit is to take the difference between two input voltages and amplify them at the output.

But what if there is some **difference** between the values of the resistors such that all the resistor values are now different from each other?

Repeating the same procedure above, find the V_{out} in terms of V_1 and V_2 for the circuit below:



Apply KCL at v^- :

$$\frac{V_1 - v^-}{R_1} + \frac{V_{out} - v^-}{R_2} = 0$$

$$\left(\frac{R_2}{R_1 + R_2} \right) V_1 + \left(\frac{R_1}{R_1 + R_2} \right) V_{out} = v^- \quad (3)$$

Now at v^+ where R_3 and R_4 are in series and we can use voltage divider rule:

$$v^+ = \left(\frac{R_4}{R_3 + R_4} \right) V_2 \quad (4)$$

Using the fact that $v^+ = v^-$ due to the infinite open loop gain of the op amp in negative feedback, we can equate (3) and (4):

$$\begin{aligned}\left(\frac{R_4}{R_3 + R_4}\right)v_2 &= \left(\frac{R_2}{R_1 + R_2}\right)v_1 + \left(\frac{R_1}{R_1 + R_2}\right)v_{out} \\ \left(\frac{R_1}{R_1 + R_2}\right)v_{out} &= \left(\frac{R_4}{R_3 + R_4}\right)v_2 - \left(\frac{R_2}{R_1 + R_2}\right)v_1 \\ v_{out} &= \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right)v_2 - \frac{R_2}{R_1}v_1\end{aligned}$$

The gain expression takes the form:

$$V_{out} = G_1 V_2 - G_2 V_1$$

$$G_1 = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right)$$

$$G_2 = \frac{R_2}{R_1}v_1$$

If we group the inputs into differential and common, we then get: $V_{out} = G_{dm} V_{dm} + G_{cm} V_{cm}$

Note here that:

- (1) $V_{dm} = (V_2 - V_1)/2$, where $V_2 = -V_1$
- (2) $V_{cm} = V_2 = V_1$

G_{cm} is known as the common mode gain, while G_{dm} is the differential mode gain.

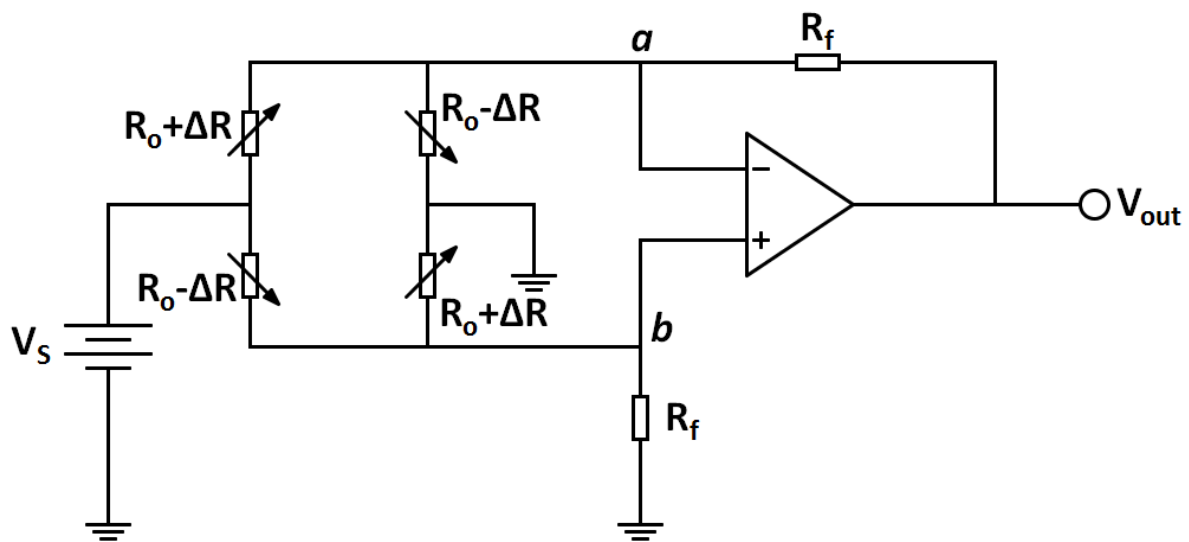
In the ideal case when resistors R_1 and R_3 as well as R_2 and R_4 are matched to each other perfectly, then G_{cm} will be zero. The amplifier will only amplify difference in voltage between the two inputs. If the inputs were exactly the same, then the output would be zero. This is the ideal case.

