

EE3109 Applied Electromagnetics
2024/25 Semester B
Assignment 1B Solution

5. A 2 GHz linearly polarized plane wave travels in wet soil ($\epsilon_r = 4, \sigma = 5 \times 10^3$ [S/m]). At $t = 0$ and at the origin ($z = 0$), the electric field is $\mathbf{E} = 2 (\hat{\mathbf{x}} - \hat{\mathbf{y}})$ [V/m]. The plane wave travels in the -z-direction.
- Is the soil a good dielectric, a good conductor or neither?
 - What is the attenuation constant in [dB/m]?
 - What is the propagation constant?
 - What is the complex permittivity?
 - How fast does the wave travel in the soil?
 - Write the magnetic field phasor in the soil.
 - Write the time-domain electric field in the soil.
 - How far will the wave travel in the soil before the electric field decays to 2% of its original value?

Solution:

$$(a) \frac{\sigma}{\omega\epsilon'} = \frac{5 \times 10^3}{(4\pi \times 10^9)(8.854 \times 10^{-12})(4)} = 1.124 \times 10^4 \gg 1 \Rightarrow \text{Good conductor}$$

$$(b) \alpha = \sqrt{\frac{\omega\sigma\mu}{2}} = \sqrt{\pi f \sigma \mu} = \sqrt{\pi \times 2 \times 10^9 \times 4\pi \times 10^{-7} \times 5 \times 10^3} = 2\pi \times 10^3 \approx 6289.7 \text{ [Np/m]}$$

Converting to dB/m: $\alpha = 6289.7 \times 8.686 = 54632.33 \text{ [dB/m]}$

$$(c) \beta = \alpha = \sqrt{\frac{\omega\sigma\mu}{2}} = 2\pi \times 10^3 \approx 6289.7 \text{ [rad/m]}; \quad \gamma = \alpha + j\beta = 6289.7(1 + j) \text{ [1/m]}$$

$$(d) \epsilon' = \epsilon_0 \epsilon_r = 4\epsilon_0 = 3.542 \times 10^{-11};$$

$$\epsilon'' = \frac{2\alpha\beta}{\omega^2\mu_0} = \frac{2(6289.7)^2}{4\pi^2(2 \times 10^9)^2(4\pi \times 10^{-7})} = 3.979 \times 10^{-7}$$

$$\epsilon_c = \epsilon' - j\epsilon'' = 3.542 \times 10^{-11} - j3.979 \times 10^{-7} \text{ [F/m]}$$

$$(e) v_p = \frac{\omega}{\beta} = \frac{4\pi \times 10^9}{2\pi \times 10^3} = 2.0 \times 10^6 \text{ [m/s]}$$

- (f) For the -z-travelling wave, the magnetic field phasor has the form

$$\mathbf{H} = H_0 e^{+\alpha z} e^{+j\beta z} \hat{\mathbf{h}}$$

α and β are already found,

$$\eta_{soil} = \frac{(1+j)\alpha}{\sigma} = \frac{(1+j)(2\pi \times 10^3)}{5 \times 10^3} = 1.256(1+j)$$

$$H_0 = \frac{E_0}{\eta_{soil}} = \frac{E_0}{1.256(1+j)} = \frac{2\sqrt{2}}{1.256(1+j)} = 1.592 \angle (-45^\circ) \text{ [A/m]}, \text{ and}$$

$$\hat{\mathbf{h}} = \hat{\mathbf{k}} \times \hat{\mathbf{e}} = (0, 0, -1) \times \frac{1}{\sqrt{2}}(1, -1, 0) = \frac{1}{\sqrt{2}}(-1, -1, 0).$$

Putting everything together, we have

$$\mathbf{H} = 1.592 \angle (-45^\circ) e^{+(2\pi \times 10^3)z} e^{+j(2\pi \times 10^3)z} \left(\frac{1}{\sqrt{2}}(-1, -1, 0) \right) \text{ [A/m]}$$

$$= 1.126 \angle (-45^\circ) e^{+(2\pi \times 10^3)z} e^{+j(2\pi \times 10^3)z} (-1, -1, 0) \text{ [A/m]}.$$

(g) For the -z-travelling wave, the time-domain electric field has the form

$$\mathbf{E} = |E_0|e^{+\alpha z} \cos(\omega t + \beta z + \angle E_0) \hat{\mathbf{e}}.$$

From the question, $|E_0|\hat{\mathbf{e}} = 2 (\hat{\mathbf{x}} - \hat{\mathbf{y}})$ [V/m], $\angle E_0 = 0^\circ$, α and β are already found, and $\omega = 2\pi f = 1.257 \times 10^{10}$ Hz. Putting everything together, we have

$$\mathbf{E} = 2e^{+(2\pi \times 10^3)z} \cos(1.257 \times 10^{10}t + 2\pi \times 10^3 z) (\hat{\mathbf{x}} - \hat{\mathbf{y}}) \text{ [V/m]}.$$

(h) $0.02 = e^{+(2\pi \times 10^3)z} \Rightarrow z = \frac{1}{2\pi \times 10^3} \ln(0.02) = -0.622 \text{ [mm]}.$

Therefore, the wave only has to travel 0.622 [mm] into the soil.

