

EE3114 HW-7 Solutions

7-1

$$sI - A = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 0 & s \end{bmatrix}$$

$$(a) \quad (sI - A)^{-1} = \frac{1}{s^2} \begin{bmatrix} s & 1 \\ 0 & s \end{bmatrix} = \begin{bmatrix} \frac{1}{s} & \frac{1}{s^2} \\ 0 & \frac{1}{s} \end{bmatrix}$$

$$\phi(t) = L^{-1}\{(sI - A)^{-1}\} = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix}.$$

$$(b) \quad \det(\lambda I - A) = |\lambda I - A| = \begin{vmatrix} \lambda & -1 \\ 0 & \lambda \end{vmatrix} = 0$$
$$\lambda^2 = 0$$

$$(c) \quad \text{solve } \lambda^2 = 0$$

$$\lambda_1 = \lambda_2 = 0$$

$$(d) \quad x(t) = \phi(t)x(0) = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} t \\ 1 \end{bmatrix}$$

(e) the roots of the characteristic equation is $s_1=s_2 = 0$;

So the system is margin stable (not stable).

7-2

(a)

$$sI - A = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} s & -1 \\ 1 & s \end{bmatrix}$$

$$(sI - A)^{-1} = \frac{1}{s^2 + 1} \begin{bmatrix} s & 1 \\ -1 & s \end{bmatrix} = \begin{bmatrix} \frac{s}{s^2 + 1} & \frac{1}{s^2 + 1} \\ \frac{-1}{s^2 + 1} & \frac{s}{s^2 + 1} \end{bmatrix}$$

$$\phi(t) = L^{-1}\{(sI - A)^{-1}\} = \begin{bmatrix} \cos(t) & \sin(t) \\ -\sin(t) & \cos(t) \end{bmatrix}.$$

(b)

$$\det(\lambda I - A) = |\lambda I - A| = \begin{vmatrix} \lambda & -1 \\ 1 & \lambda \end{vmatrix} = \lambda^2 + 1$$

(c)

$$\lambda^2 + 1 = 0$$

$$\lambda_1 = i, \lambda_2 = -i$$

(d)

$$x(t) = \phi(t)x(0) = \begin{bmatrix} \cos(t) & \sin(t) \\ -\sin(t) & \cos(t) \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos(t) \\ -\sin(t) \end{bmatrix}$$

(e) two poles is $s_1 = i, s_2 = -i$;

It is marginal stable (unstable).

7-3

$$G(s) = \frac{Y(s)}{X(s)} = \frac{X(s)}{U(s)} = \frac{(s+1)}{(s^4 + 3s^3 - 4s)}$$

$$\frac{X(s)}{U(s)} = \frac{1}{(s^4 + 3s^3 - 4s)}$$

$$\frac{Y(s)}{X(s)} = s+1$$

so

$$a_1 = 3, a_2 = 0, a_3 = -4, a_4 = 0;$$

$$b_0 = 0, b_1 = 0, b_2 = 0, b_3 = 1, b_4 = 1;$$

$$\beta_0 = b_0 = 0,$$

$$\beta_1 = b_1 - a_1\beta_0 = 0 - 0 = 0,$$

$$\beta_2 = b_2 - a_1\beta_1 - a_2\beta_0 = 0 - 0 - 0 = 0,$$

$$\beta_3 = b_3 - a_1\beta_2 - a_2\beta_1 - a_3\beta_0 = 1,$$

$$\beta_4 = -2$$

Canonical form

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 4 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ -2 \end{bmatrix} u$$

$$y = [1 \quad 0 \quad 0 \quad 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + [0]u$$

Not unique. Other form state space equation may also be right.