But we see that in the real practical case when there is some difference in resistor values, the output will not be zero even when the two input are identical. This has implications for the differential amplifier:

Whatever internal resistance of the sources will cause a difference to appear between R_1 and R_3 .

Application: Bridge with differential amplifier

The following is circuit used for pressure sensors (e.g. for measuring pressure in tires). The working principle involves the conversion of pressure on a membrane into a resistance change in resistors on the membrane. This resistance change is then converted to a voltage output and amplified using the circuit below.



Typically, the pressure sensor is designed so that: $\Delta R \ll R_0$. Find V_{out} in terms of R_f , R_0 , ΔR , and V_S .

Apply KCL at node a:

$$\frac{V_s - v^-}{R_o + \Delta R} + \frac{V_{out} - v^-}{R_f} = \frac{v^-}{R_o - \Delta R}$$
$$\frac{V_s}{R_o + \Delta R} + \frac{V_{out}}{R_f} = v^- \left(\frac{1}{R_o - \Delta R} + \frac{1}{R_o + \Delta R} + \frac{1}{R_f} \right) \quad (1)$$

Apply KCL at node b:

$$\frac{V_s - v^+}{R_o - \Delta R} = \frac{v^+}{R_o + \Delta R} + \frac{v^+}{R_f}$$
$$\frac{V_s}{R_o - \Delta R} = v^+ \left(\frac{1}{R_o - \Delta R} + \frac{1}{R_o + \Delta R} + \frac{1}{R_f} \right) \quad (2)$$

Assume: $v^- = v^+$ (infinite open loop gain in negative feedback)

Combine equations (1) and (2):

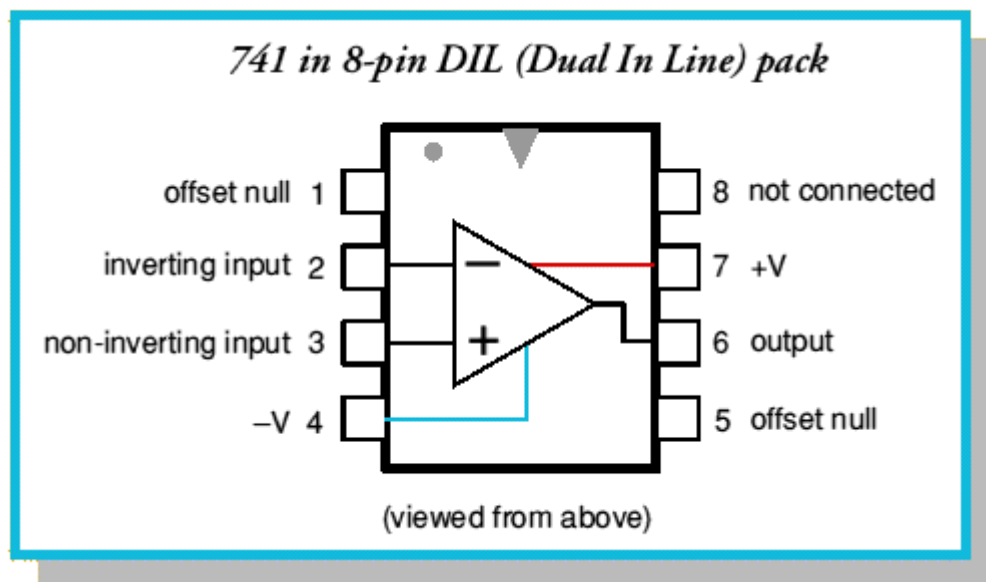
$$\frac{V_s}{R_o + \Delta R} + \frac{V_{out}}{R_f} = \frac{V_s}{R_o - \Delta R} \Rightarrow V_{out} = \left(\frac{2R_f}{R_o} \right) \left(\frac{2(\Delta R)}{R_o^2 - (\Delta R)^2} \right) V_s$$

But since given that $\Delta R \ll R_o$:

$$V_{out} \approx \left(\frac{2R_f}{R_o} \right) \left(\frac{\Delta R}{R_o} \right) V_s$$