6. The electric field of a plane wave travelling in free space in +z direction is

$$\mathbf{E}(z, t) = \cos(\omega t - kz + 30^\circ) \hat{\mathbf{x}} - \sin(\omega t - kz - 150^\circ) \hat{\mathbf{y}} \text{ [V/m]}$$

- What is the polarization of this plane wave?
- Trace the time-evolution of the electric field at the plane $z = 0$.
- Trace the time-evolution of the magnetic field at the plane $z = 0$.
- What is the amplitude of the magnetic field at time $t = 0$ and location $z = 0$?

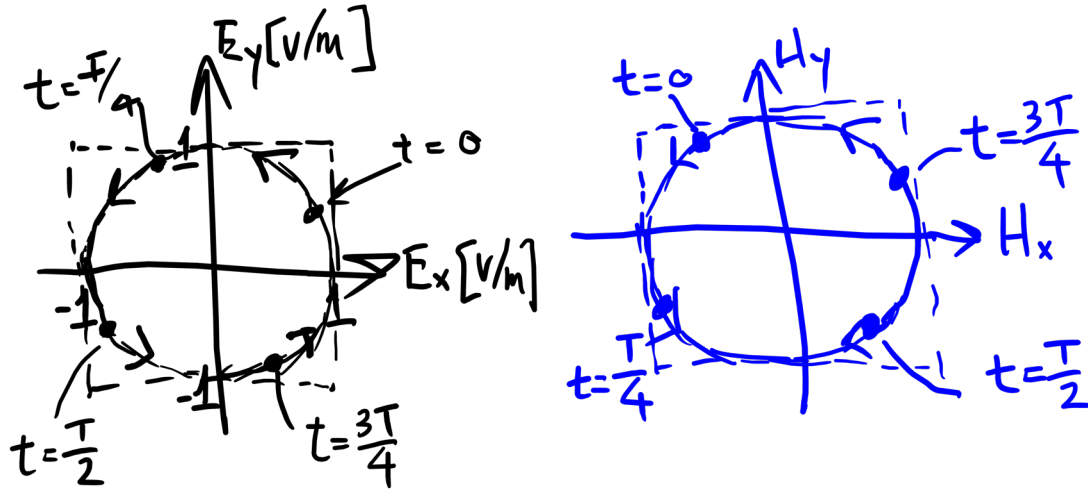
Solution:

- We rewrite the electric field as

$$\begin{aligned} \mathbf{E}(z, t) &= \cos(\omega t - kz + 30^\circ) \hat{\mathbf{x}} + \sin(\omega t - kz + 30^\circ) \hat{\mathbf{y}} \\ &= \cos(\omega t - kz + 30^\circ) \hat{\mathbf{x}} + \cos(\omega t - kz - 60^\circ) \hat{\mathbf{y}} \end{aligned}$$

Since $a_x = a_y$ and $\delta = -90^\circ$, the wave is RHC polarized.

- Please see the time-evolution plots below for both $\mathbf{E}$ and $\mathbf{H}$. Note that $\mathbf{H}$ always follows the same time-evolution pattern $\mathbf{E}$ as $\mathbf{E}$, $\mathbf{H}$, and the propagation direction $\hat{\mathbf{z}}$ form a right-hand triplet at all times.



In part (c) above (drawn in blue), the H-field magnitude is $|H| = \frac{|E|}{\eta_0} = \frac{1}{377} = 2.652 \text{ [mA/m]}$.

The electric and magnetic fields at different phase points are shown below to help with marking:

ωt	E_x	E_y
0°	$\cos(0^\circ + 30^\circ) = 0.8660$	$\cos(0^\circ - 60^\circ) = 0.5$
90°	-0.5	0.8660
180°	-0.8660	-0.5
270°	0.5	-0.8660

ωt	$H_x/ H $	$H_y/ H $
0°	$\cos(0^\circ + 30^\circ + 90^\circ) = -0.5$	$\cos(0^\circ - 30^\circ + 90^\circ) = 0.8660$
90°	-0.8660	-0.5
180°	0.5	-0.8660
270°	0.8660	0.5

- Since this is a circularly polarized wave, the magnitude of the magnetic field remains constant at all times across all of space. So the magnitude is 2.652 [mA/m] as calculated above.