III. Op amps in real life

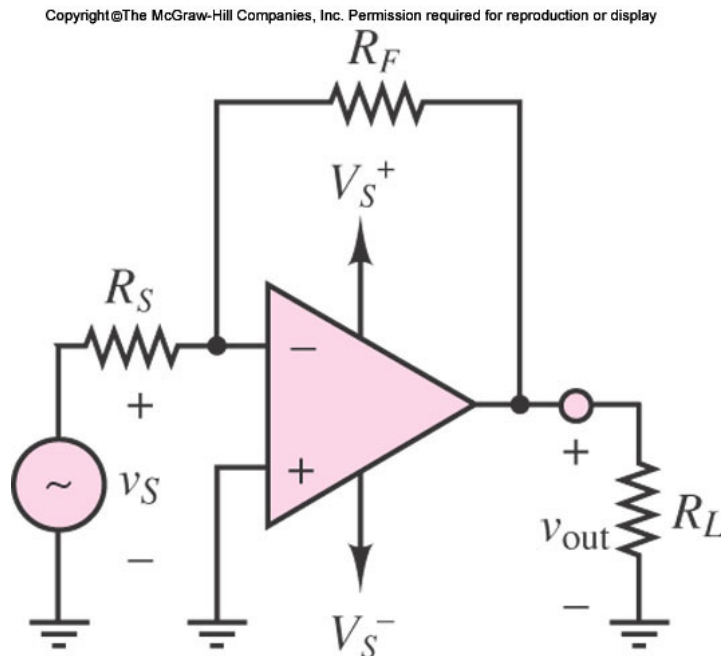
Op amps are actually electronic chips that are kept in a package. The various terminals of the op amp are then connected to different pins on the package. The figure below shows one example:



One may see that the package has 8 pins, of which only 7 are used (1 pin has no connection – pin 8). To build the circuits mentioned so far, we only need to consider 5 of the pins.

The pins -V and +V each refer to the negative and the positive supply. For the op amp to work, power needs to be supplied to the -V and +V pins.

The following figure then shows an inverting amplifier with the power supply connections included now. You might realize that we did not consider the power supply pins when deriving expressions for v_{out} .



You find that the gain expression for the circuit is still:

$$\frac{V_{out}}{V_S} = -\frac{R_F}{R_S}$$

In practice, we need to supply power otherwise the op amp will not work and will not get these expressions of V_{out} .

So what then is the effect of the power supply pins apart from giving power to the op amp for it work?

IV. Saturation

From the expression of $V_{\text{out}}/V_{\text{in}}$, you cannot see whether there is a limit on the maximum V_{out} . In a real op amp, V_{out} cannot exceed either of the supply voltages.

Hence if V_{S+} is 15V and V_{S-} is -15V, then V_{out} cannot be higher than 15V. It also cannot be lower than -15V.

Example on saturation

You have built an inverting op amp with gain of 10 and you use power supply of 15V to the op amp.

Case 1: $V_s = 2V$

If we multiply 2V by -10, we should be -20V. But this is lower than the positive supply. So V_{out} will be -15V. In the labs, you will find that the minimum V_{out} does not even reach V_{S-} .

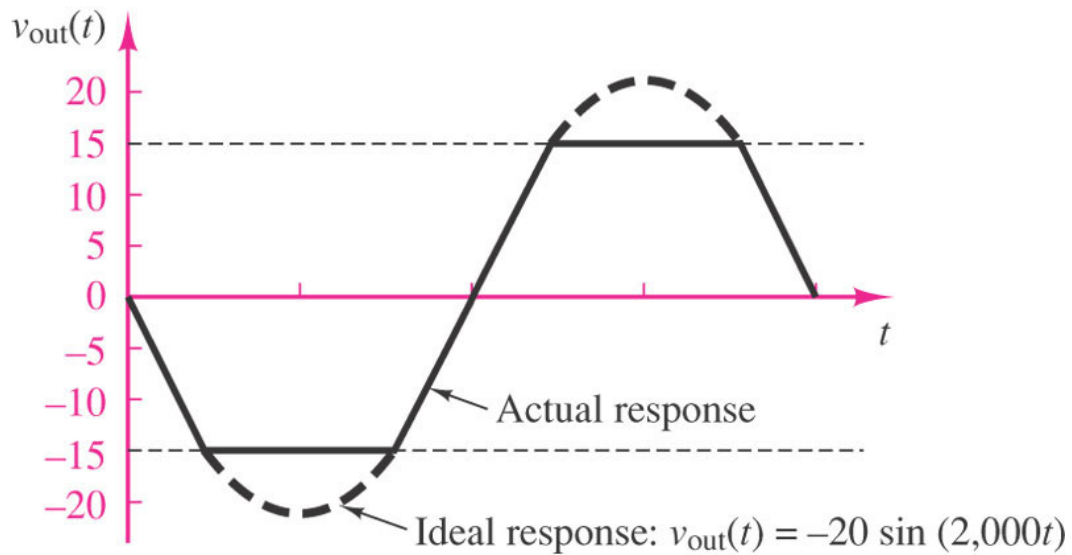
Case 2: $V_s = -2V$

$V_{\text{out}} = 15V$. In the labs, you will find that the maximum V_{out} does not even reach V_{S+} .

Case 3: $V_s = 2*\sin(2000t) V$

When V_{out} reaches V_{S+} or V_{S-} , it gets chopped off.

Copyright © The McGraw-Hill Companies, Inc. Permission required for reproduction or display



Op amps are capable of amplifying signals within the range of their supply voltages.

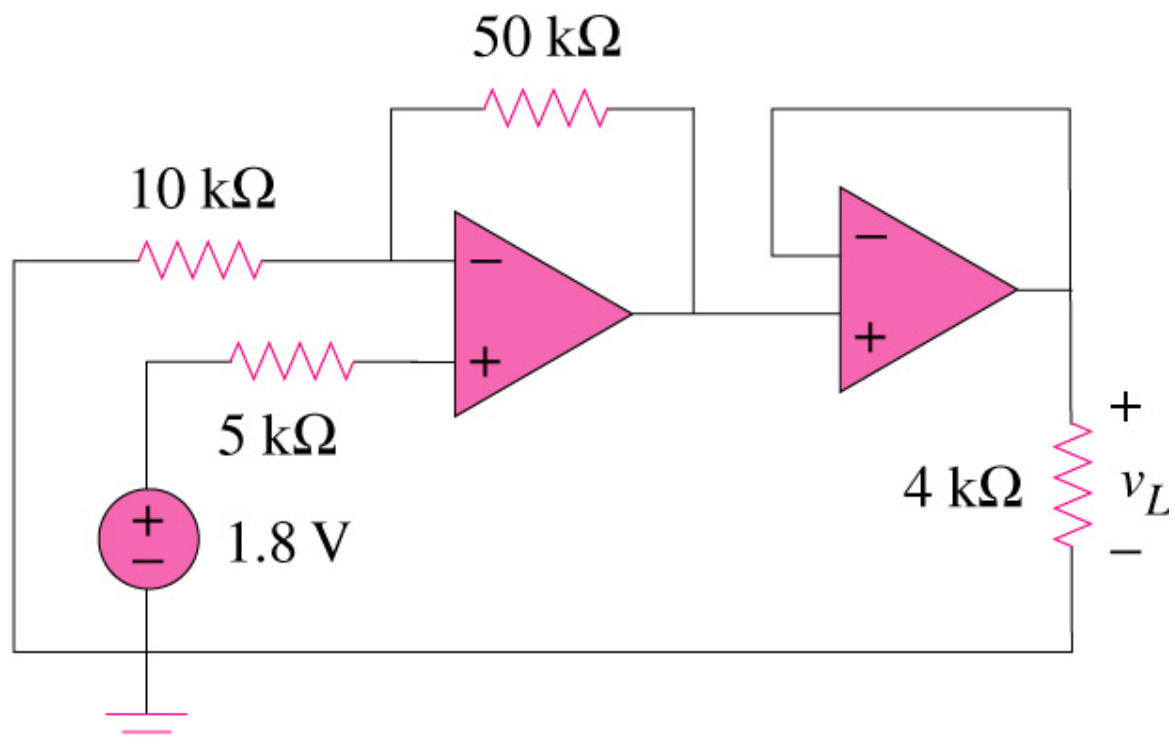
The output cannot exceed the supply voltages while the input is also capped by the supply voltages.

V. Cascaded amplifiers

The configurations above represent the most common ones used for amplifier circuits. We can then combine them, taking the output of one amplifier stage and then feeding it to the input of a second stage. This is known as cascading. The following are just a few examples.

Example on cascaded amplifiers 1

Identify the types of amplifiers in the circuit below.



First stage: Non-inverting

Second stage: Follower

Non-inverting amplifier followed by a follower

Gain of the first stage:

$$\text{Gain} = 1 + 50/10 = 6$$

Gain of the second stage:

$$\text{Gain} = 1$$

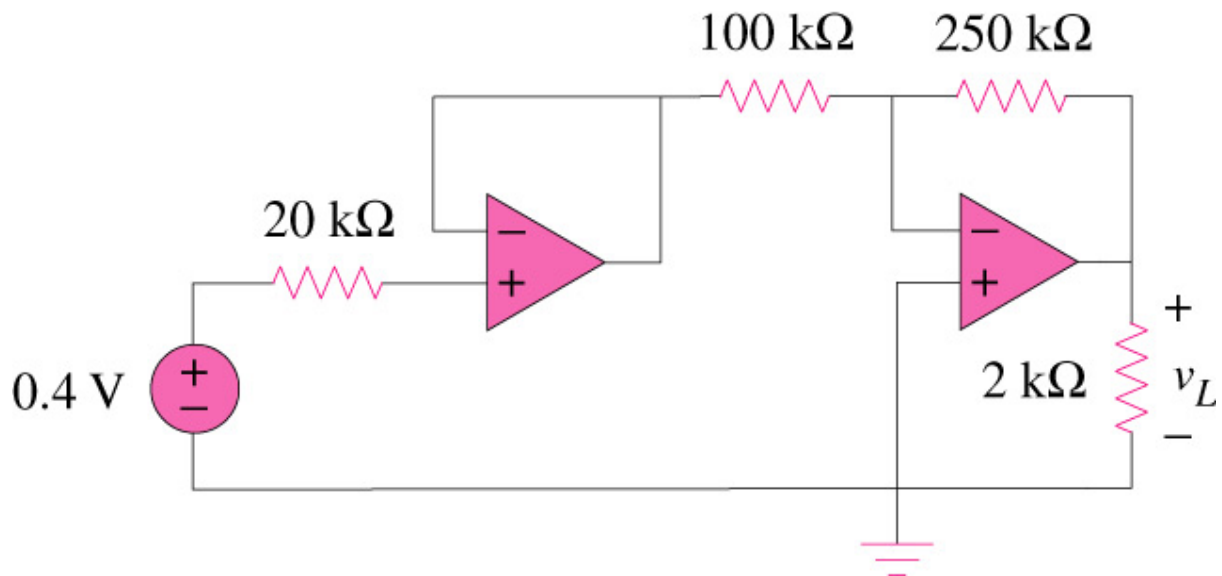
Multiply the 2 stages to find the total gain:

$$V_L = (1.8) * (6) * (1) = 10.8V$$

$$V_L = 10.8V$$

Example on cascaded amplifiers 2

Identify the types of amplifiers in the circuit below.



First stage: Follower

Second stage: Inverting

Inverting amplifier with follower at the input

Gain of the first stage:

$$\text{Gain} = 1$$

Gain of the second stage:

$$\text{Gain} = -250/100 = -2.5$$

Multiply the 2 stages to find the total gain:

$$V_L = 0.4 * 1 * (-2.5) = -1V$$

$$V_L = -1V$$

What if the follower was not used, so that the source of 0.4V and 20k Ω were connected directly to the inverting amplifier?

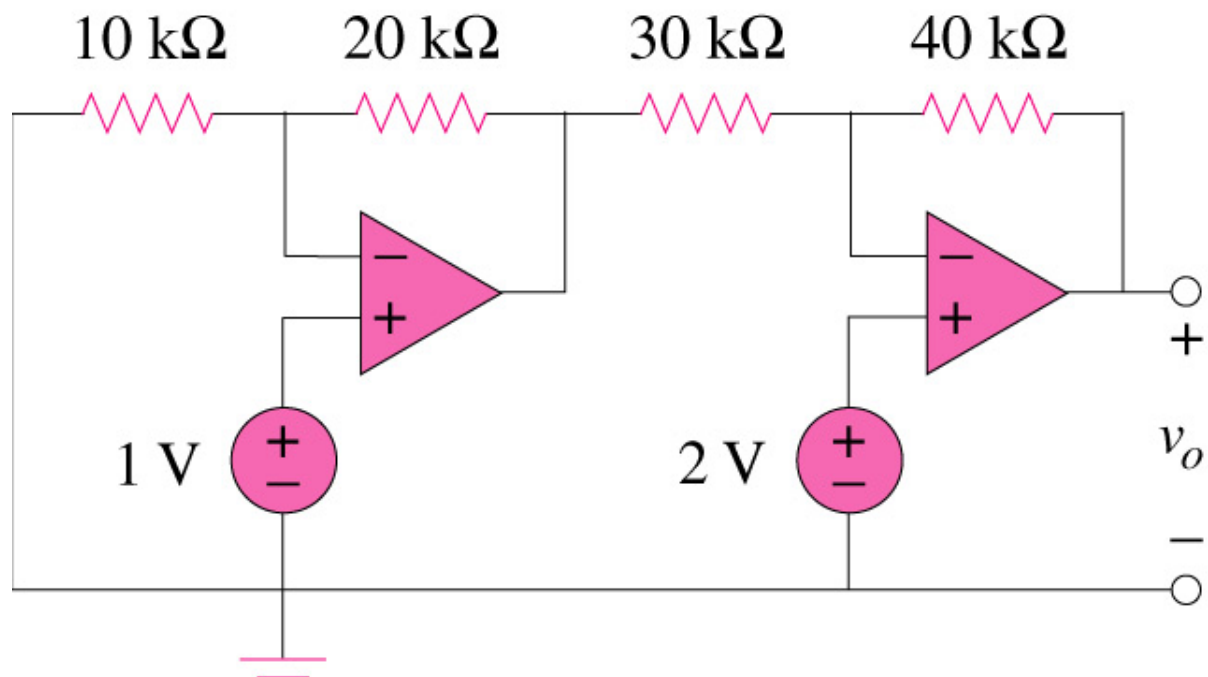
Re-calculate the gain expression for the second stage with 20k Ω included in series with 100k Ω :

$$\begin{aligned} V_L &= -\frac{250}{(100 + 20)} \times 0.4 \\ &= -0.83V \end{aligned}$$

The source follower isolates the 20k Ω resistor from the limited input resistance of the inverting amplifier (here it is 100k Ω).

Example on cascaded amplifiers 3

Identify the types of amplifiers in the circuit below.



First stage: Non-inverting

Second stage: Inverting with V^+ terminal at 2V

Gain of 1st stage:

$$\text{Gain} = (20/10) + 1 = 3$$

$$G_1 = 3$$

Output of first stage = 3V

Gain of 2nd stage:

Inverting amplifier with gain = $-40/30 = -4/3$

$G_2 = -4/3$ with an offset of 2V

Given the 2V offset at V_+ , the input of the second stage can be readjusted to $3V - 1V = 1V$

Final output voltage, $v_o = (1)*(-4/3) + 2 = (2/3)V$

Alternatively,

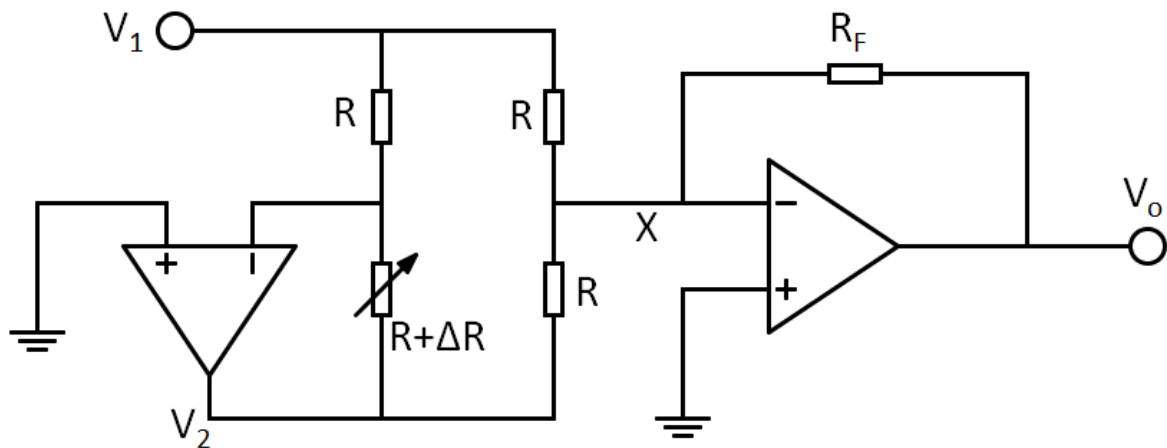
Apply KCL at V_- of second amplifier:

$$\frac{3V - 2V}{30k} + \frac{V_{out} - 2V}{40k} = 0$$

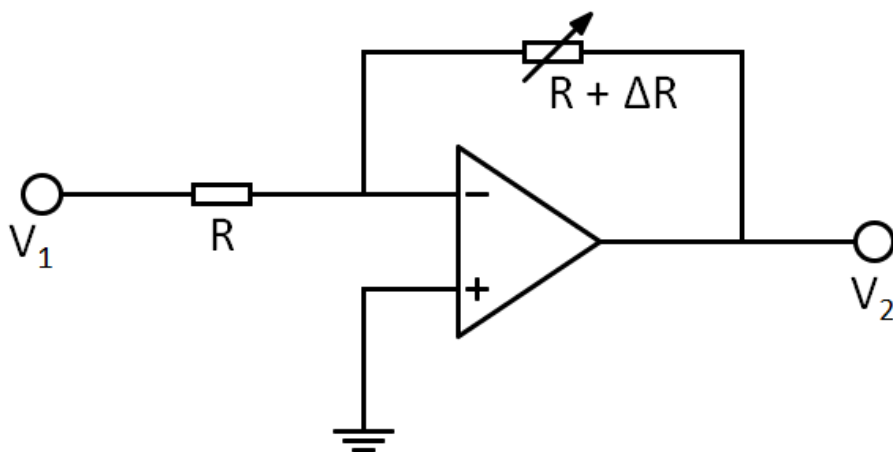
Bear in mind that V_- has a voltage of 2V.

Example on cascaded amplifiers 4 *Lab 3

Find V_o in terms of V_1 , R , ΔR , and R_F .



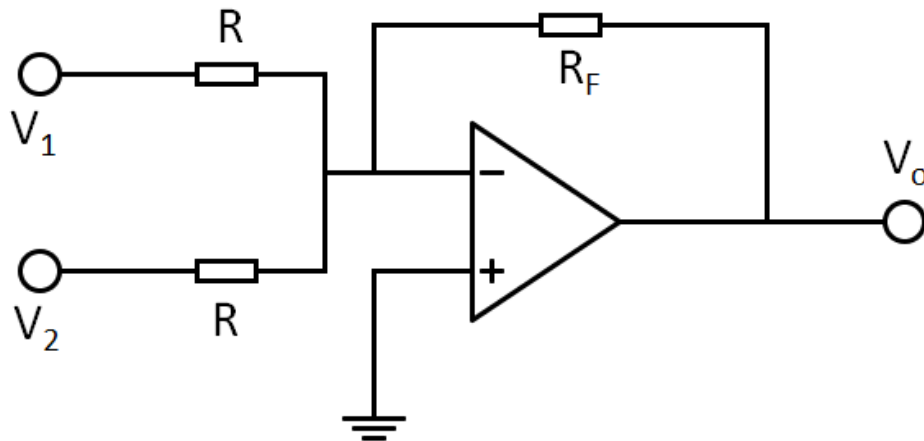
Inverting op amp circuit:



$$\frac{V_2}{V_1} = -\left(\frac{R + \Delta R}{R}\right)$$

$$\Rightarrow V_2 = -\left(1 + \frac{\Delta R}{R}\right)V_1 \quad (1)$$

Summing amplifier



$$V_o = -\left[\left(\frac{R_F}{R}\right)V_1 + \left(\frac{R_F}{R}\right)V_2\right] = -\left(\frac{R_F}{R}\right)(V_1 + V_2) \quad (2)$$

Combining both stages

Substitute (1) into (2):

$$\begin{aligned} V_o &= -\left(\frac{R_F}{R}\right)\left[V_1 + -\left(1 + \frac{\Delta R}{R}\right)V_1\right] \\ &= \left(\frac{R_F}{R}\right)\left(\frac{\Delta R}{R}\right)V_1 \end{aligned}$$

Summary

- 1) Remember the 3 characteristics of an ideal op amp
- 2) The output voltage is limited by the power supply
- 3) Op amps can be cascaded and combined